Elastic Full Waveform Modeling and Inversion in the Frequency Domain

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Outline

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Introduction

Objectives

Reconstruction of Earth physical parameters

- Waves velocities V_p, V_s (or combination)
- Density ρ
- Attenuation
- Anisotropy

Method

- Full-waveform inversion (Tarantola, 1984)
- Frequency domain (Pratt and Worthington, 1990)
- Active seismic Wide-aperture acquisition

Elastic Full Waveform Inversion main issues

Forward problem : Elastic wave equation

- Computationally efficient and accurate simulation of complex-wave propagation in heterogenous environments
- Liquid/solid contact with high Poisson's ratio for offshore problems

Ill-posed and highly non-linear inverse problem

- Multi-parameters inversion, with different sensitivities and signatures in data
- Inaccuracies of the starting model and lack of low frequencies, particularly for V_S
- Complex waves phenomena (converted waves, multiples, surface-waves...)
- Sensitivity to noise

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Theory : Forward problem

- 1^{st} order velocity-stress system for 2D P-SV waves in the frequency domain

$$\begin{aligned} -i\omega\rho\mathbf{V} &= \nabla\cdot\sigma + \rho\vec{f} \\ -i\omega\sigma &= \mathbf{c} : \nabla\mathbf{V} - i\omega\sigma_0 \end{aligned}$$

- Equations solved with low order finite element Discontinuous Galerkin method
  - Medium is discretized with triangular mesh
  - Physical properties are constant in each cell
- PML absorbing conditions (Berenger, 1994)

Theory : Forward problem

$$\begin{bmatrix} -i\omega & 0 & -\frac{1}{\rho}\frac{\partial}{\partial x} & 0 & -\frac{1}{\rho}\frac{\partial}{\partial z} \\ 0 & -i\omega & 0 & -\frac{1}{\rho}\frac{\partial}{\partial z} & -\frac{1}{\rho}\frac{\partial}{\partial x} \\ -(\lambda+2\mu)\frac{\partial}{\partial x} & -\lambda\frac{\partial}{\partial z} & -i\omega & 0 & 0 \\ -\lambda\frac{\partial}{\partial x} & -(\lambda+2\mu)\frac{\partial}{\partial z} & 0 & -i\omega & 0 \\ -\mu\frac{\partial}{\partial z} & -\mu\frac{\partial}{\partial x} & 0 & 0 & -i\omega \end{bmatrix} \begin{bmatrix} V_x \\ V_z \\ \sigma_{xx} \\ \sigma_{zz} \\ \sigma_{xz} \end{bmatrix} = \begin{bmatrix} f_x \\ f_z \\ \sigma_{xx_0} \\ \sigma_{zz_0} \\ \sigma_{xz_0} \end{bmatrix}$$

Linear system resolution per frequency done with the parallel direct solver MUMPS (Amestoy et al., 2006)

$$\mathbf{Au} = \mathbf{b}$$

- **A** : impedance matrix, forward problem operator
- **u** : solution vector, velocity/stress wavefield
- **b** : excitation vector

Outline

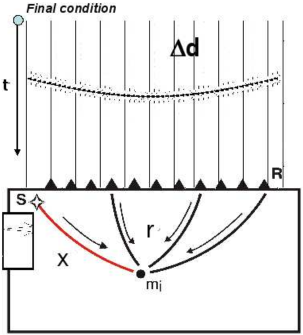
Theory : Inverse problem

- Classically : least-square functional in the frequency-domain (Pratt and Worthington, 1990)

$$\mathcal{C} = \sum_{k=1}^{ns} \frac{1}{2} \Delta \mathbf{d}_k^t \overline{\Delta \mathbf{d}}_k,$$

- Local optimization based on the gradient direction computed with the adjoint-state method : two simulations per shot for the forward and the adjoint wavefields

$$\begin{aligned} \mathcal{G}_{m_i} &= -\sum_{k=1}^{ns} \Re\left\{ \frac{\partial \mathbf{d}_{calc_k}}{\partial m_i} \overline{\Delta \mathbf{d}}_k \right\} \\ &= -\sum_{k=1}^{ns} \Re\left\{ \mathbf{u}_k^t \frac{\partial \mathbf{A}^t}{\partial m_i} \overline{\lambda}_k \right\} \end{aligned}$$



