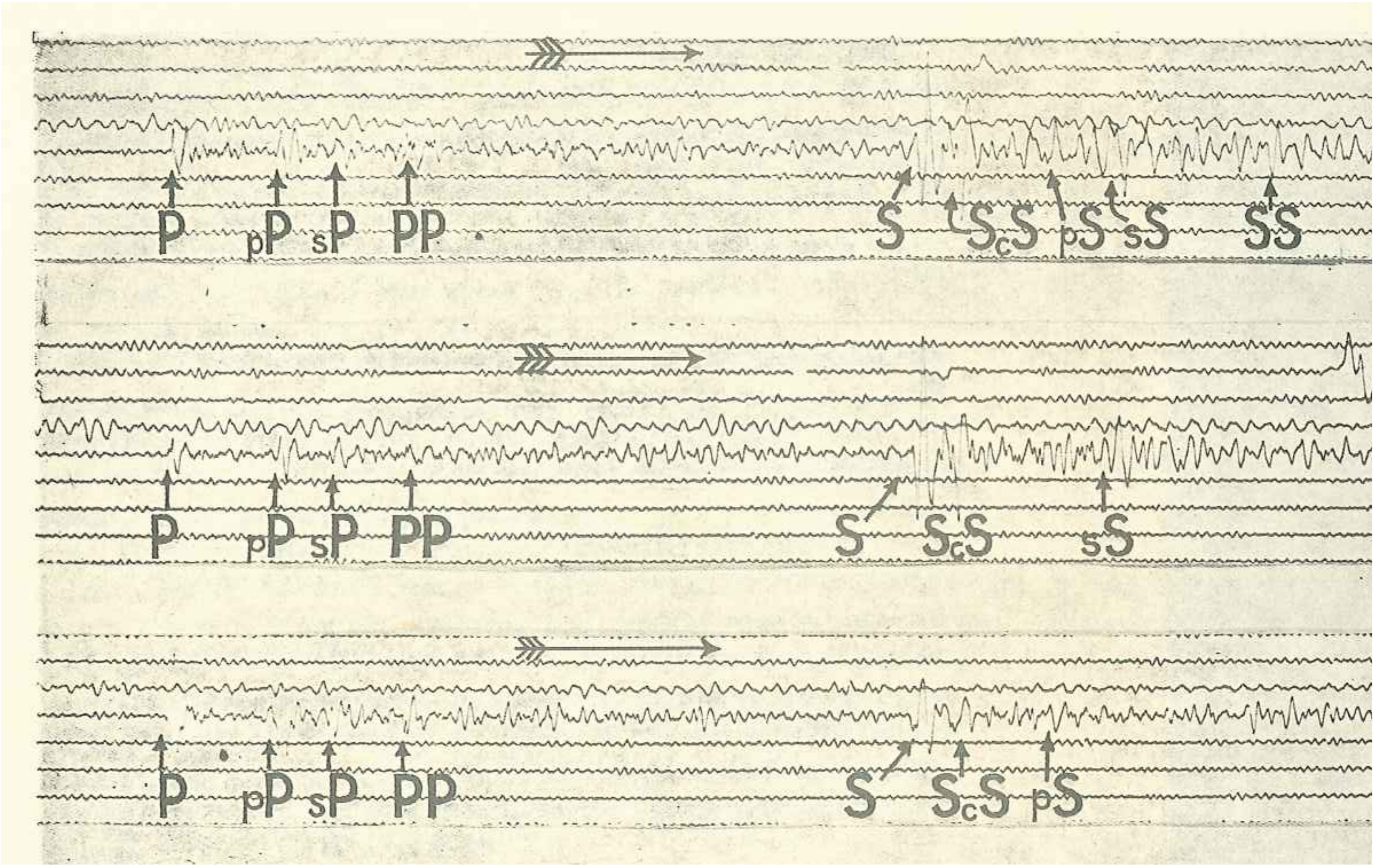
The trouble with travel times

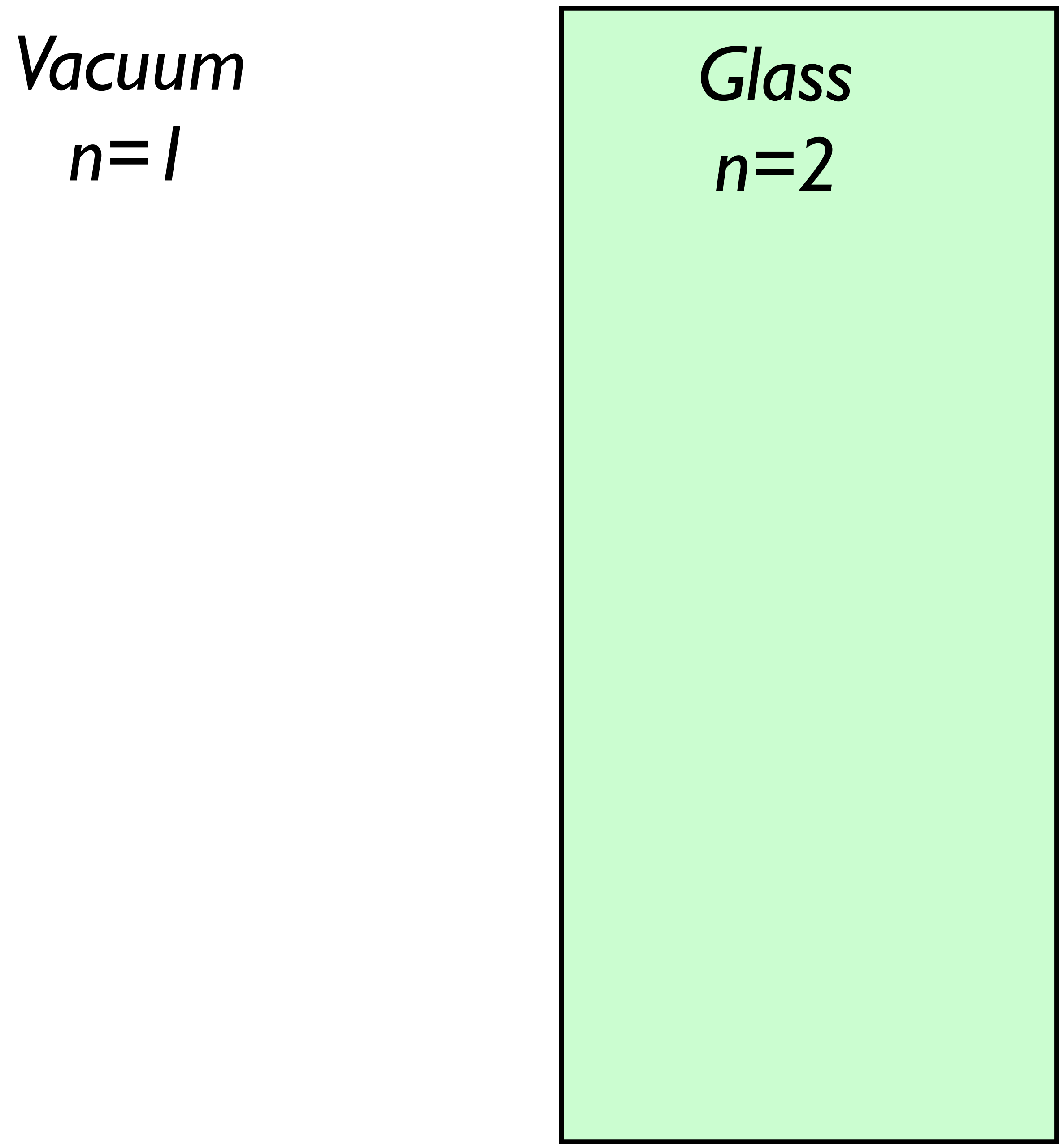
Guust Nolet
GeoAzur

onset time



From Jeffreys, "The Earth"

