Dynamic Rupture Modeling of Earthquake Faulting with the ADER-DG method

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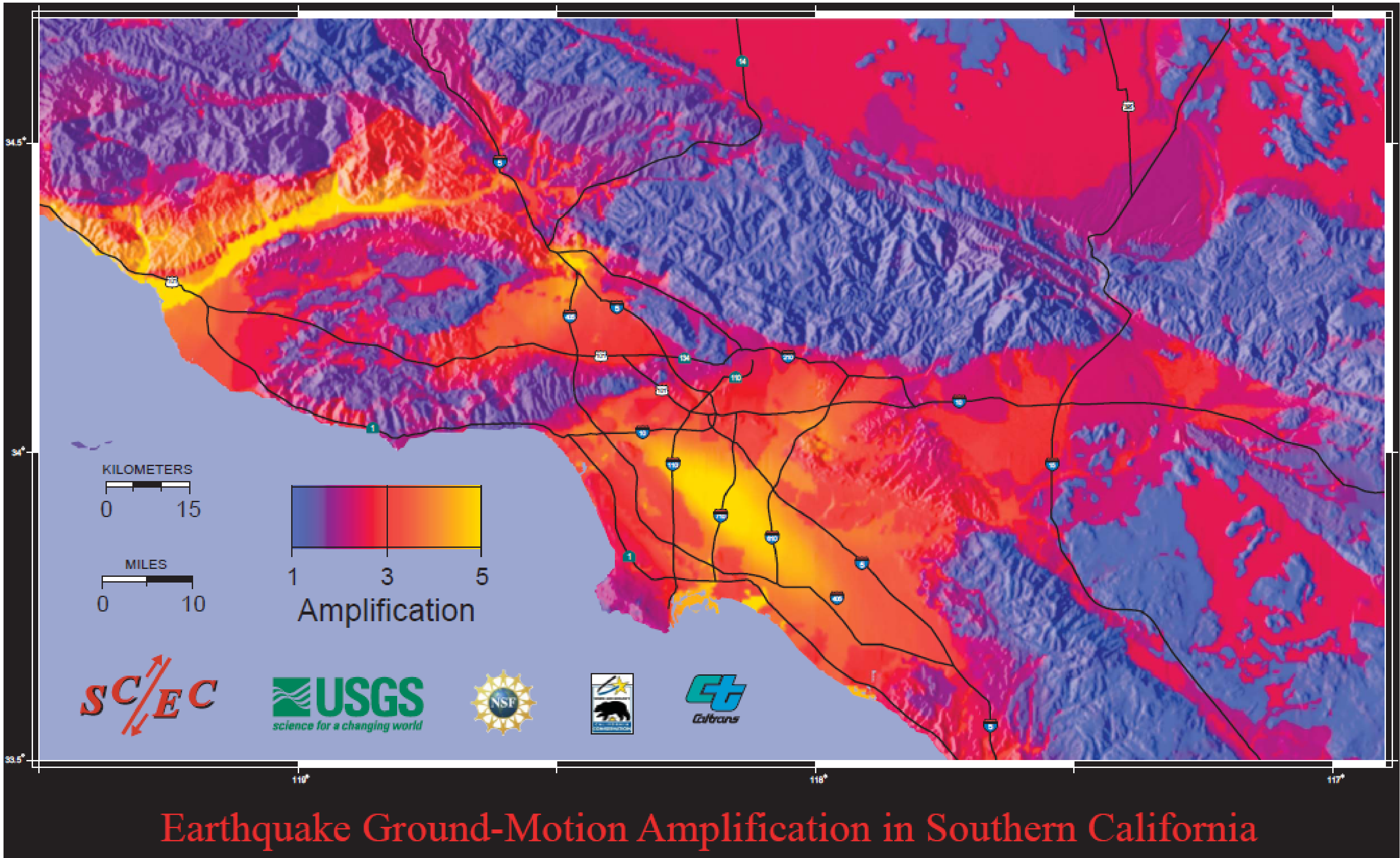
What is a spontaneous DR simulation?

Application:

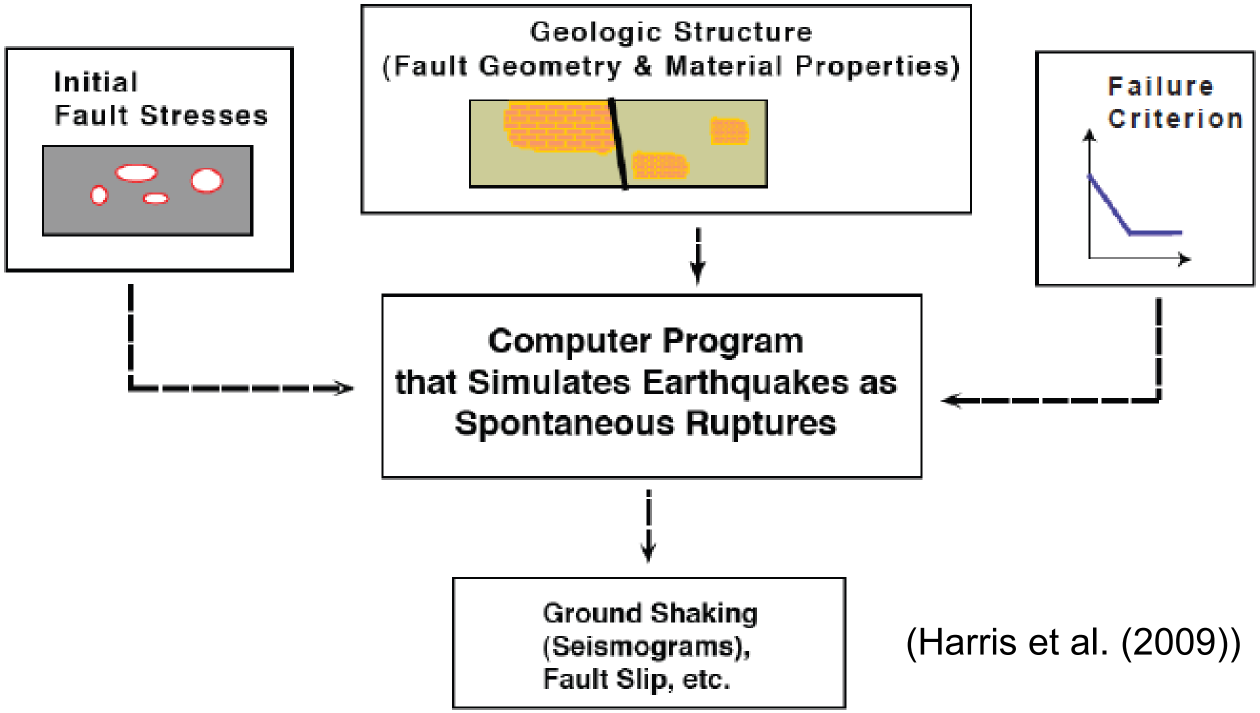
- Understanding physics of earthquake initiation, propagation, and restarting effects
- Ground motion prediction
- Hazard assessments
- Seismic risk



What is a spontaneous DR simulation?

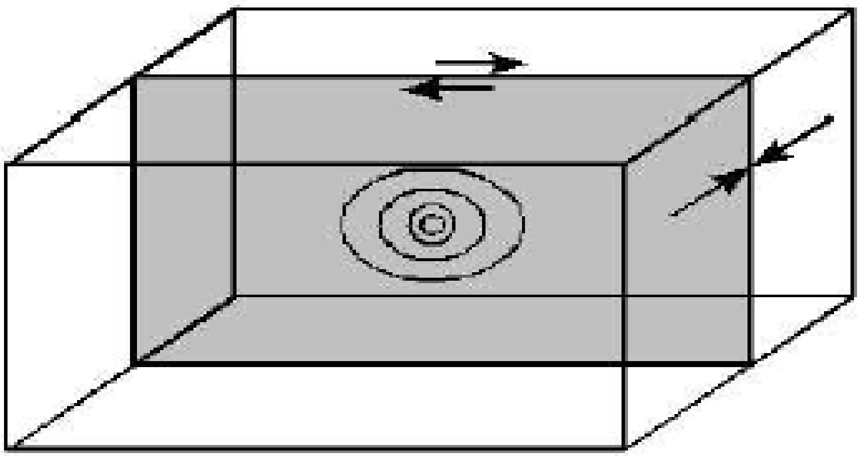
Basic ingredients:

- linear elastic medium (wave equation)
- a pre-existing fault (slip plane)
- initial conditions (stress)
- friction: non linear relation between fault stress and slip

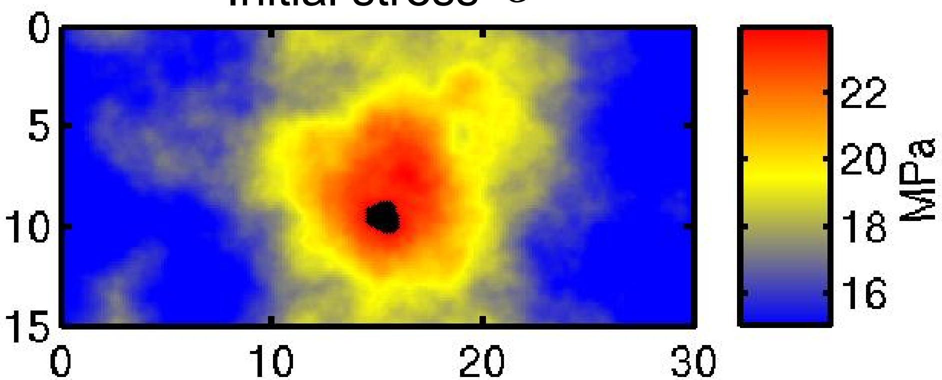


(Harris et al. (2009))