Optimization scheme

- Based on Newton's equation

$$\mathbf{B}\Delta\mathbf{m} \quad = \quad -\mathcal{G}$$

- $\mathbf{B}$ is the Hessian matrix (second derivative of $\mathcal{C}$) (Pratt et al., 1998)
  - Remove the geometrical spreading effects from the gradient (diagonal terms)
  - Take into account parameters cross-correlations : acts as a deconvolving operator on the gradient (off-diagonal terms)
  - Scales the relative weight of each parameter class : multiparameter inversion scaling
- Methods
  - Newton and Gauss-Newton : full Hessian or approximated Hessian (costly)
  - Gradient or conjugate-gradient : $\mathbf{B} = \beta$
  - Preconditionned gradient or conjugate-gradient : $\mathbf{B} = \textit{diag}\tilde{\mathbf{B}}$
  - Quasi-Newton methods : economical approximation of Hessian $\mathbf{B}$ (L-BFGS Nocedal, 1980)

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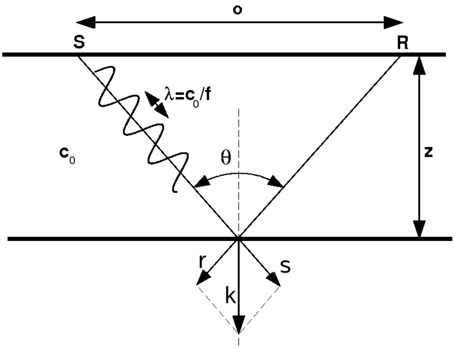
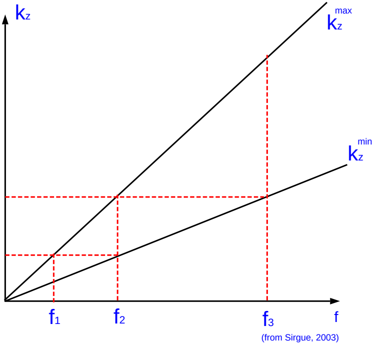
Efficient multiscale frequency-domain FWI algorithm

- FWI is based on diffraction tomography and acts as a spatial inverse Fourier transform Devaney (1982)
- Spatial resolution of FWI (*cf.* Sirgue and Pratt, 2004)

$$\vec{\mathbf{k}} = \frac{2\omega}{c_0} \cos\left(\theta/2\right) \vec{\mathbf{n}}, \qquad (1)$$





Efficient multiscale frequency-domain FWI algorithm

Efficient frequency-domain FWI algorithms use only few discrete frequencies from low to high frequencies for wide aperture acquisitions (Sirgue and Pratt, 2004; Brenders and Pratt, 2007a)

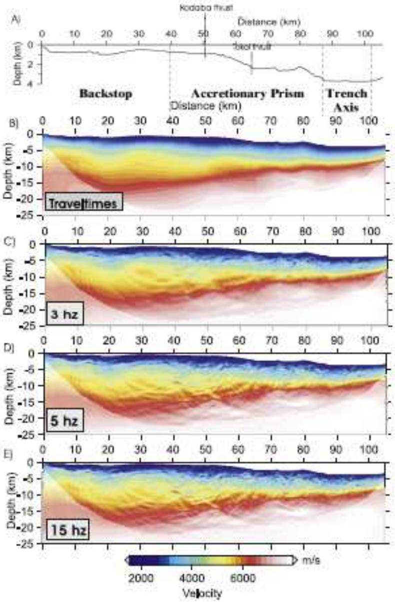
- multiscale approach and mitigation of non-linearities
- efficiency