The speed of a photon in glass



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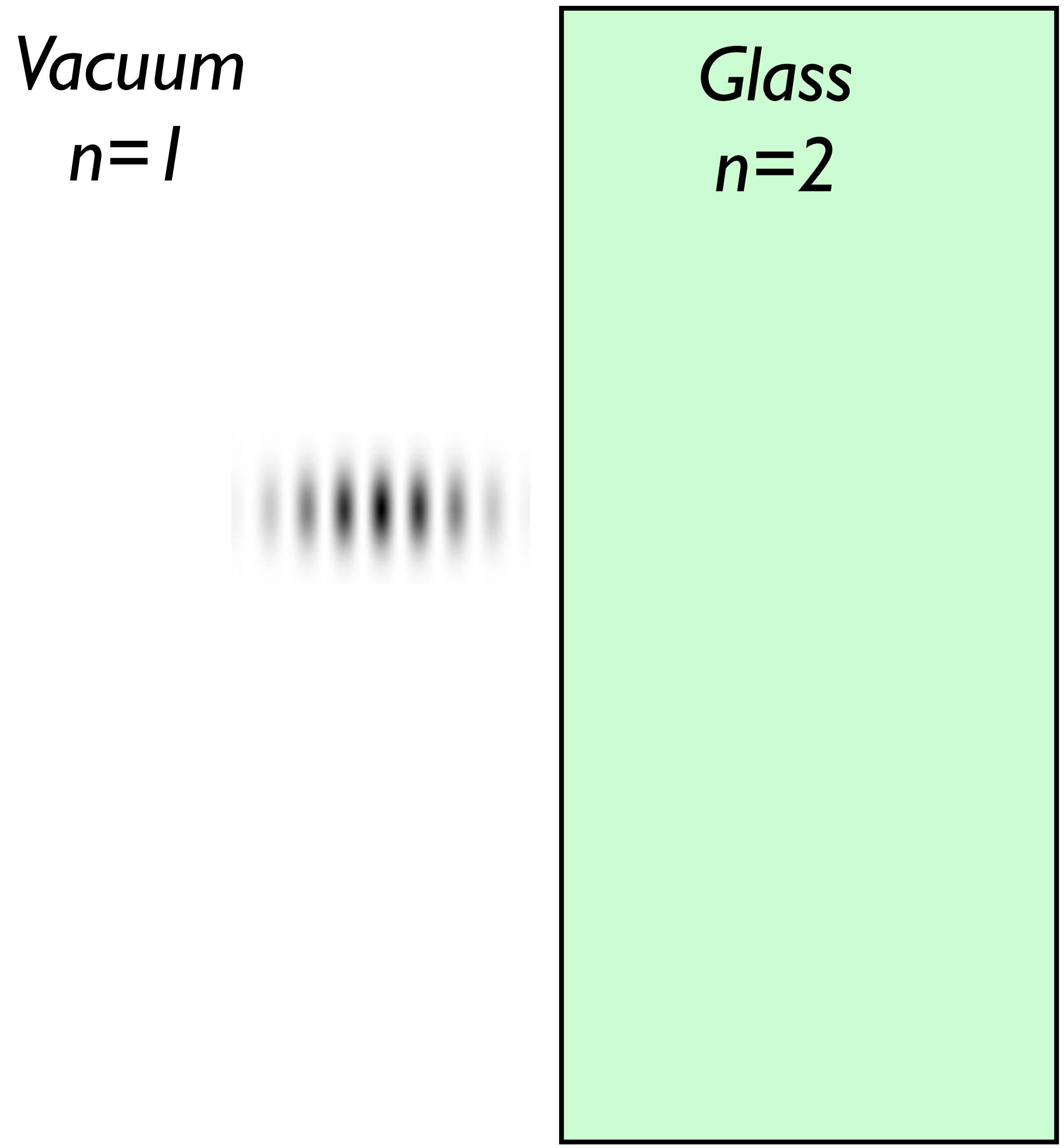
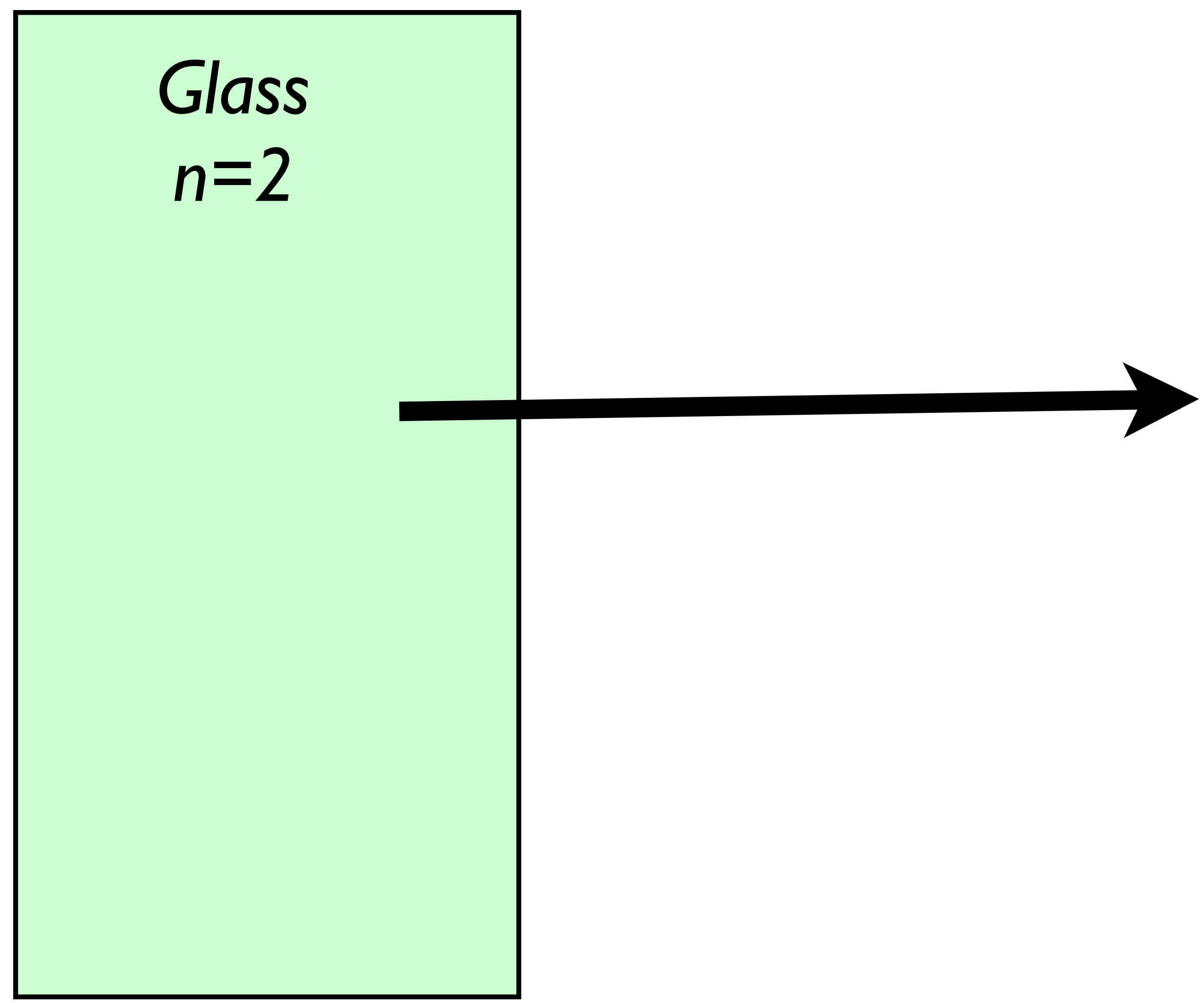