Planar strike-slip fault



Initial stress σ



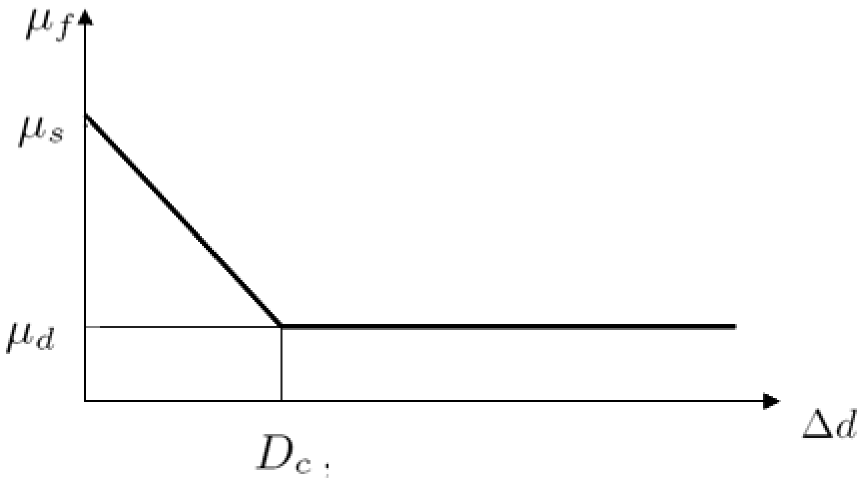
(Ampuero, talk at SPICE meeting)

Failure criterion:

Coulomb friction model

$$|\sigma_{xy}| \leq \mu_f \sigma$$

$$(|\sigma_{xy}| - \mu_f \sigma)\Delta v = 0$$



$$\mu_f = \begin{cases} \mu_s - \dfrac{\mu_s - \mu_d}{D_c}\Delta d & \text{if } \Delta d < D_c\,, \\ \mu_d & \text{if } \Delta d \geq D_c\,. \end{cases}$$

Linear Slip Weakening friction law
(laboratory experiments
– space for improvements!)

σ_{xy} shear strength

μ_f friction coefficent

σ normal stress

Δd slip

Δv slip rate

D_c critical slip distance

Provides:

- initial rupture
- arrest of sliding
- reactivation of slip

2D wave equations in velocity-stress formulation

$$\frac{\partial}{\partial t}\sigma_{xx} - (\lambda + 2\mu)\frac{\partial}{\partial x}u - \lambda\frac{\partial}{\partial y}v = 0\,,$$

$$\frac{\partial}{\partial t}\sigma_{yy} - \lambda\frac{\partial}{\partial x}u - (\lambda + 2\mu)\frac{\partial}{\partial y}v = 0\,,$$

$$\frac{\partial}{\partial t}\sigma_{xy} - \mu(\frac{\partial}{\partial x}v + \frac{\partial}{\partial y}u) = 0\,,$$

$$\rho\frac{\partial}{\partial t}u - \frac{\partial}{\partial x}\sigma_{xx} - \frac{\partial}{\partial y}\sigma_{xy} = 0\,,$$

$$\rho\frac{\partial}{\partial t}v - \frac{\partial}{\partial x}\sigma_{xy} - \frac{\partial}{\partial y}\sigma_{yy} = 0\,,$$

more compact form:

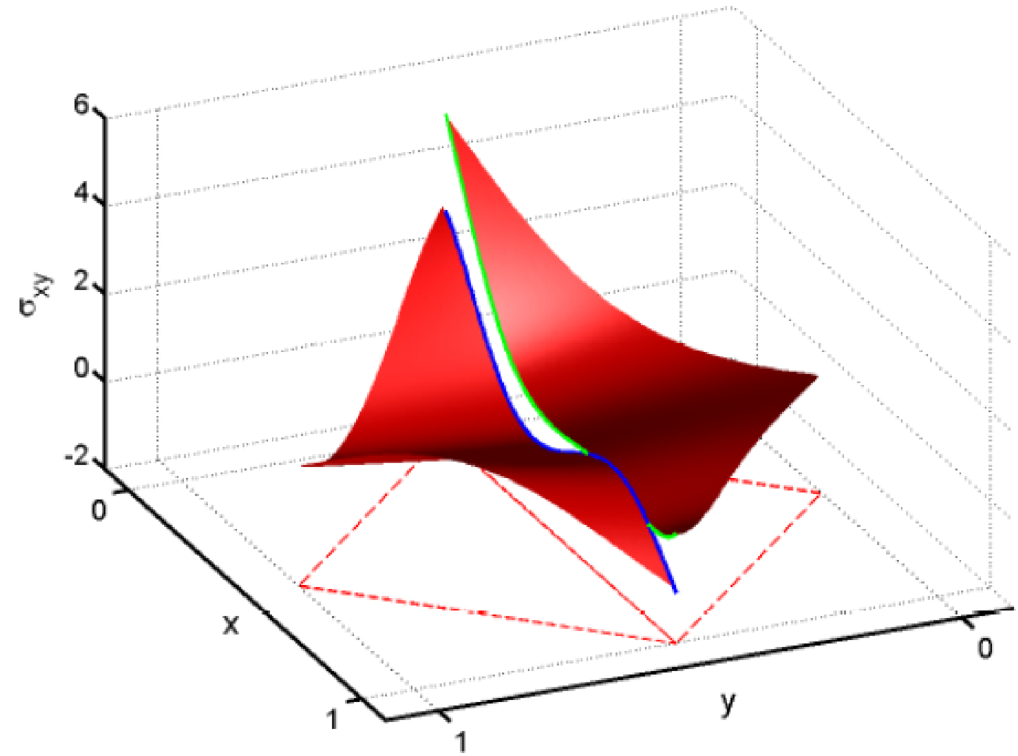
$$\frac{\partial Q_p}{\partial t} + A_{pq}\frac{\partial Q_q}{\partial x} + B_{pq}\frac{\partial Q_q}{\partial y} = 0$$ with $\mathbf{Q} = (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, u, v)^T$

Discontinuous Galerkin Approach

Numerical approximation of the solution:

$$\left(Q_h^{(m)}\right)_p (\xi, \eta, t) = \hat{Q}_{pl}^{(m)}(t)\Phi_l(\xi, \eta)$$

- Φ_l are othogonal basis functions



Integrating the governing equations in space and time in the Discontinuous Galerkin (DG) framework gives

$$\int_t^{t+\Delta t}\int_{\mathcal{T}^{(m)}} \Phi_k \frac{\partial Q_p}{\partial t}\, dV\, dt + \sum_{j=1}^{3} \mathcal{F}_{pk}^{j} - \int_t^{t+\Delta t}\int_{\mathcal{T}^{(m)}} \left(\frac{\partial \Phi_k}{\partial x} A_{pq} + \frac{\partial \Phi_k}{\partial y} B_{pq}\right) Q_q\, dV\, dt = 0$$

where the numerical flux is given by $\mathcal{F}_{pk} = A_{pr}\int_t^{t+\Delta t}\int_S \Phi_k \tilde{Q}_r\, dS\, dt$

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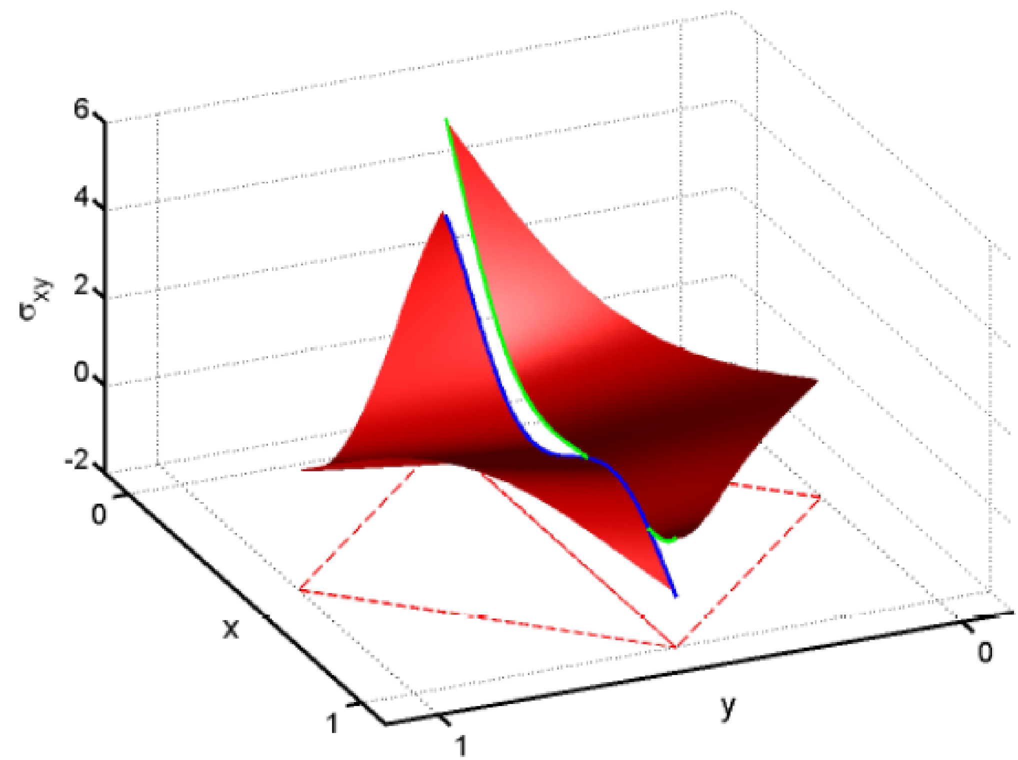