Algorithm

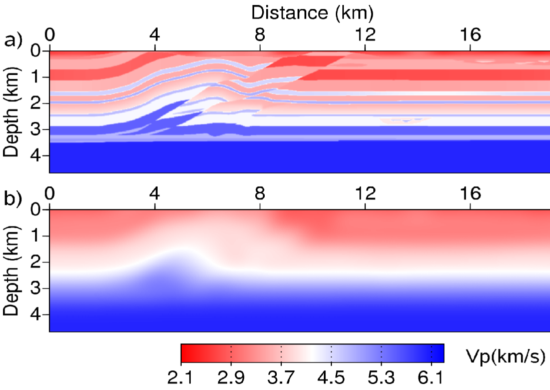
1: **for** *frequency* = *frequency_1* to *frequency_n* **do**
2:    **while** (NOT *convergence* AND $iter < niter_{max}$) **do**
3:       Build gradient vector $\mathcal{G}_{\mathbf{m}}^{(k)}$
4:       Build perturbation vector $\delta\mathbf{m}$
5:       Update model $\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} + \alpha\delta\mathbf{m}$
6:    **end while**
7: **end for**

Efficient multiscale frequency-domain FWI algorithm

Crustal imaging on the Nankai thrust, Japan (Operto et al., 2006)



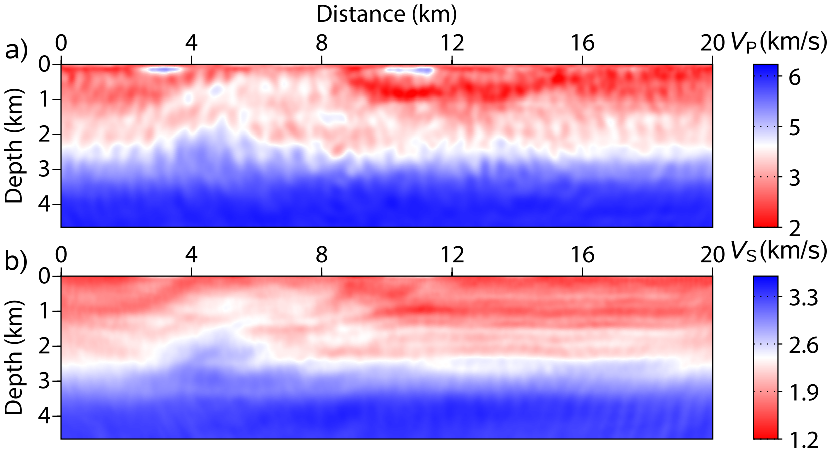
Overthrust model and acquisition set up



- Constant Poisson ratio (0.24), constant density
- 199 explosive sources 25 m below free surface
- 198 receivers recording horizontal and vertical components of velocity

Test1 : Raw data inversion (all arrivals). Successive inversion of single frequencies without data damping

5 frequency groups : [1.7], [2.5], [3.5], [4.7], [7.2] Hz
No time damping preconditioning

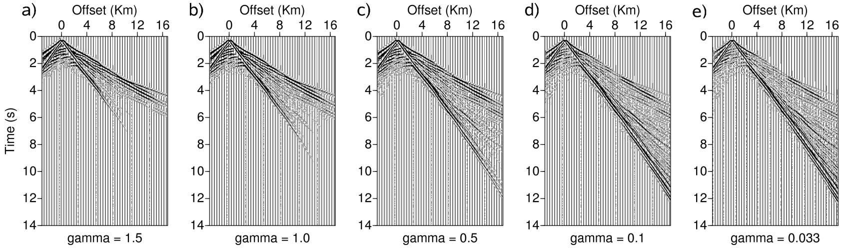


Efficient multiscale frequency-domain FWI algorithm

Complex frequency preconditioning allows to select arrivals from the first arrival (Shin et al., 2002; Brenders and Pratt, 2007b).

- Remove complex late arrivals
- An heuristics to select apertures in the data

$$F(\omega + i\gamma)e^{\gamma t_0} = \int_{-\infty}^{+\infty} f(t)e^{-\gamma(t-t_0)}e^{-i\omega t}dt \tag{2}$$