The photon speed

(a) v=c
(b) v=c/2
(c) don't know



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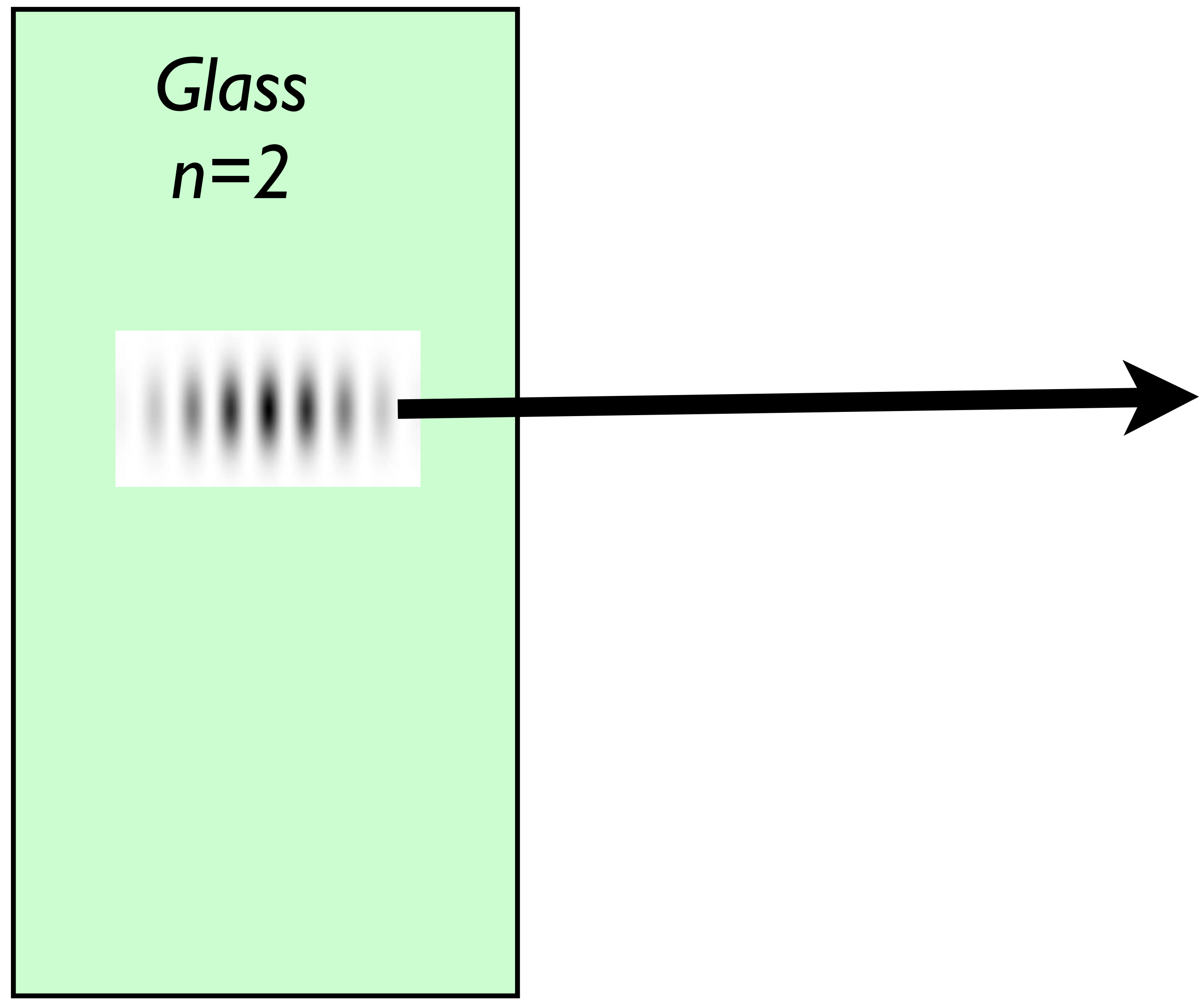
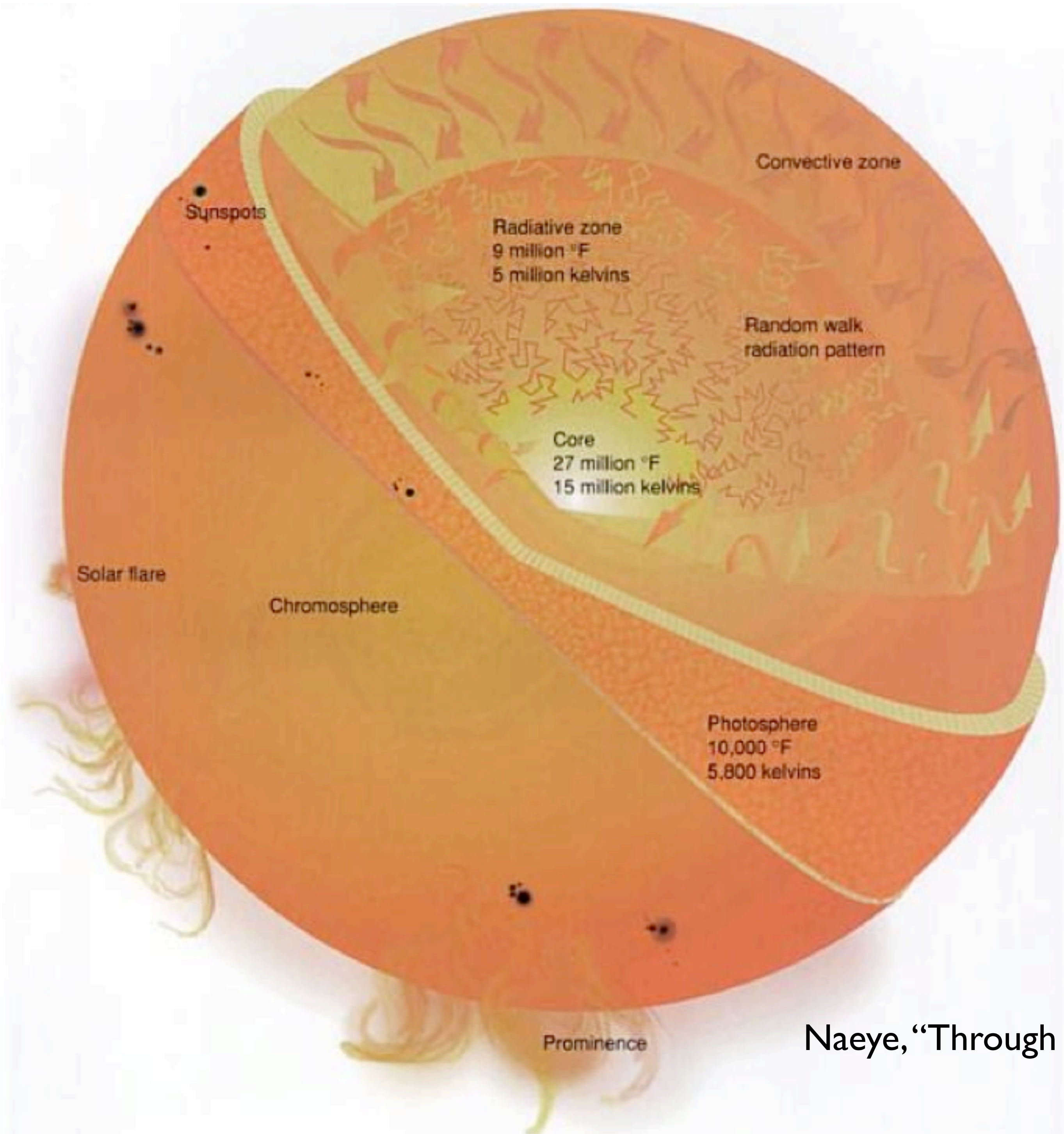
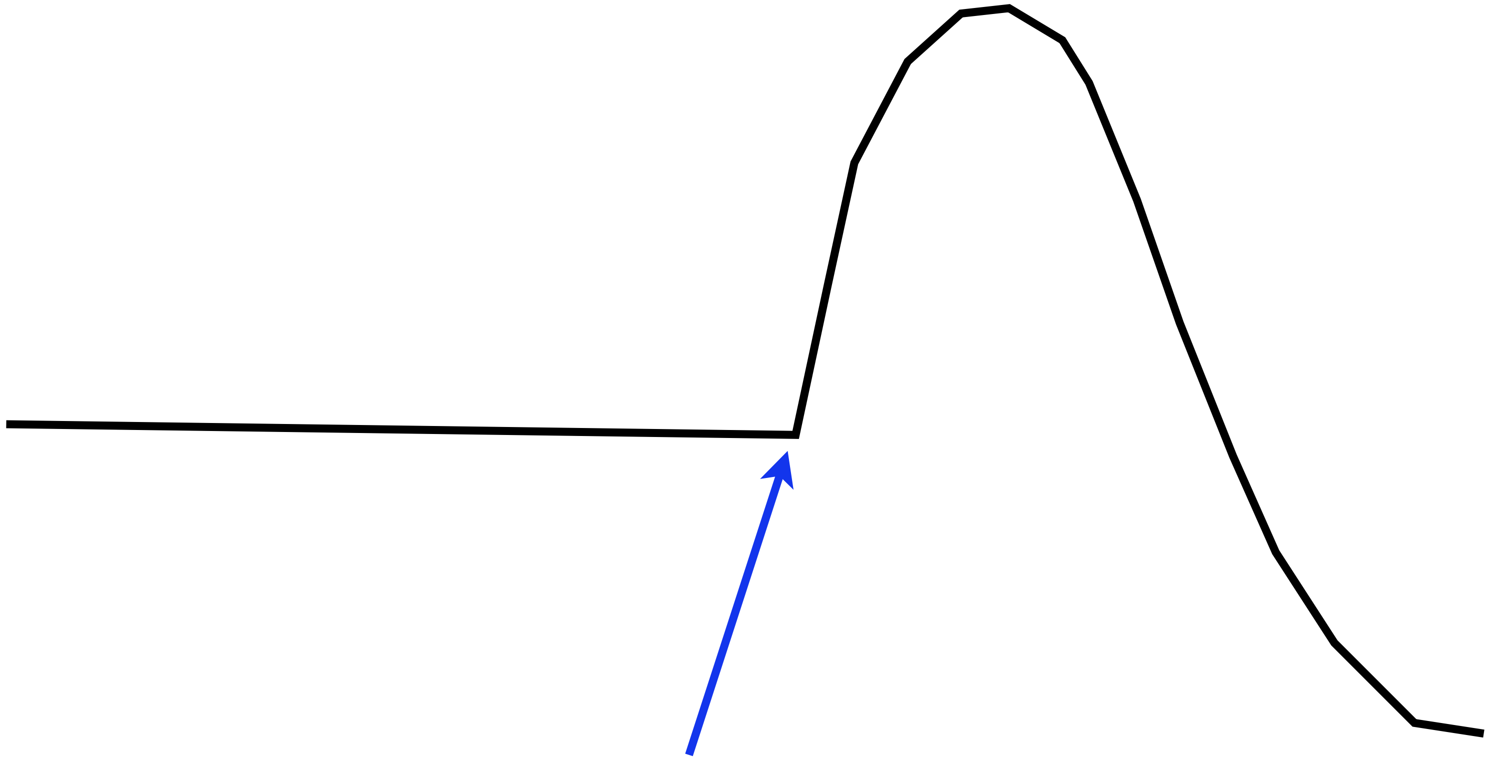