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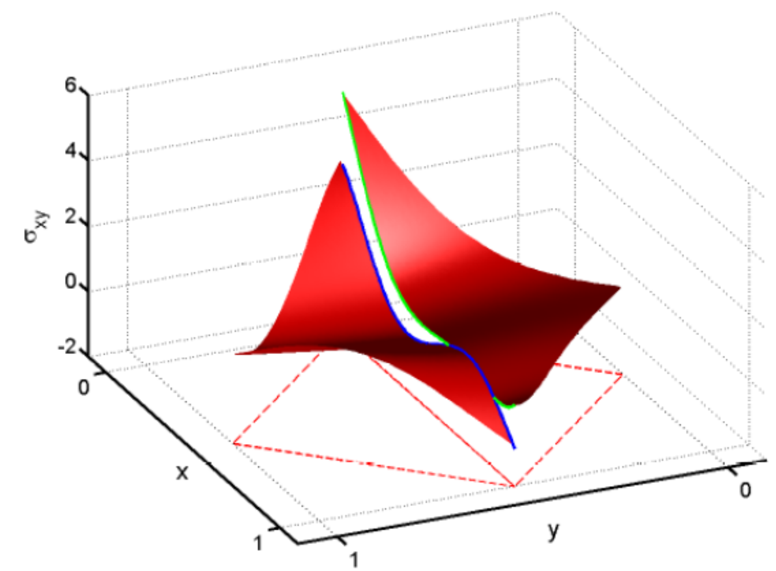
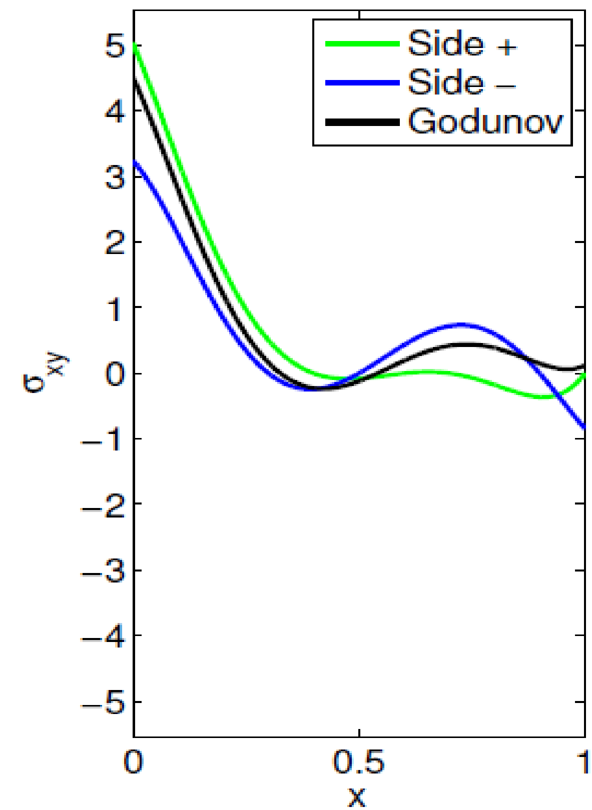
Riemann problem

Standard wave propagation!



The state of the variables at the interface are given as

$$
\begin{aligned}
2\sigma_{xx}^G &= (\sigma_{xx}^- + \sigma_{xx}^+) + \frac{\lambda + 2\mu}{c_p}(u^- - u^+) \ , \\
2\sigma_{yy}^G &= \frac{\lambda}{c_p}(u^- - u^+) + \frac{\lambda}{\lambda + 2\mu}(\sigma_{xx}^- + \sigma_{xx}^+) + 2\sigma_{yy}^+ \ , \\
2\sigma_{xy}^G &= (\sigma_{xy}^- + \sigma_{xy}^+) + \frac{\mu}{c_s}(v^- - v^+) \ , \\
2u^G &= (u^- + u^+) + \frac{c_p}{\lambda + 2\mu}(\sigma_{xx}^- - \sigma_{xx}^+) \ , \\
2v^G &= (v^- + v^+) + \frac{c_s}{\mu}(\sigma_{xy}^- - \sigma_{xy}^+) \ ,
\end{aligned}
$$

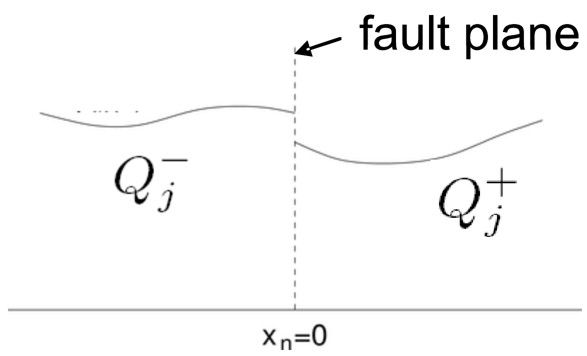


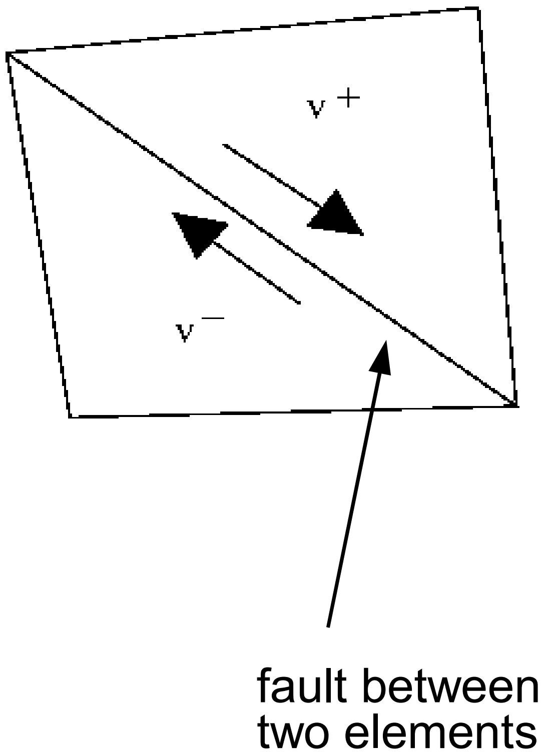
How to implement dynamic rupture?

Treat dynamic rupture as a 'boundary condition' using the flux term!

We need a new traction
respecting the failure criterion...

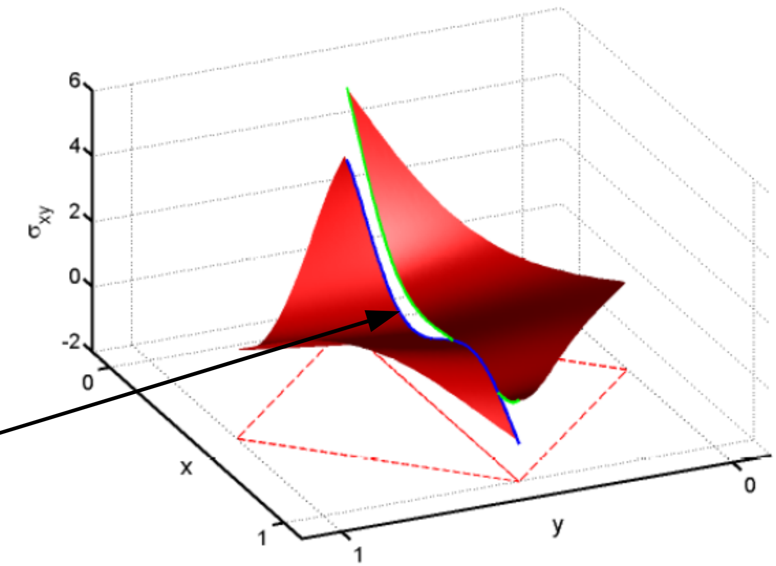
We need fault parallel velocities
in opposite directions...





Riemann problem

At fault interface!



The state of the variables at the interface are given as

$$2\sigma_{xx}^G = (\sigma_{xx}^- + \sigma_{xx}^+) + \frac{\lambda + 2\mu}{c_p}(u^- - u^+) \ ,$$

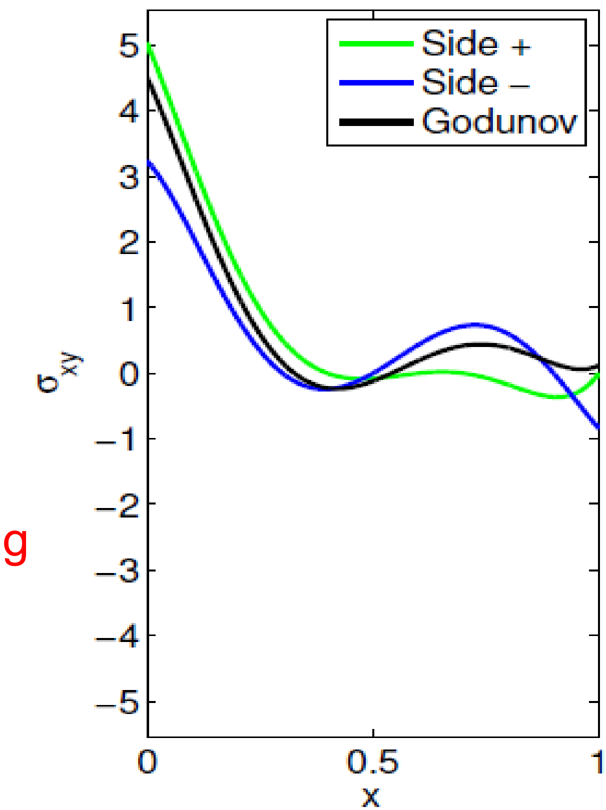
$$2\sigma_{yy}^G = \frac{\lambda}{c_p}(u^- - u^+) + \frac{\lambda}{\lambda + 2\mu}(\sigma_{xx}^- + \sigma_{xx}^+) + 2\sigma_{yy}^+ \ ,$$

$$2\sigma_{xy}^G = (\sigma_{xy}^- + \sigma_{xy}^+) + \frac{\mu}{c_s}(v^- - v^+) \ ,$$

$$2u^G = (u^- + u^+) + \frac{c_p}{\lambda + 2\mu}(\sigma_{xx}^- - \sigma_{xx}^+) \ ,$$

$$2v^G = (v^- + v^+) + \frac{c_s}{\mu}(\sigma_{xy}^- - \sigma_{xy}^+) \ ,$$

Take imposed values regarding failure criterion!