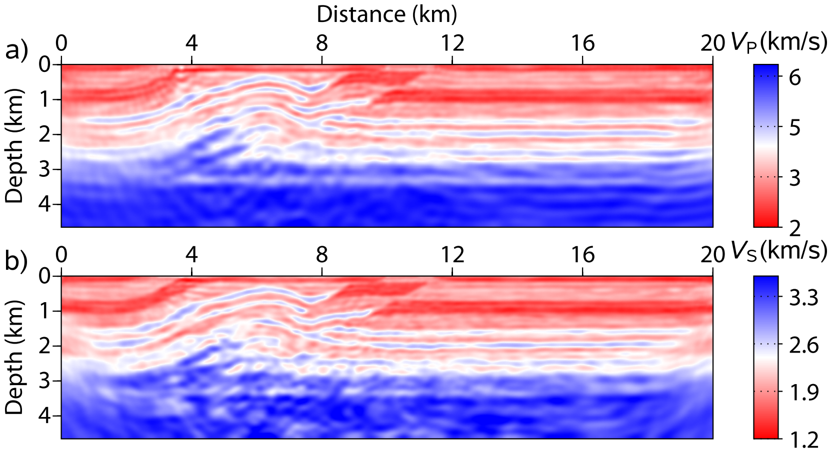
Efficient multiscale frequency-domain FWI algorithm

Two-levels hierarchical algorithm (Brossier et al., 2009)

for *frequency* = *frequency_1* to *frequency_n* **do**
  for *data_damping* = $high_{damping}$ to $low_{damping}$ **do**
    while (NOT *convergence* AND $iter < niter_{max}$) **do**
      Build gradient vector $\mathcal{G}_{\mathbf{m}}^{(k)}$
      Build perturbation vector $\delta\mathbf{m}$
      Update model $\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} + \alpha\delta\mathbf{m}$
    end while
  end for
end for

Test2 : Successive frequencies, preconditioned data

5 frequency groups : [1.7], [2.5], [3.5], [4.7], [7.2] Hz
5 damping factors per group : $\gamma = 1.5,\ 1,\ 0.5,\ 0.1,\ 0.033$



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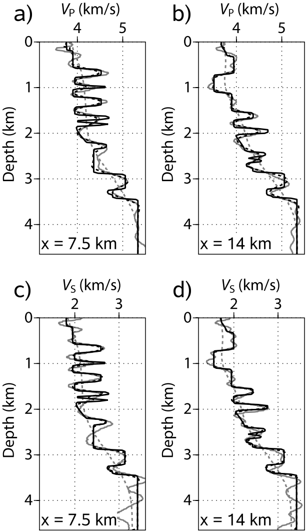
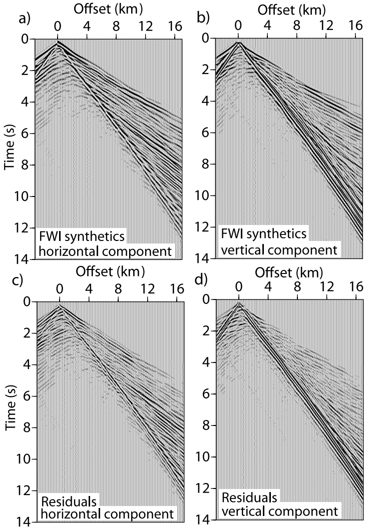
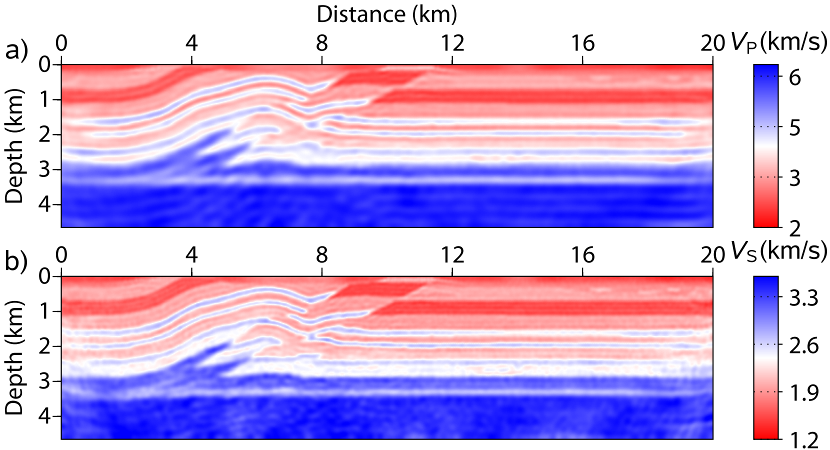