A photon in the Sun



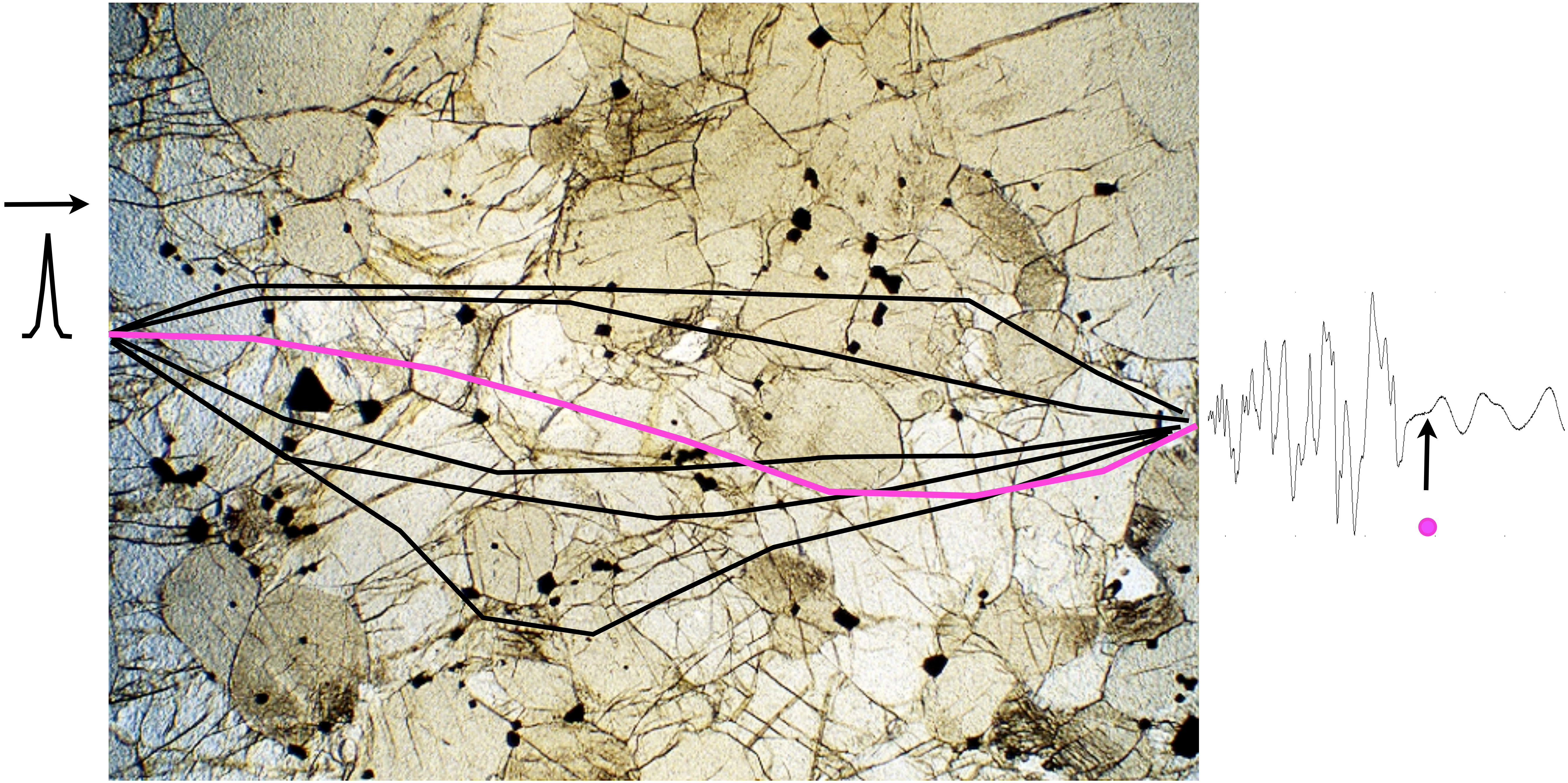
Naeye, "Through the eyes of Hubble" (CRC Press, 1998).