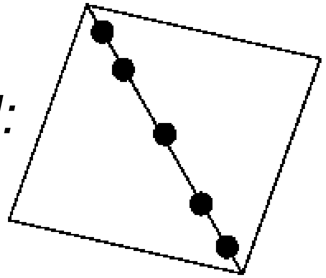
To get the imposed state vector Q_{il} we follow three steps:

1. Evaluate failure criterion & impose traction $\tilde{\sigma}_{xy}$

Substitute the Godunov state σ_{xy}^{G} from linear elasticity with an <u>imposed traction</u> $\tilde{\sigma}_{xy}$ at the fault considering the Coulomb failure criterion!

$$\tilde{\sigma}_{xy,il} = \min\left\{\sigma_{xy,il}^{G}, \mu_{f,il}(\sigma_{xx,il}^{G} + \sigma_{xx}^{0}) - \sigma_{xy}^{0}\right\}$$

<u>Side note:</u>
This is done for each Gaussian integration point *il:*



To get the imposed state vector Q_{il} we follow three steps:

1. Evaluate failure criterion & impose traction $\tilde{\sigma}_{xy}$

2. Compute fault parallel velocities and slip rate

The imposed traction provides boundary conditions for the slip rates (velocities) on both sides!

$$\tilde{v}^{+} = v^{+} + \frac{c_s}{\mu}\left(\tilde{\sigma}_{xy} - \sigma_{xy}^{+}\right)$$

$$\tilde{v}^{-} = v^{-} - \frac{c_s}{\mu}\left(\tilde{\sigma}_{xy} - \sigma_{xy}^{-}\right)$$

The <u>imposed slip rate</u> is given by

$$\Delta\tilde{v} = \tilde{v}^{+} - \tilde{v}^{-} = \left(v^{+} - v^{-}\right) + \frac{c_s}{\mu}\left[2\tilde{\sigma}_{xy} - \left(\sigma_{xy}^{-} + \sigma_{xy}^{+}\right)\right]$$

$$\Delta\tilde{v} = \frac{2c_s}{\mu}\left(\tilde{\sigma}_{xy} - \sigma_{xy}^{G}\right)$$

➔ $\Delta\tilde{v} \neq 0$ only if $\tilde{\sigma}_{xy} \neq \sigma_{xy}^{G}$

To get the imposed state vector Q_{il} we follow three steps:

1. Evaluate failure criterion & impose traction $\tilde{\sigma}_{xy}$

2. Compute fault parallel velocities and slip rate

3. Compute slip Δd and update friction coefficent

$$\mu_f = \begin{cases} \mu_s - \dfrac{\mu_s - \mu_d}{D_c} \Delta d & \text{if } \Delta d < D_c \,, \\ \mu_d & \text{if } \Delta d \geq D_c \,. \end{cases}$$

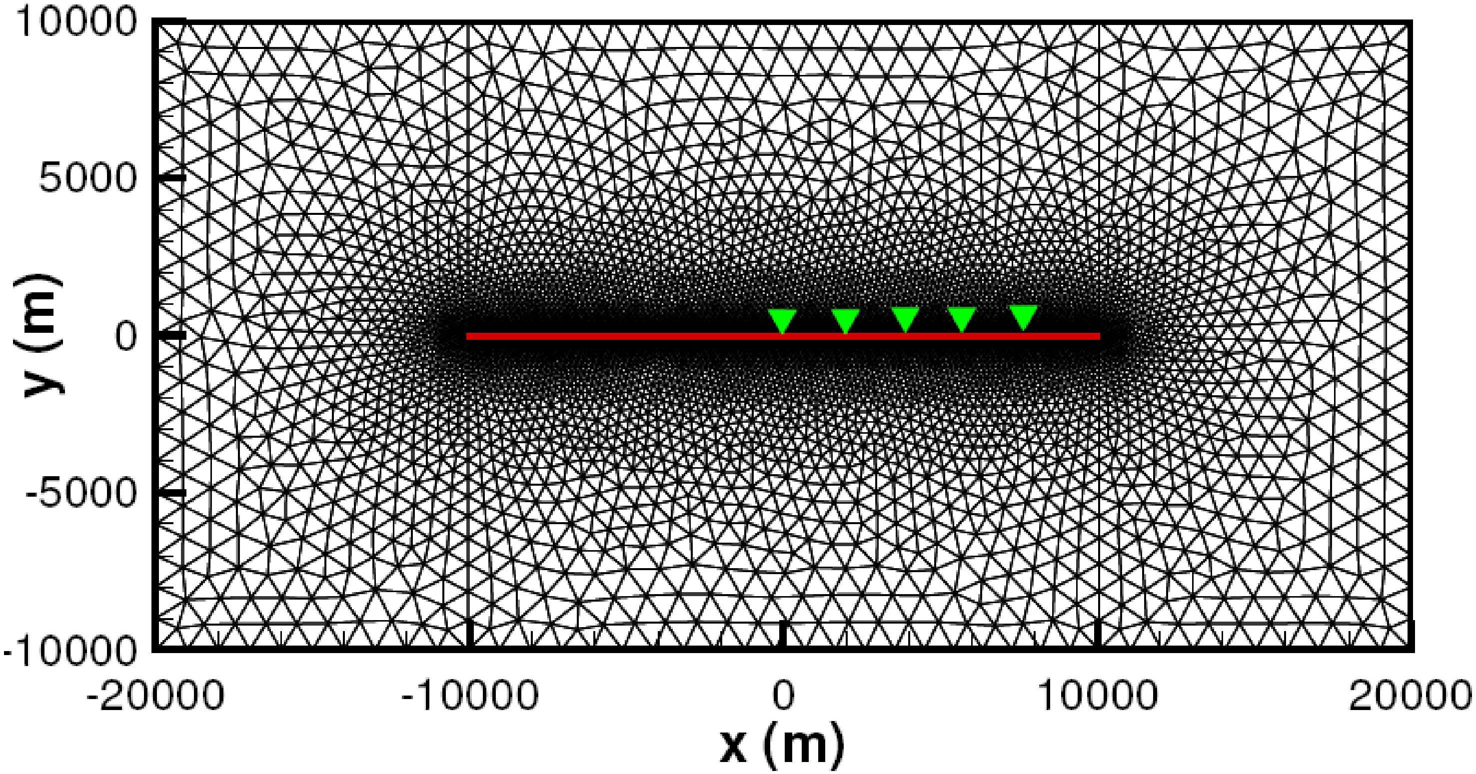
Gauss-Integration of flux:

$$\mathcal{F}_{pk} = A_{pr} \sum_{i=1}^{3N} \sum_{l=1}^{N+1} \omega_i^S \omega_l^T \Phi_k(\boldsymbol{\xi}_i) \tilde{Q}_{r,il}$$

Validation – SCEC Test Case

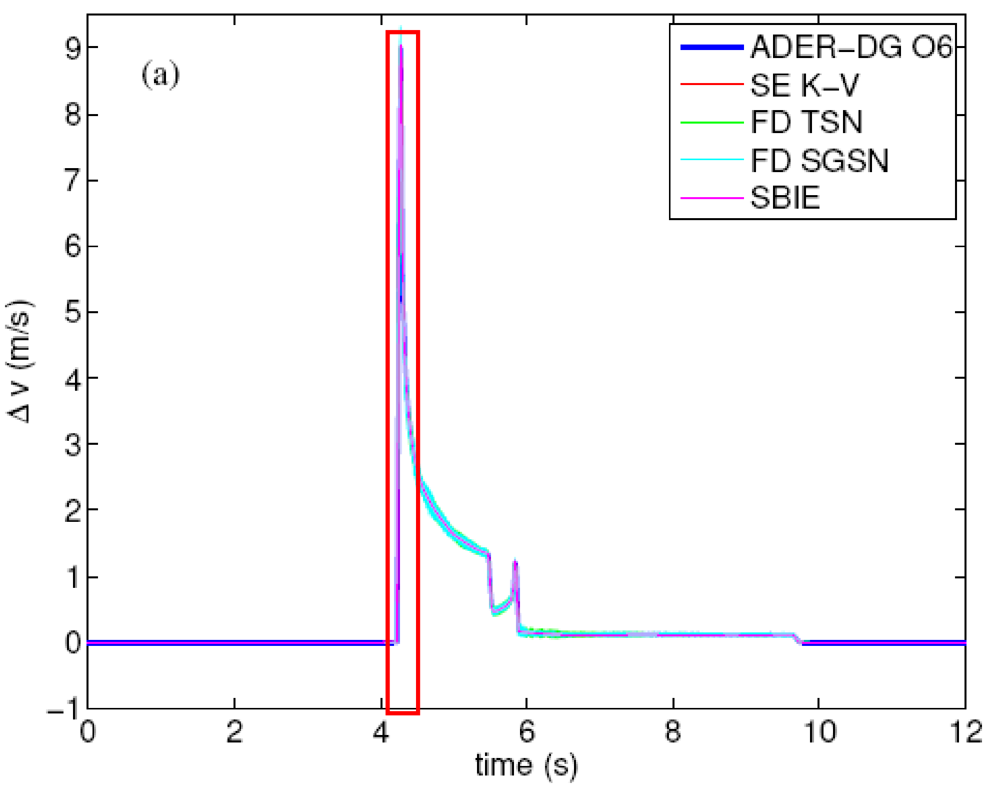
(Harris et al., 2004)