Test3 : Successive frequencies, preconditioned data without free surface

5 frequency groups : [1.7], [2.5], [3.5], [4.7], [7.2] Hz
5 damping factors per group : $\gamma = 1.5,\ 1,\ 0.5,\ 0.1,\ 0.033$



Test2 : Successive frequencies, preconditioned data

5 frequency groups : [1.7], [2.5], [3.5], [4.7], [7.2] Hz
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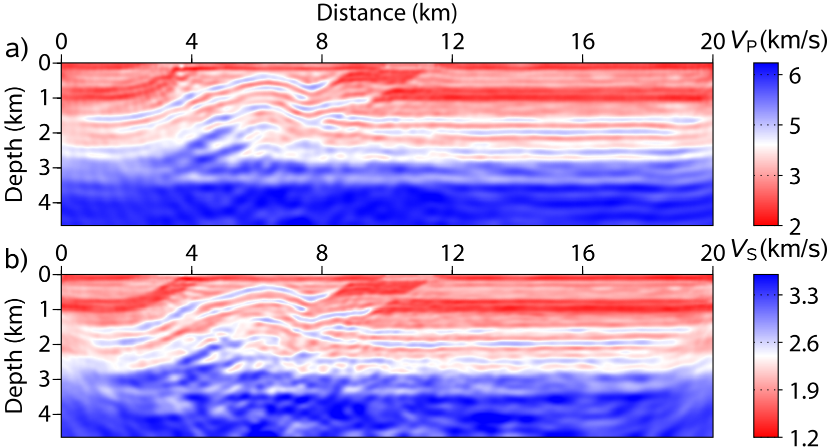
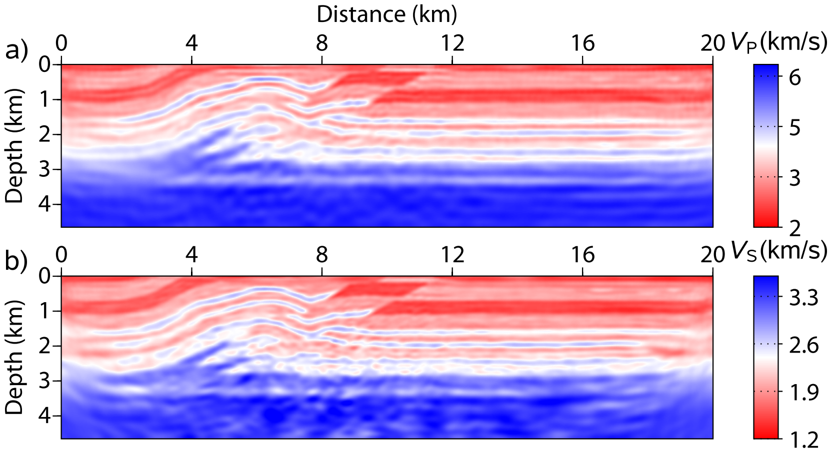


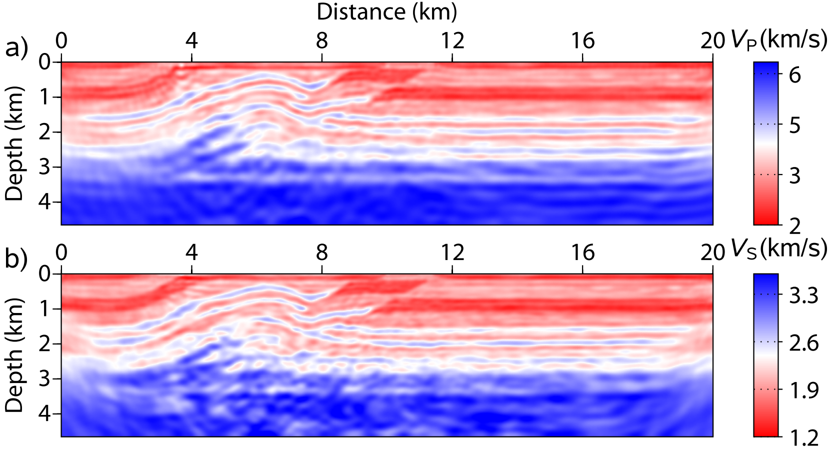
Test4 : simultaneous inversion approach, preconditioned data