The trouble with onsets



- all frequencies arrive at same time (zero phase)
- no frequencies have been attenuated away
- (and we are not even talking about instrument response...)

3D: multipathed arrivals

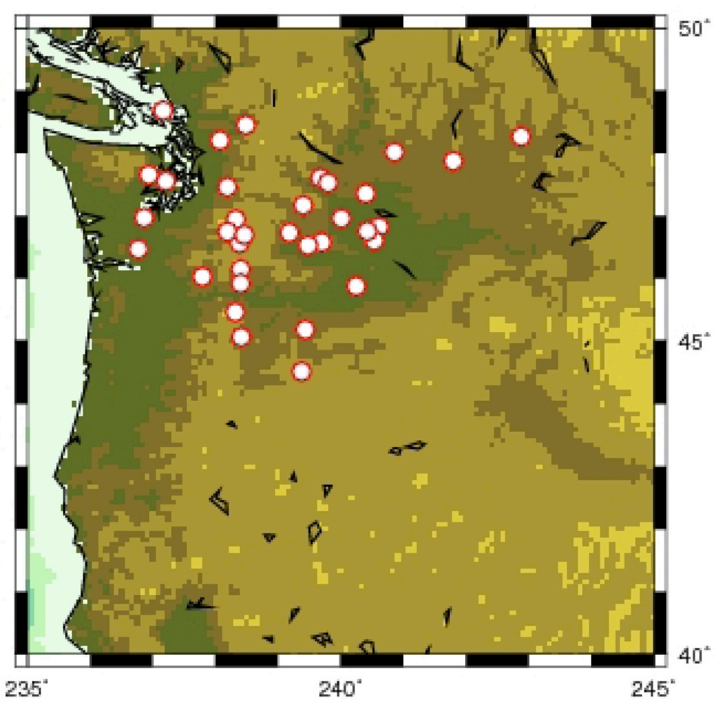


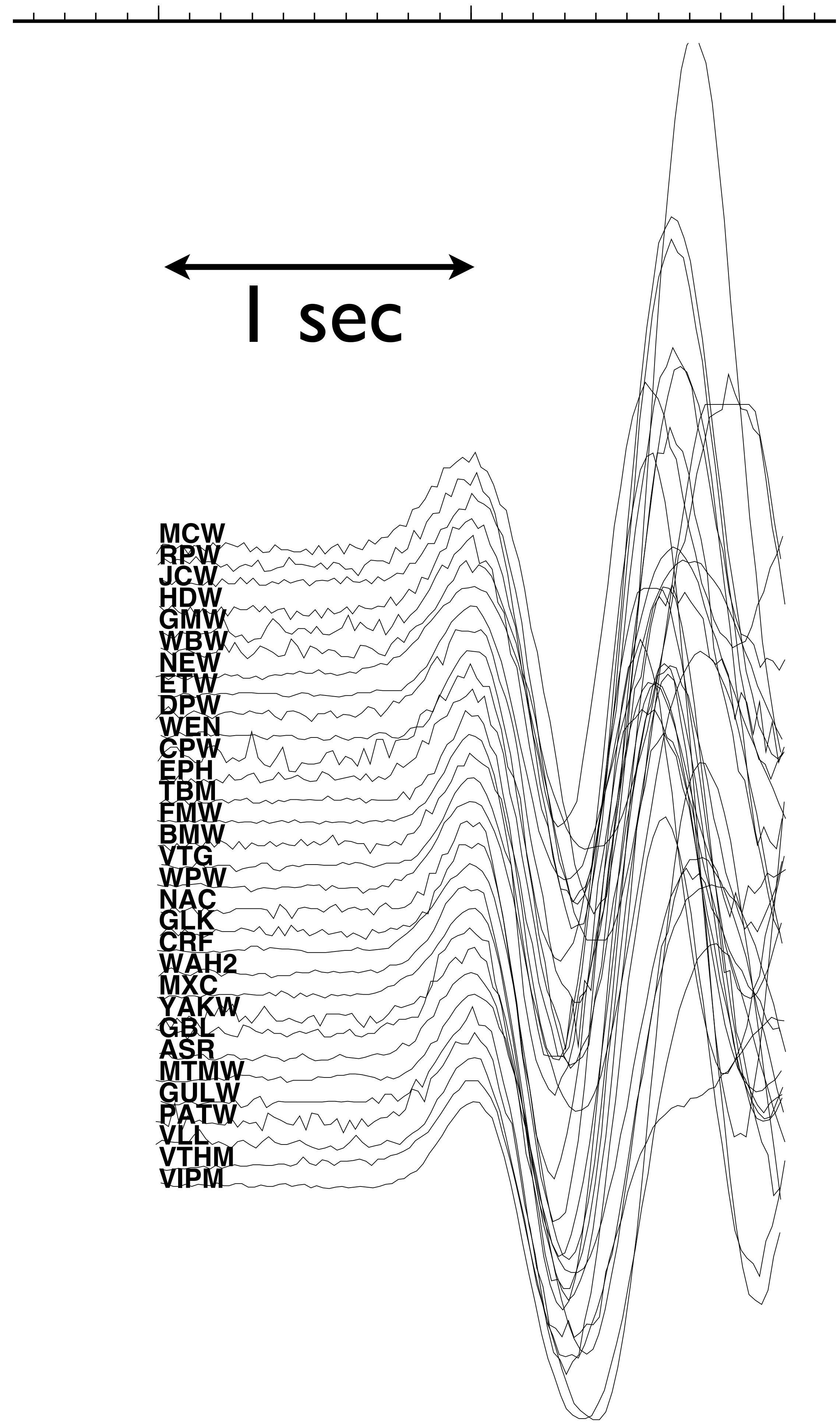
3D: Jeffreys' analysis from 1931 (!)

$$S(t) = \tfrac{1}{2}\left(1 + \mathrm{Erf}\,\frac{t'}{(2\tau t)^{\frac{1}{2}}}\right) \quad . \quad . \quad (19)$$

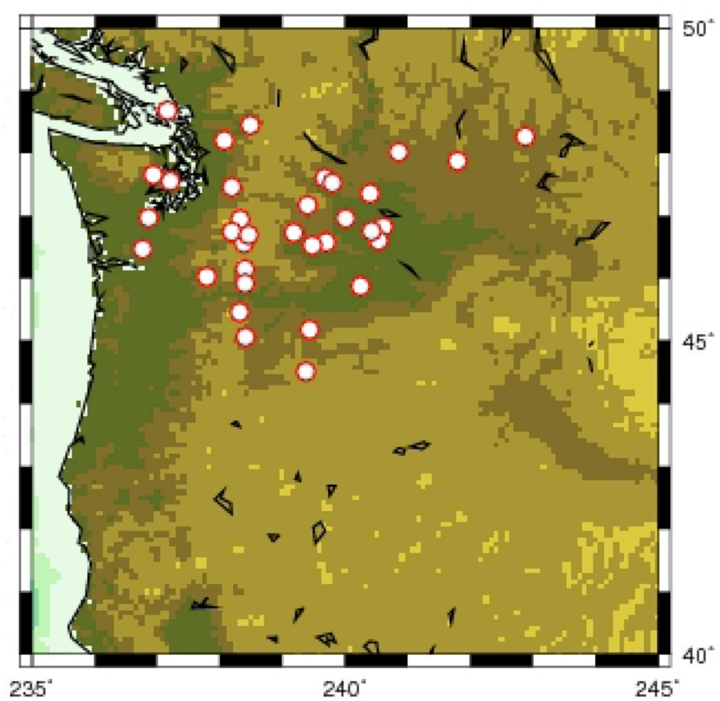
Thus the displacement, instead of being zero up to time x/c and suddenly jumping to unity, begins to be appreciable a little too early and continues to grow after time x/c ; the growth is distributed over an interval of order $(2\tau t)^{\frac{1}{2}}$.