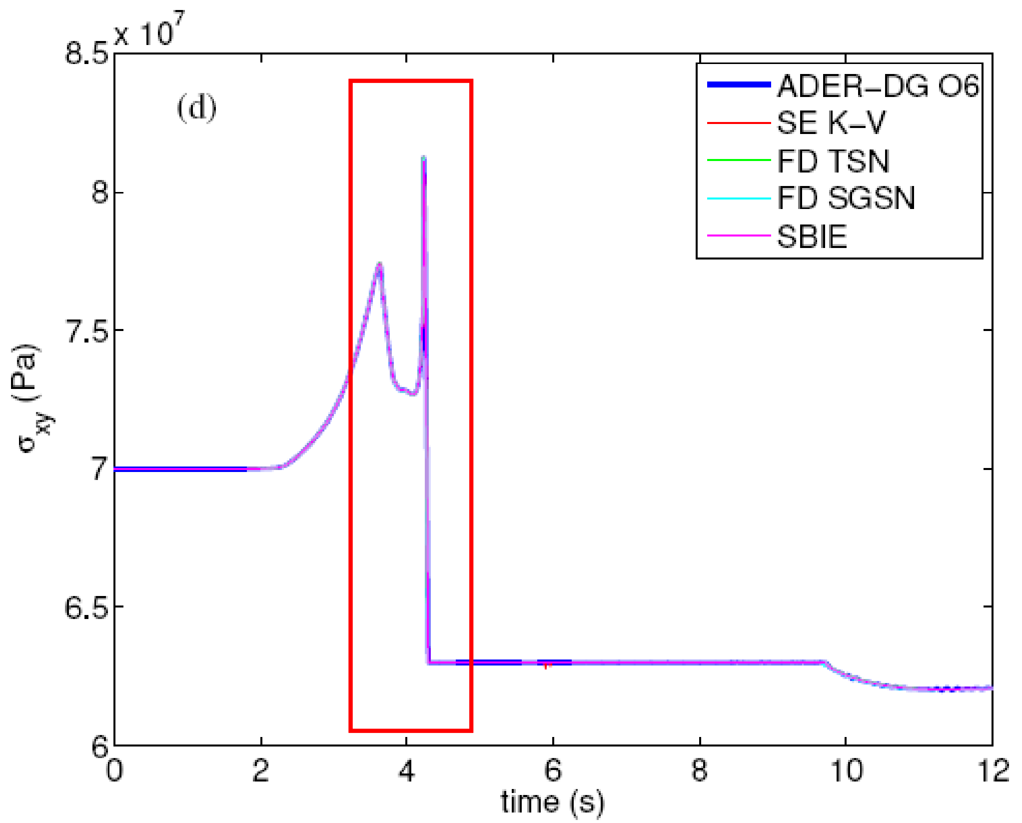
- spontaneous rupture propagation on a straight fault
- LSW friction



Validation – SCEC Test Case

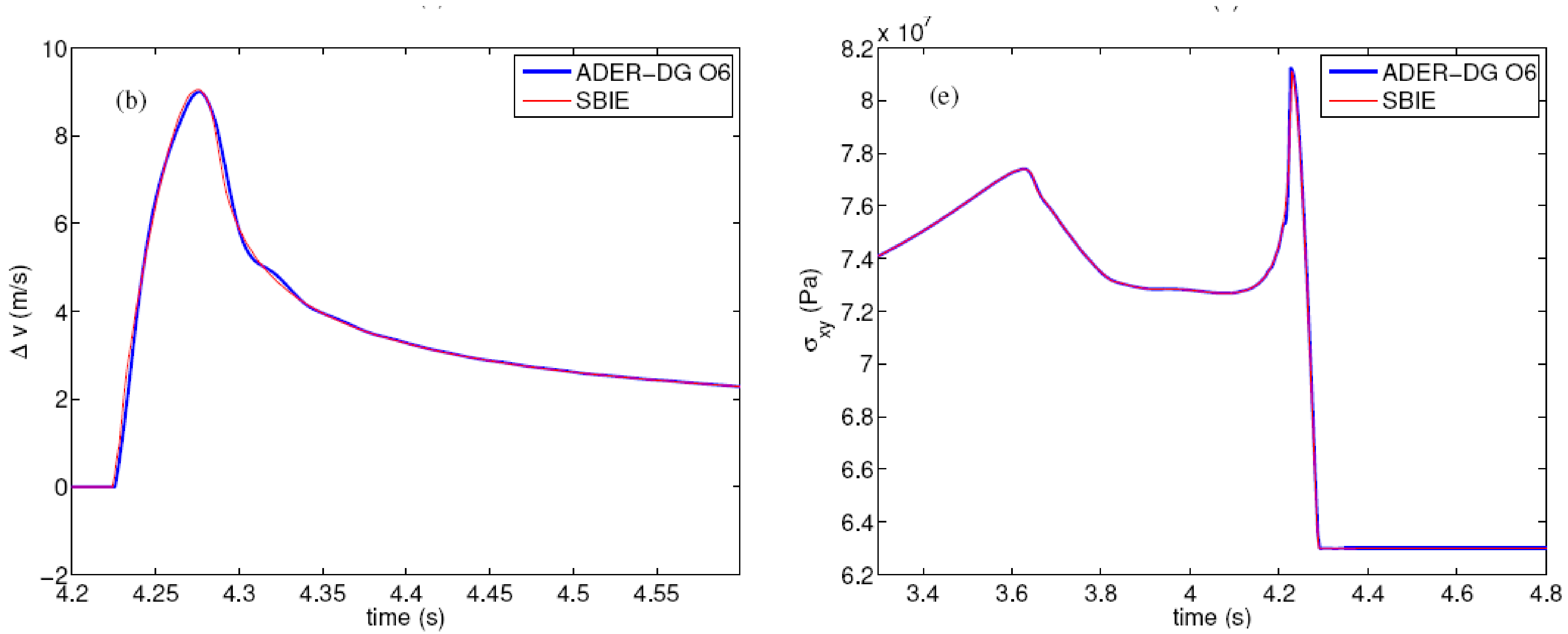
Results of slip rate and shear traction





Validation – SCEC Test Case

Results of <u>slip rate</u> and <u>shear traction</u>



Validation – SCEC Test Case

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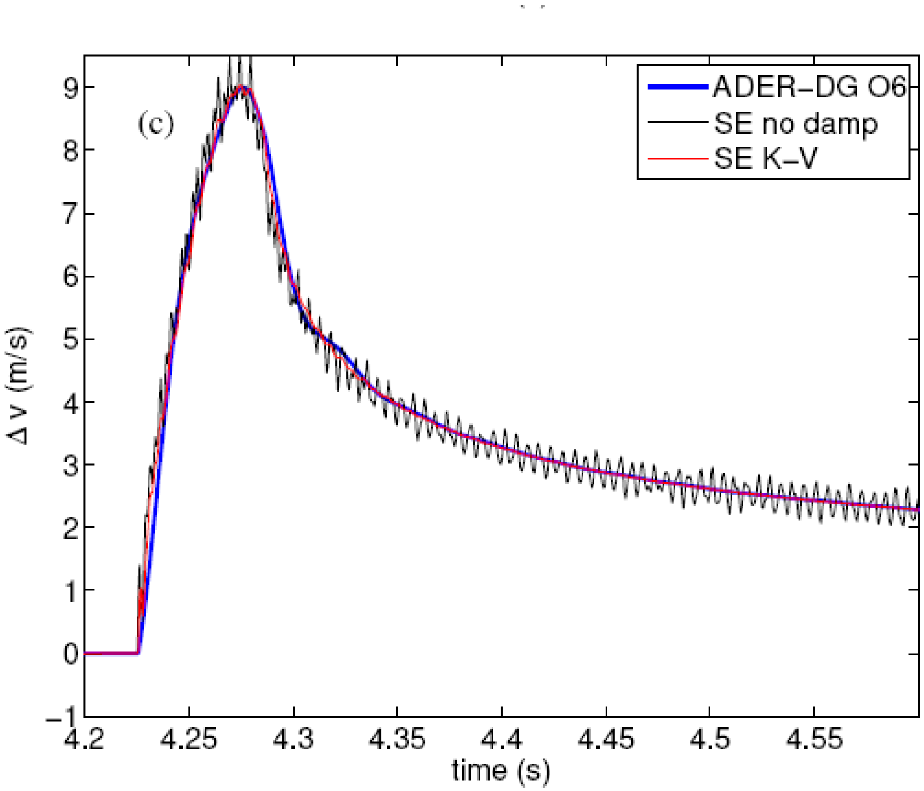