2 frequency groups : $[1.7, 2.5, 3.5], [3.5, 4.7, 7.2]$ Hz
5 damping factors per group : $\gamma = 1.5,\ 1,\ 0.5,\ 0.1,\ 0.033$

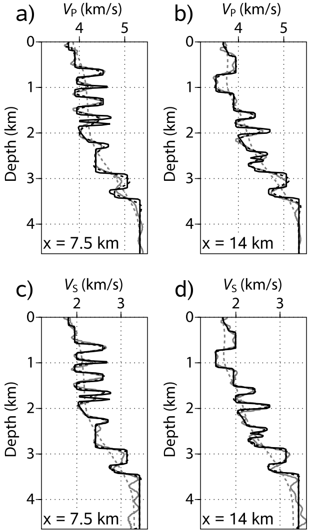


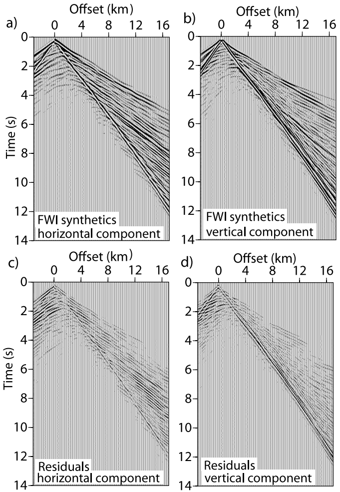
Test2 : Successive frequencies, preconditioned data

5 frequency groups : [1.7], [2.5], [3.5], [4.7], [7.2] Hz
5 damping factors per group : $\gamma = 1.5,\ 1,\ 0.5,\ 0.1,\ 0.033$



Test4 : simultaneous inversion approach, preconditioned data (5 factors)

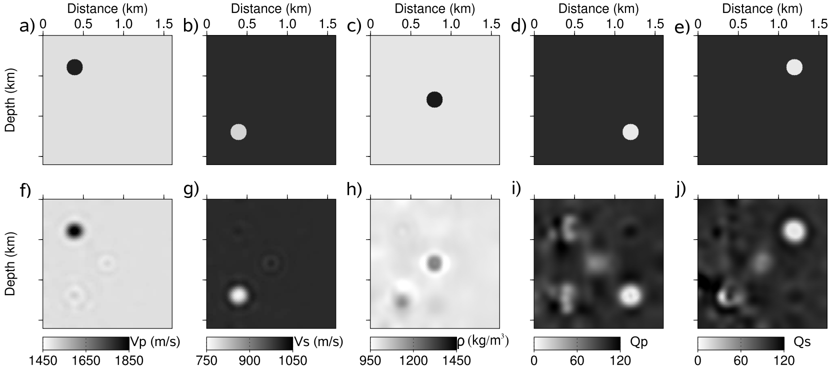




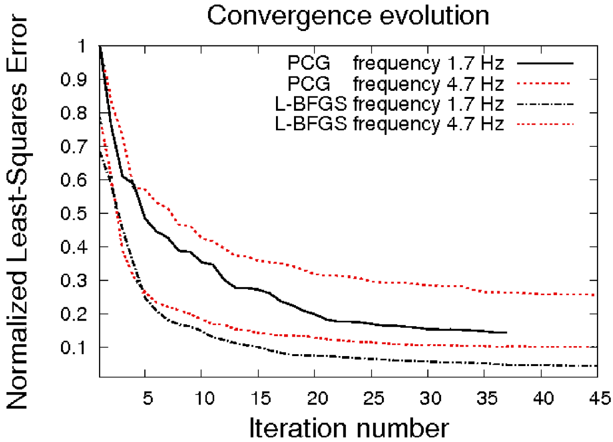
Outline

Optimization method : canonical example

- Reconstruction of 5 parameters in a canonical configuration.
  - Lamé parameters : λ and μ : $\approx \mathcal{O}(10^8 - 10^9)$
  - Density : ρ : $\approx \mathcal{O}(10^3)$
  - Attenuations : Q_p et Q_s : $\approx \mathcal{O}(10^1 - 10^2)$
- L-BFGS algorithm and a single steplength