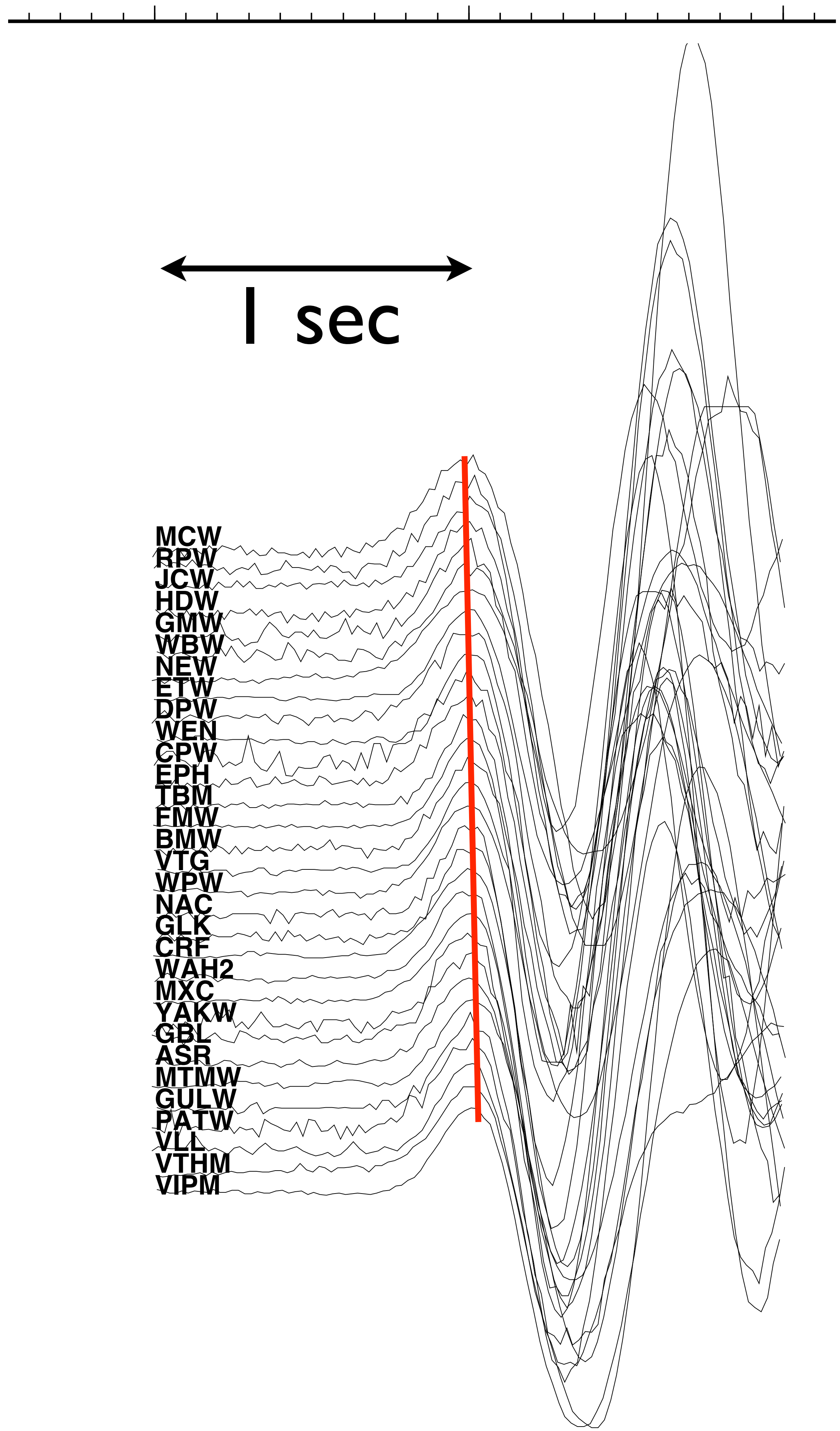
Nuclear source or δ(t)...