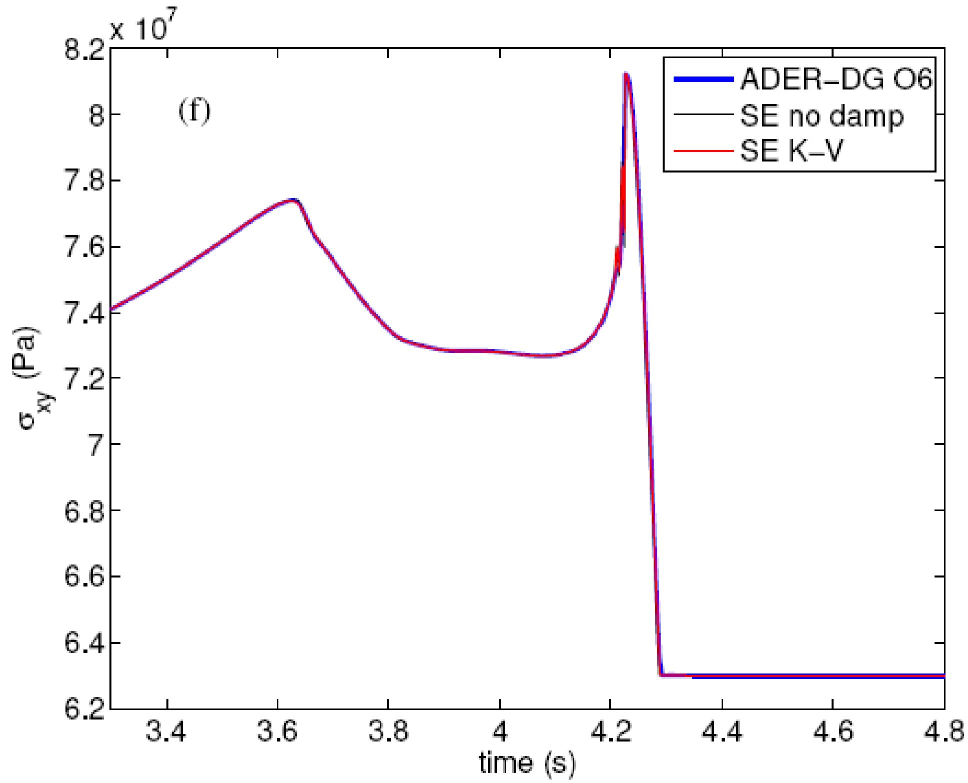

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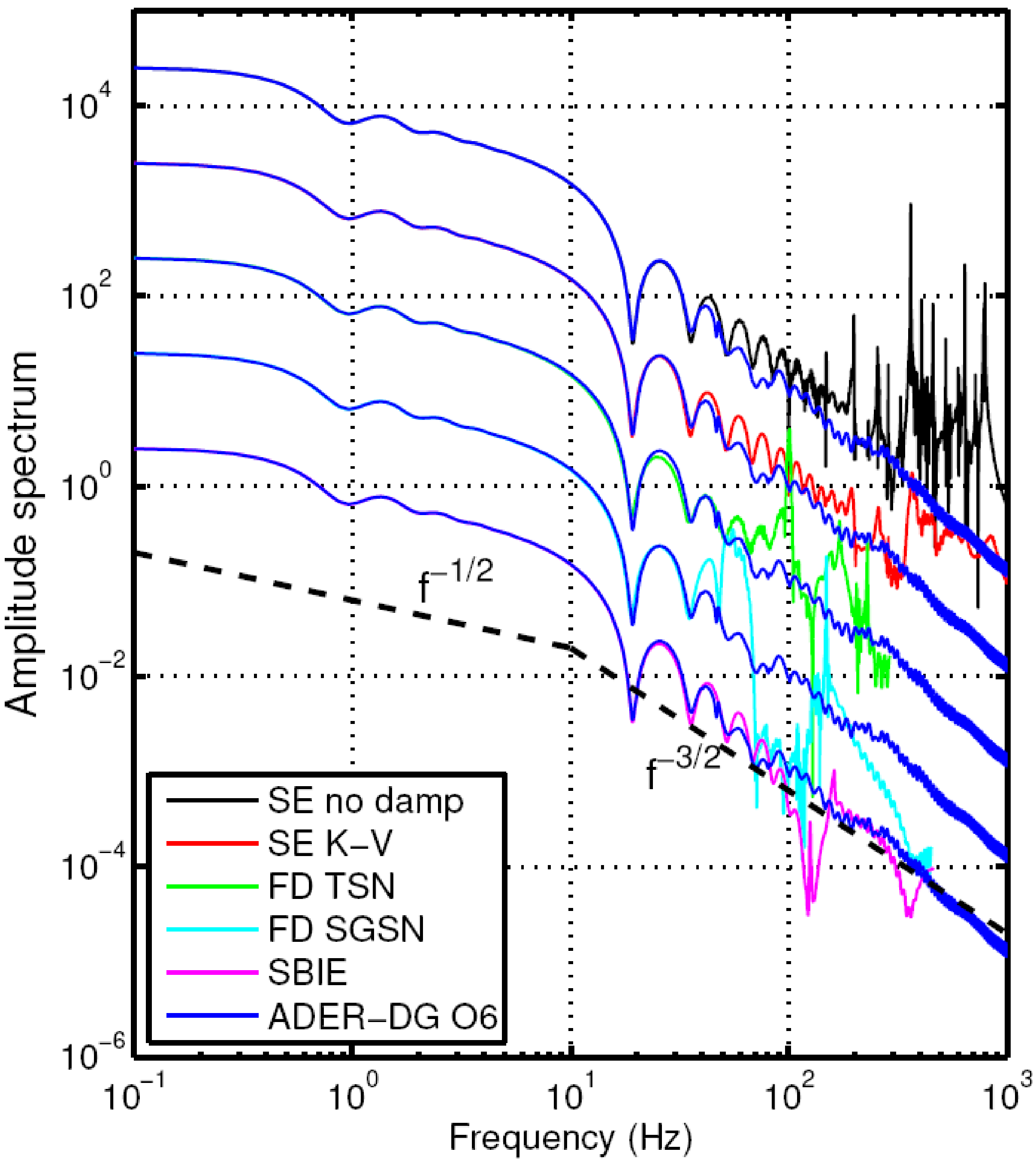
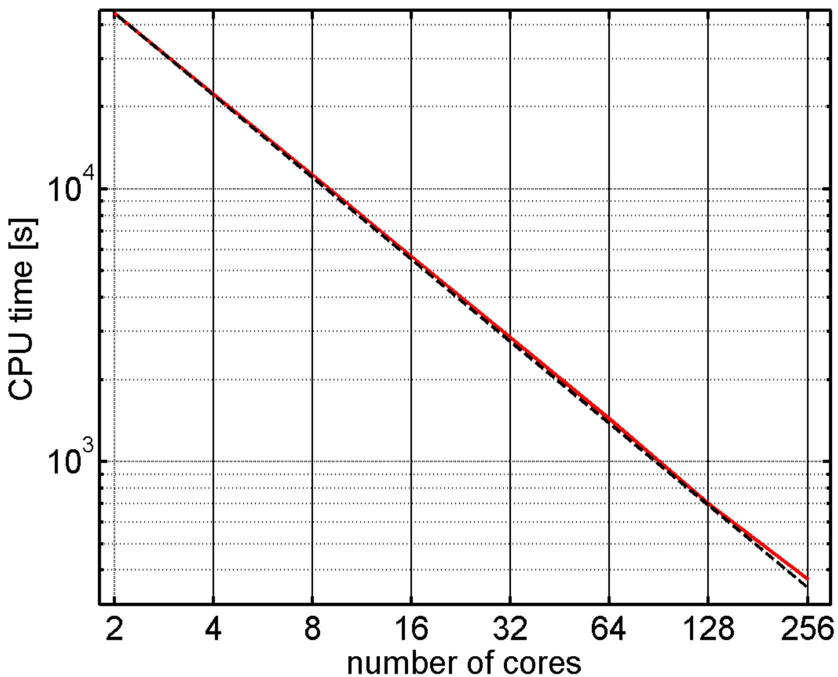
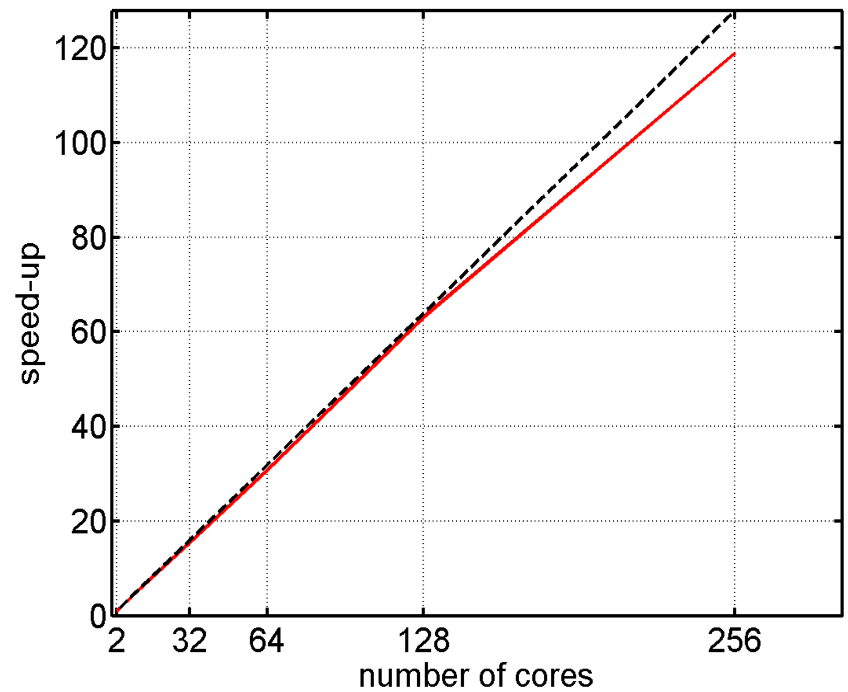


ADER-DG does not show spurious oscillations at the high frequency content of the seismograms !

Parallel properties - Speed-Up

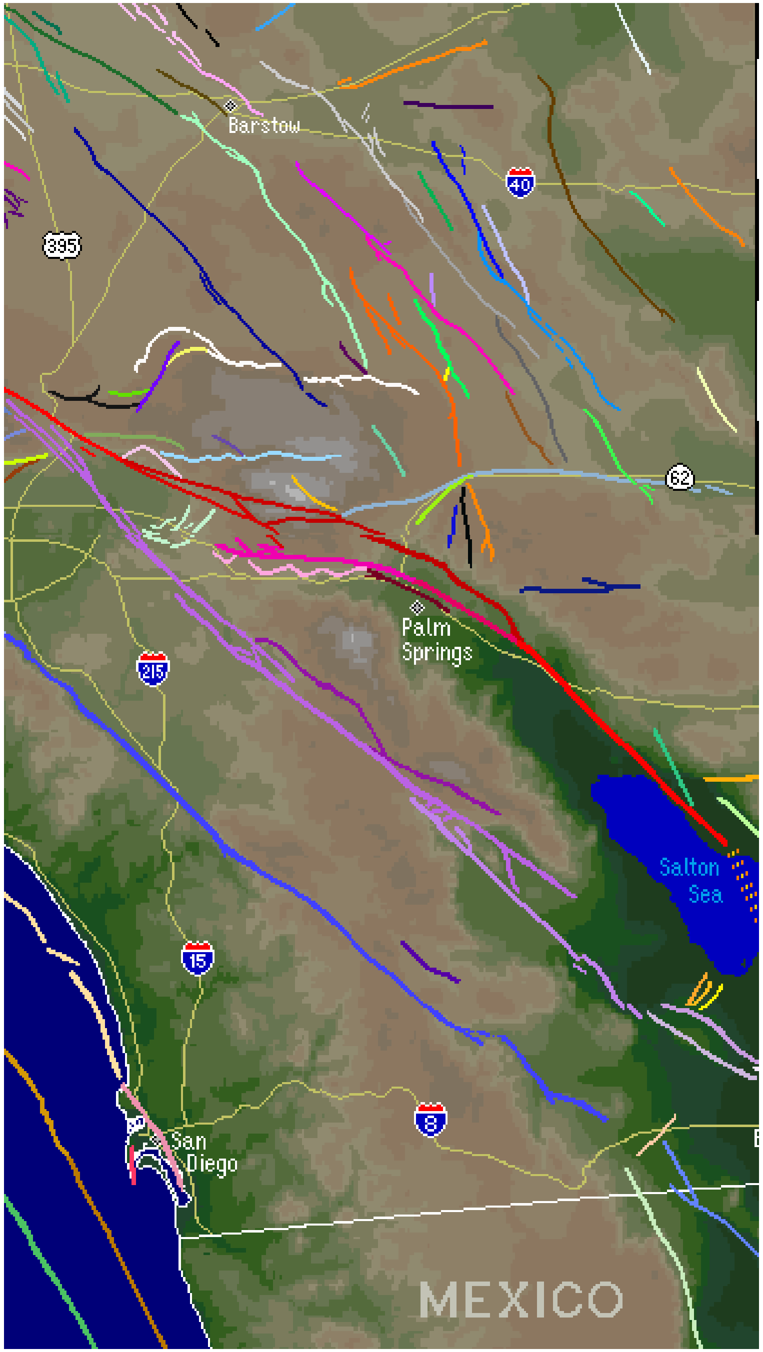




- 96,630 triangular element discretization of the SCEC test
- Approximation order of 4 in space and time
- CPU time reduction (red line) remains close to the ideal case (dashed line)
- Excellent scalability for a wide range of used cores
- Efficiency over 93 %