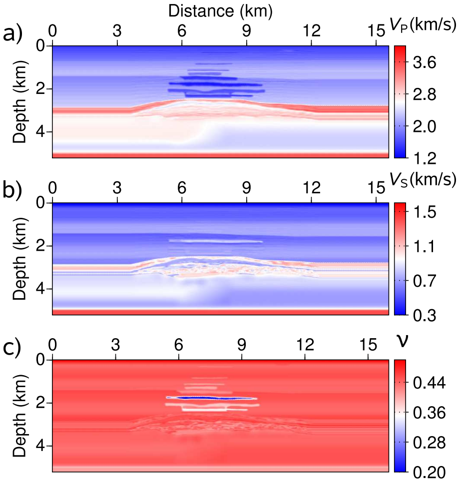
L-BFGS vs Preconditioned Conjugate Gradient



Outline

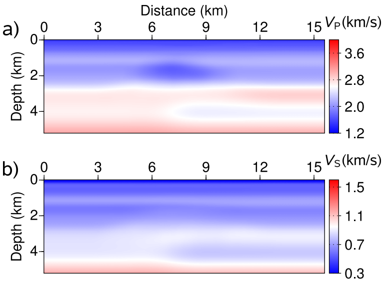
Offshore synthetic Valhall model

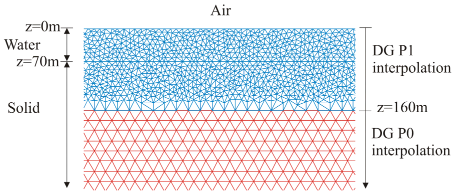
- Representative of Oil & Gas Field in North Sea (Munns, 1985)
- Shallow water (70 m) and soft seabed environment
- The acquisition mimics a permanent OBC survey (Kommedal et al., 2004)
  - 320 explosive sources each 50m, 6 m below water surface
  - 320 3-Components sensors each 50 m, located on the sea floor



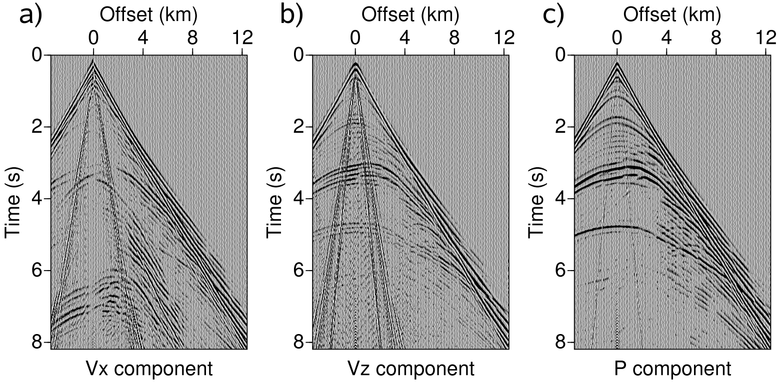
Inversion set up

- 5 frequencies inverted sequentially
  2, 3, 4, 5 and 6 Hz
- 3 time-damping factors γ used sequentially for each frequency
  $\gamma = 2, 0.33, 0.1\ s^{-1}$

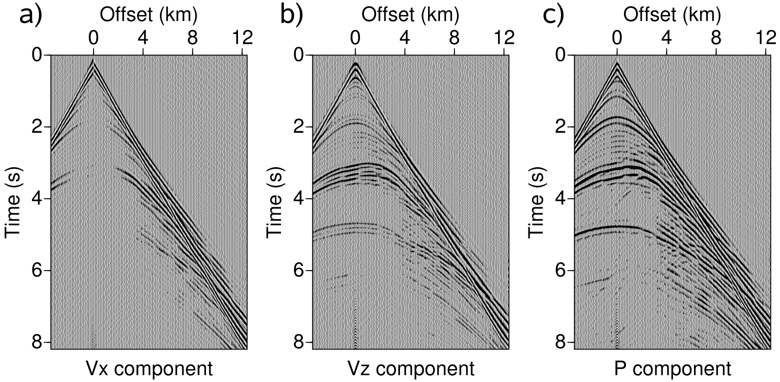




Data comparison : Elastic data



Data comparison : Acoustic data



2-steps inversion results

Hierarchical inversion procedure (closed from Sears et al., 2008, developed for time-domain FWI)