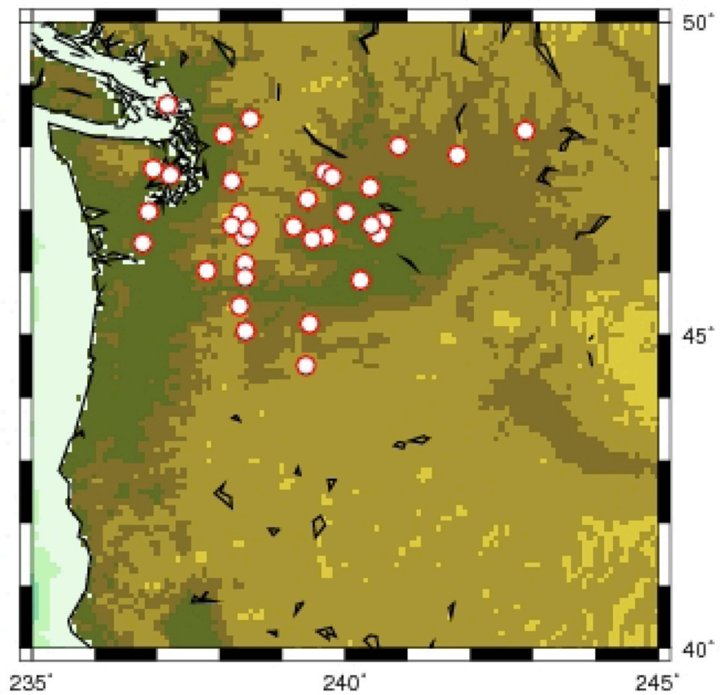


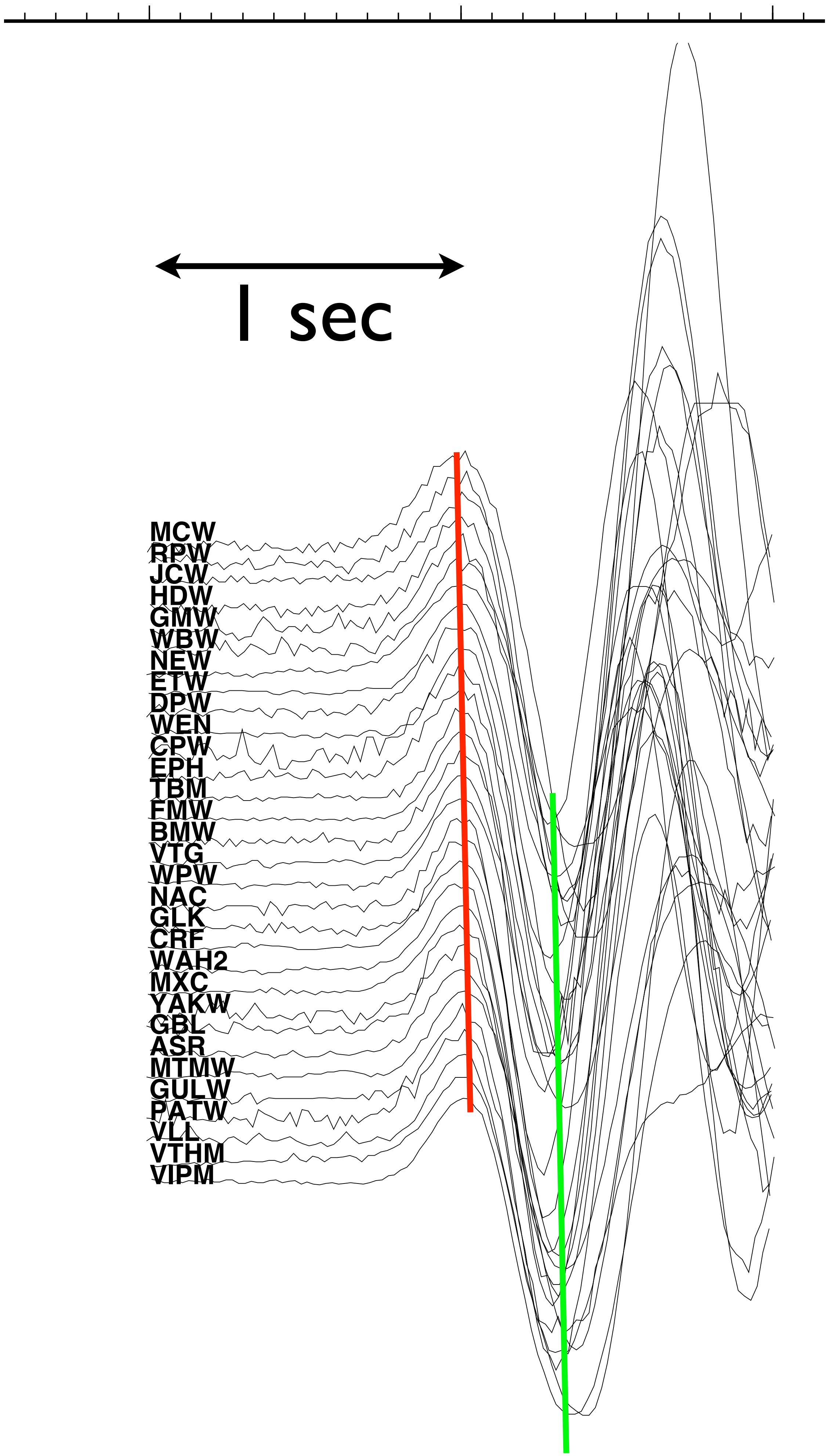
Nuclear source or δ(t)...





Nuclear source or δ(t)...





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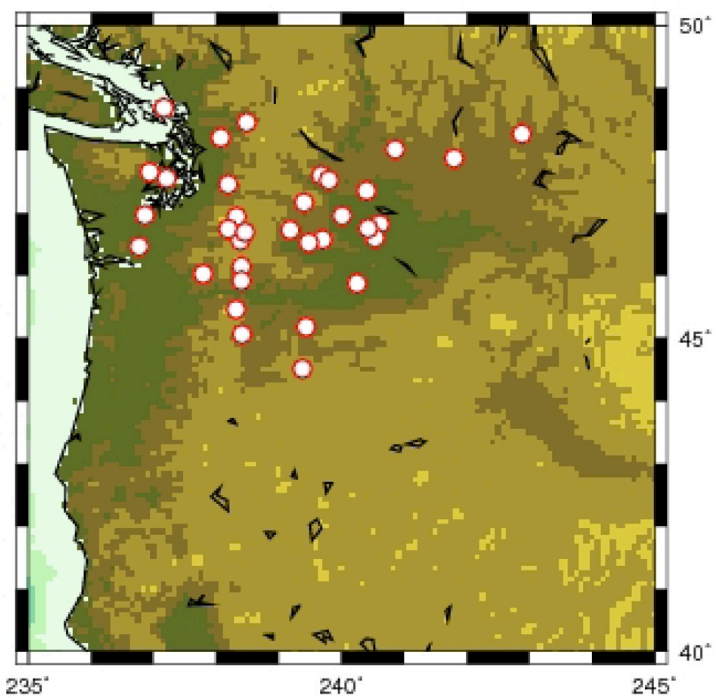
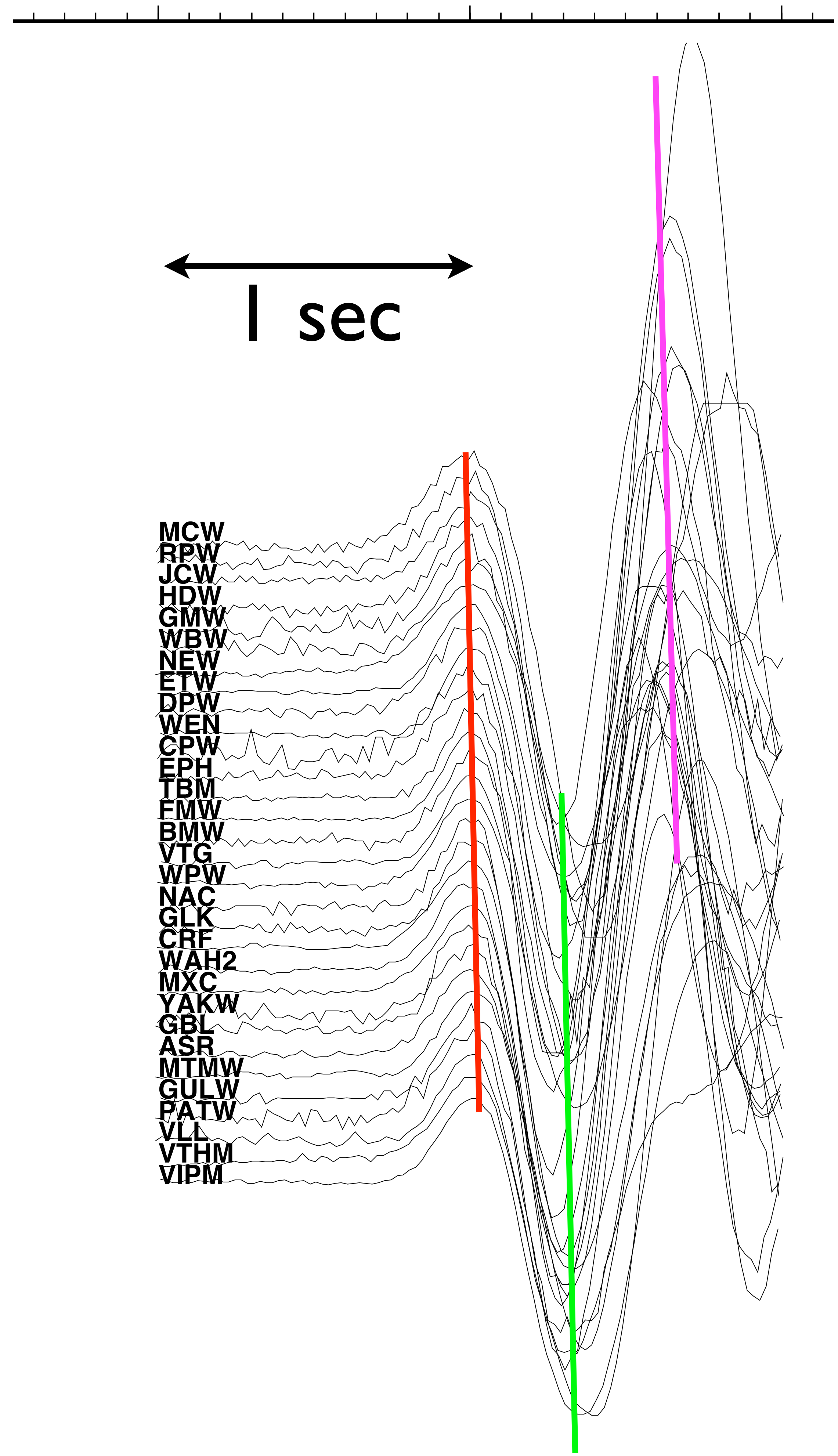
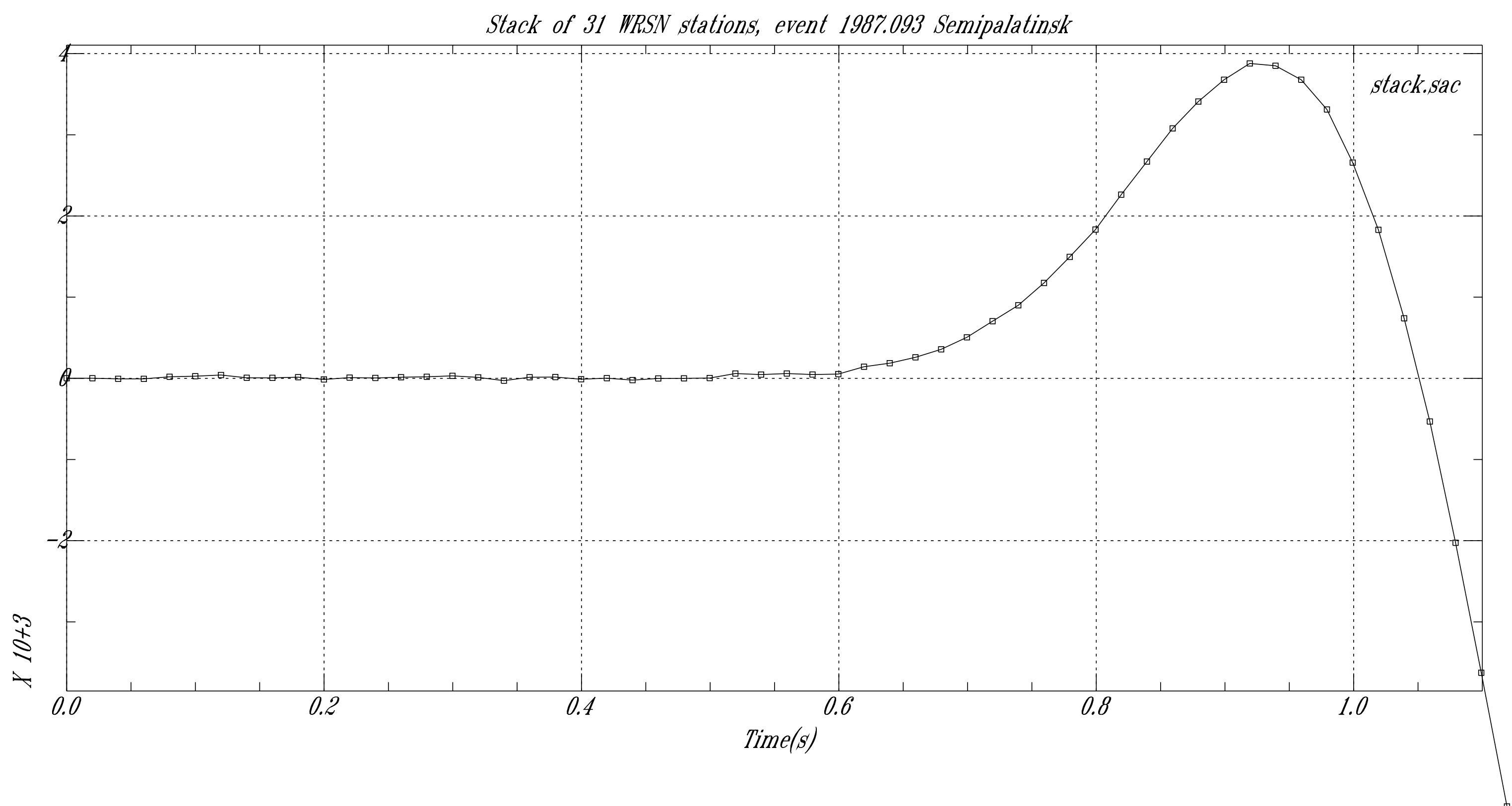