→ Addition of fault dynamics has only minor impacts on the performance

Application to the Landers Earthquake Fault System



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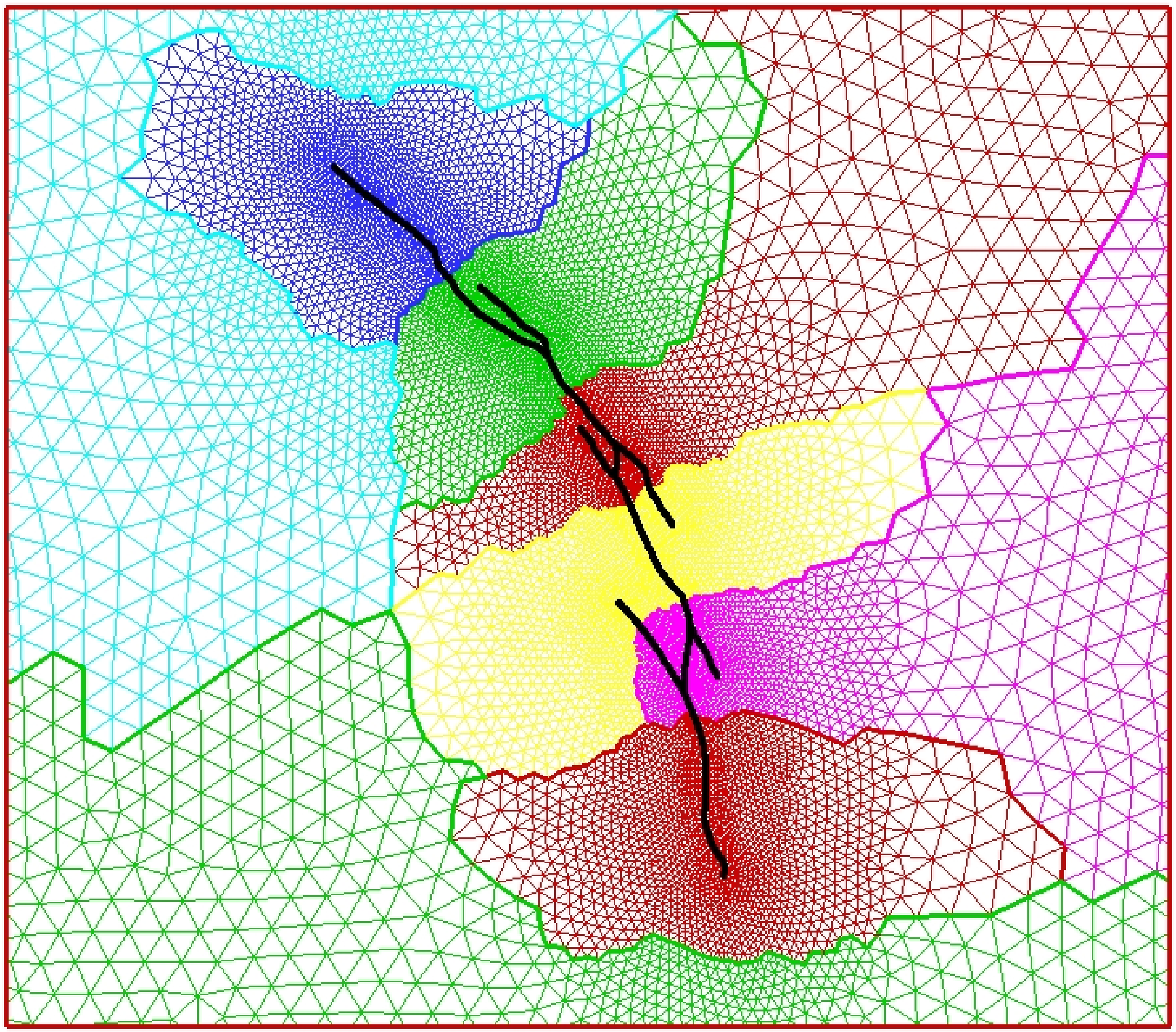
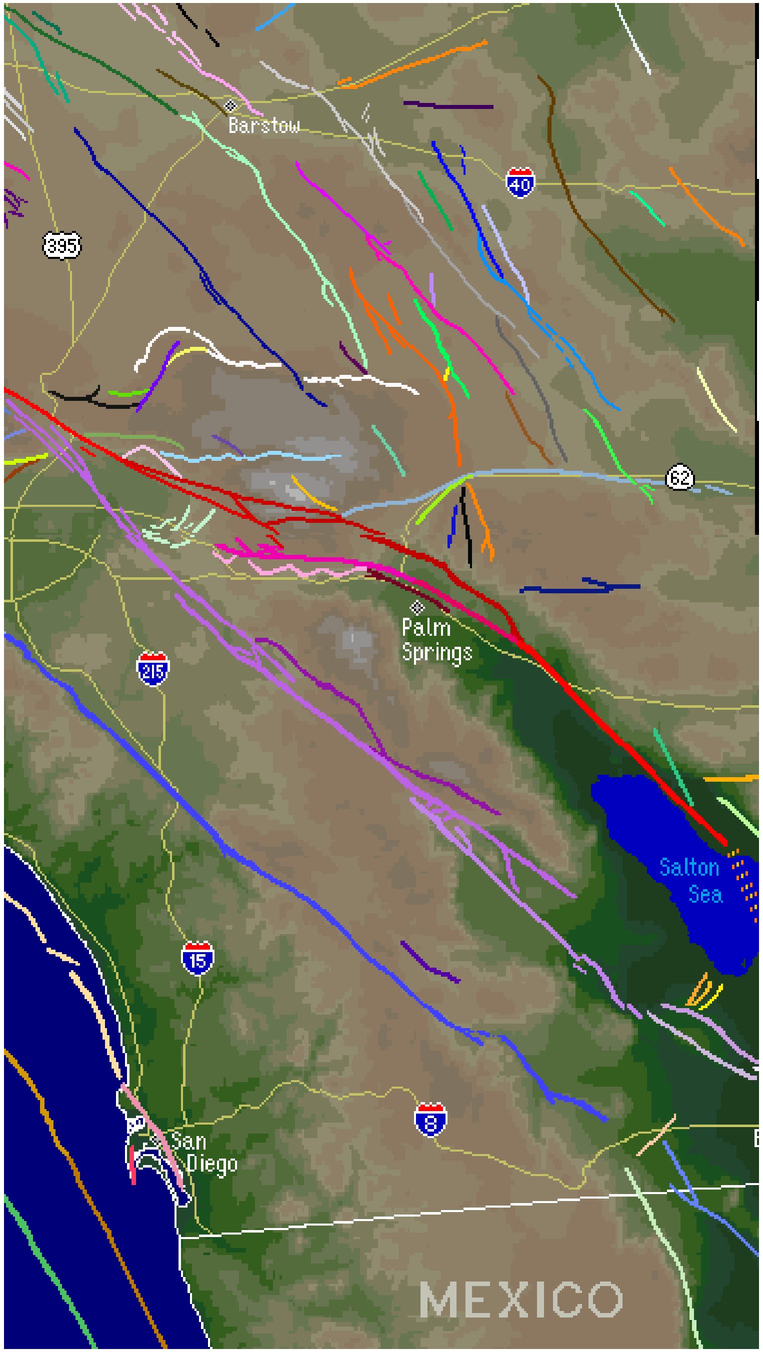
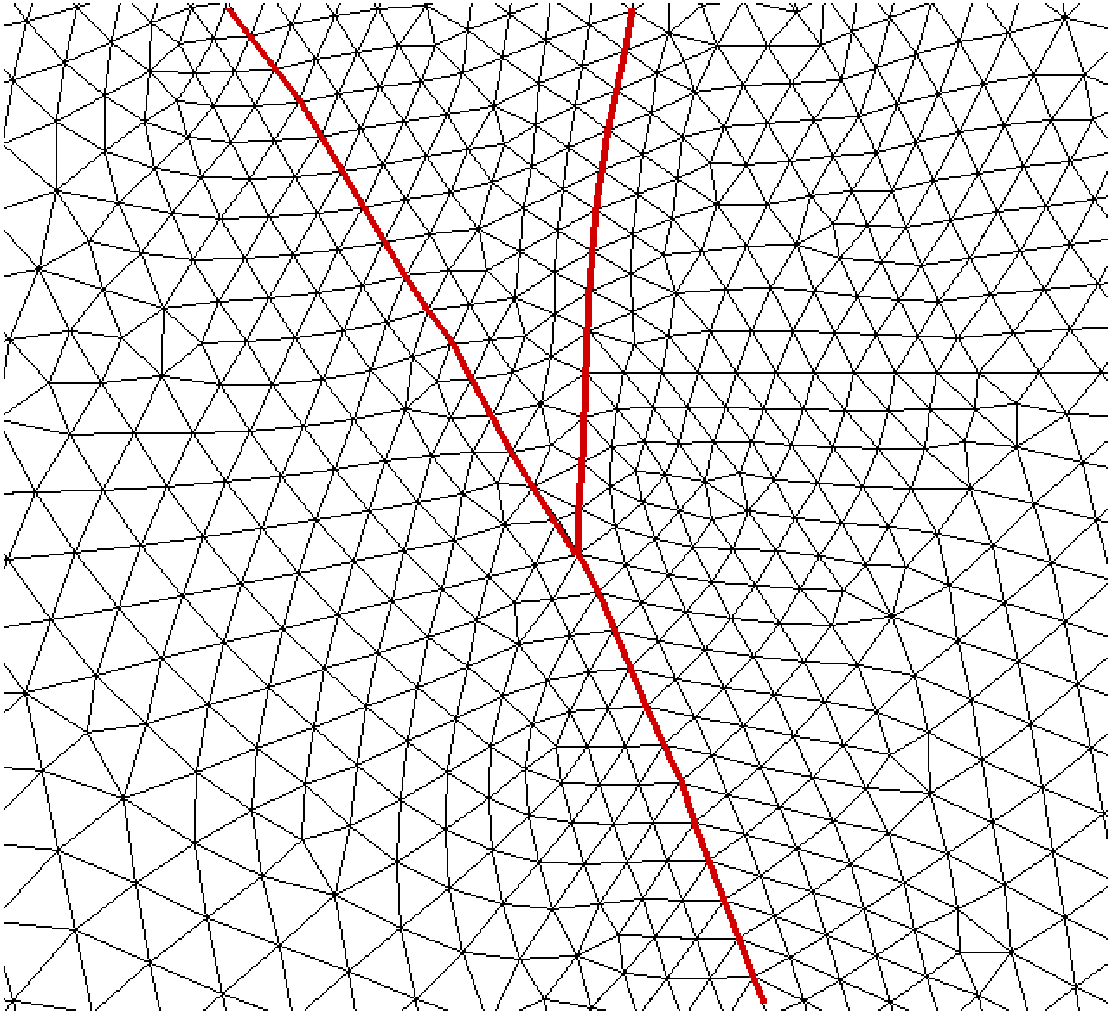
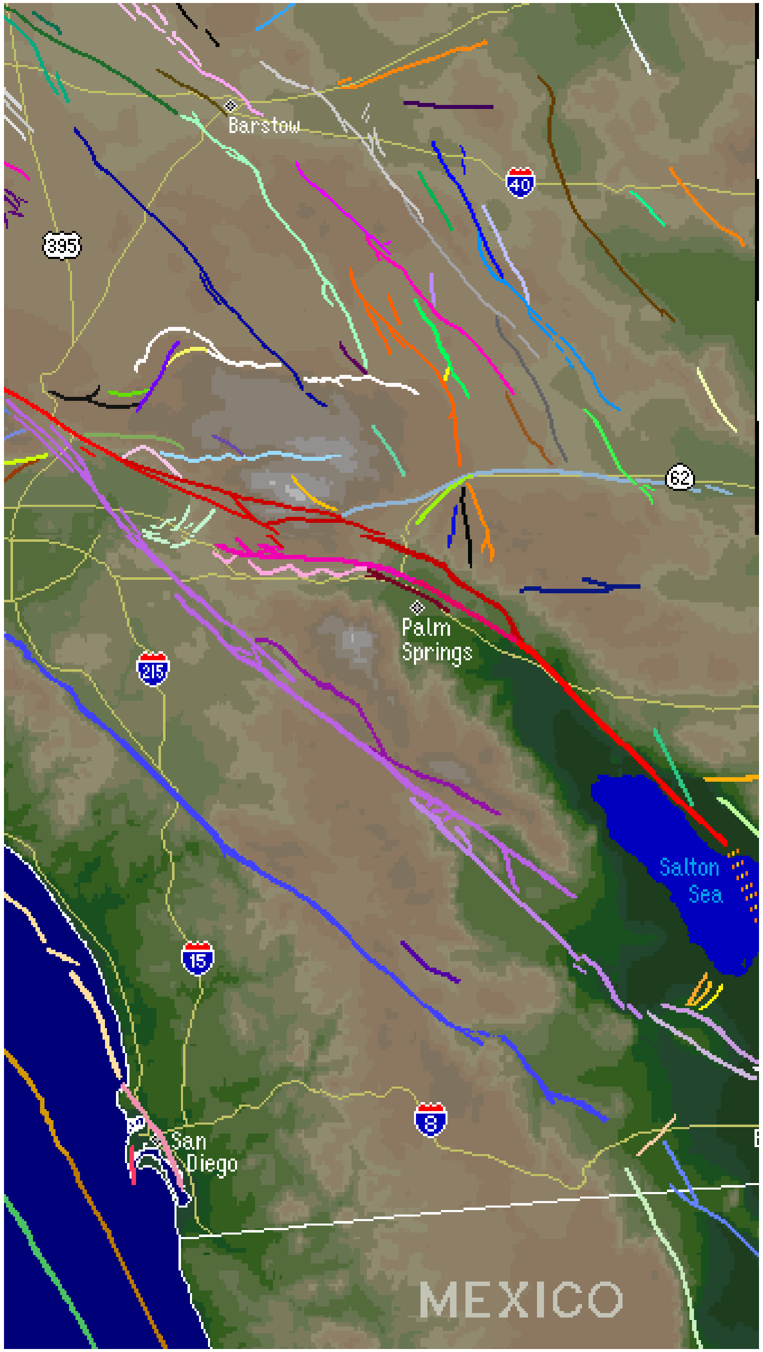