1. Inversion of V_P from hydrophone data (acoustic-like inversion)
2. Joint inversion of V_P and V_S from geophone data

2-steps inversion results

Hierarchical inversion procedure (closed from Sears et al., 2008, developed for time-domain FWI)

1. Inversion of V_P from hydrophone data (acoustic-like inversion)
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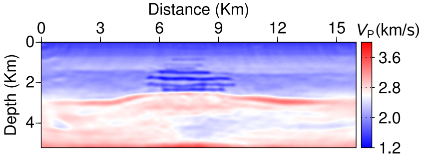

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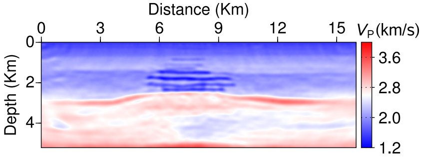

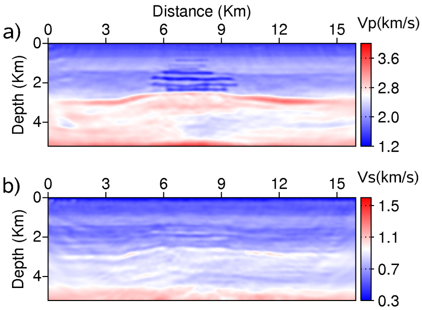

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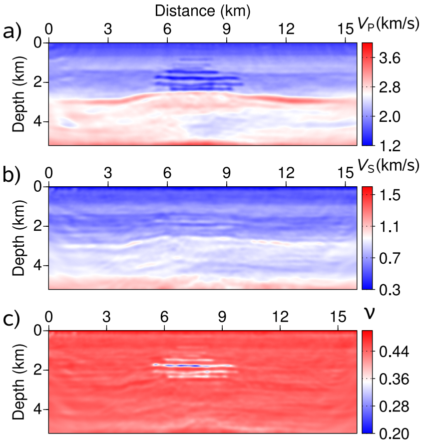
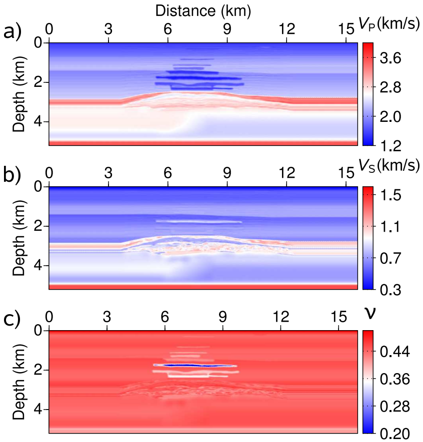


Reconstructed models



True models

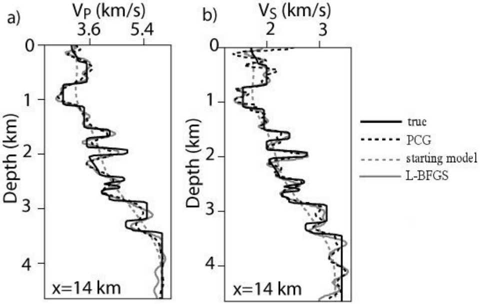
Outline

Conclusion

- Elastic multiparameter reconstruction from FWI is a highly non-linear problem
  - Effects of free surface in onshore environments
  - Low V_S signature in offshore soft-seabed environments
- Preconditioning strategies and optimal optimization are required
  - Hierarchy on frequency content of data : low to high frequency
  - Hierarchy on time-windowing through frequency-domain damping : progressive introduction of complex phases
  - Hierarchy on data components depending on parameter signatures in data
  - Hessian information through L-BFGS optimization : deconvolution of gradient and scaling

L-BFGS vs Preconditioned Conjugate Gradient

Example of vertical profiles for Overthrust Test3 : Successive frequencies.
L-BFGS improves focusing of structure and stabilize subsurface



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