Application to the Landers Earthquake Fault System

Zoom...





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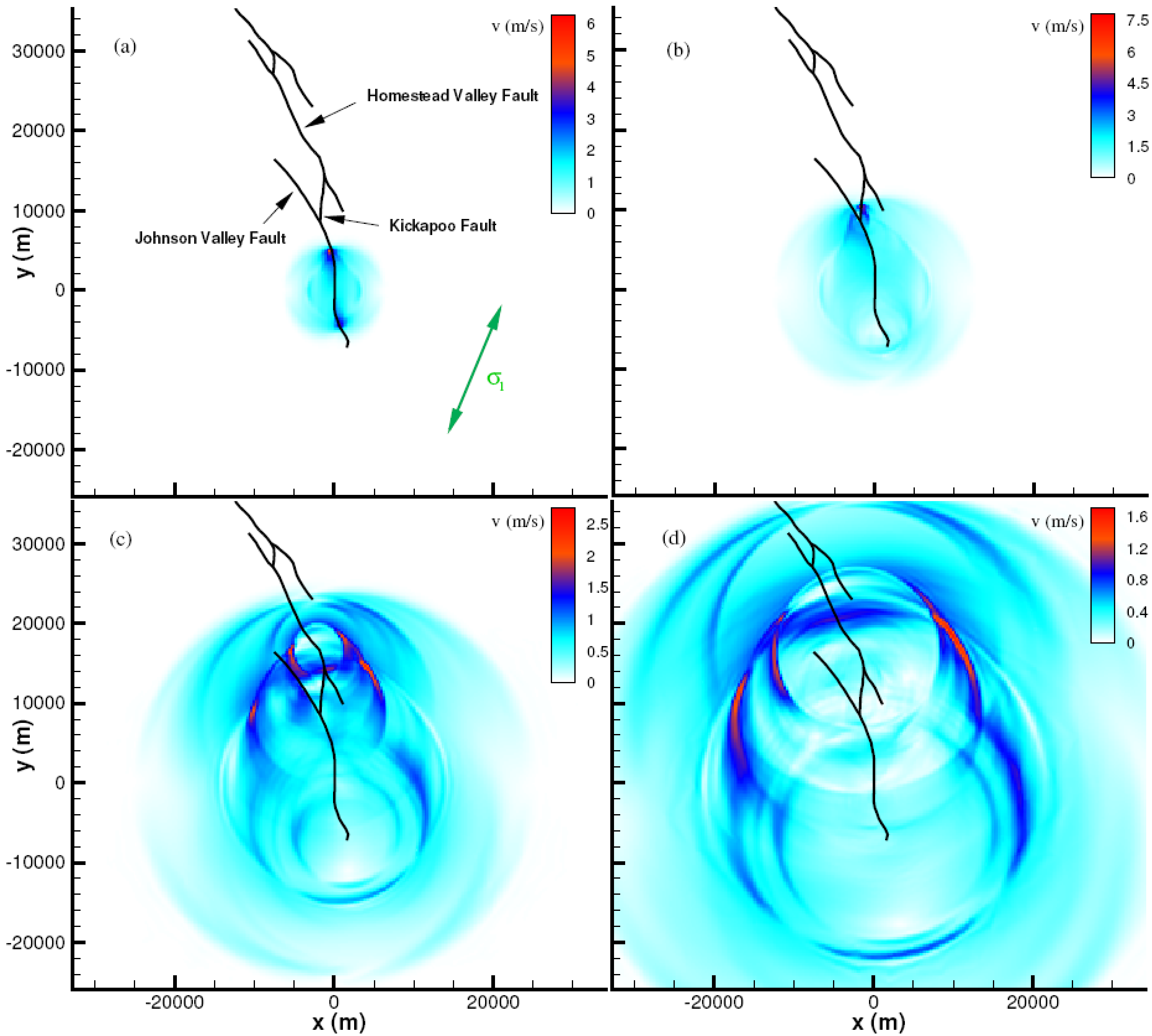

Summary

• Complex fault geometries can be modeled adequately with small elements while fast mesh coarsening is possible

• Method allows fault branching and surface rupture

• No spurious high-frequency contributions in the slip rate spectra

• High accuracy of following wave propagation

• Computationally efficient in heterogeneous media and good parallel scalability