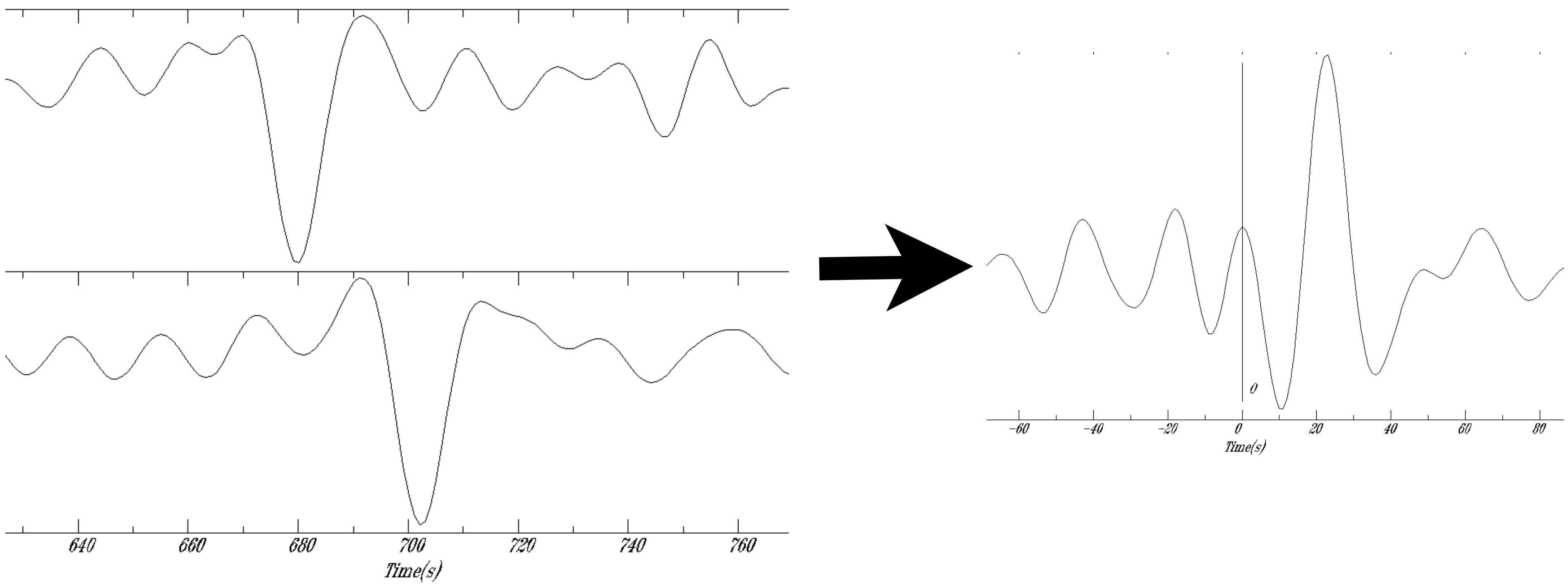


Picking the onset is at best ambiguous or inaccurate, sometimes impossible.



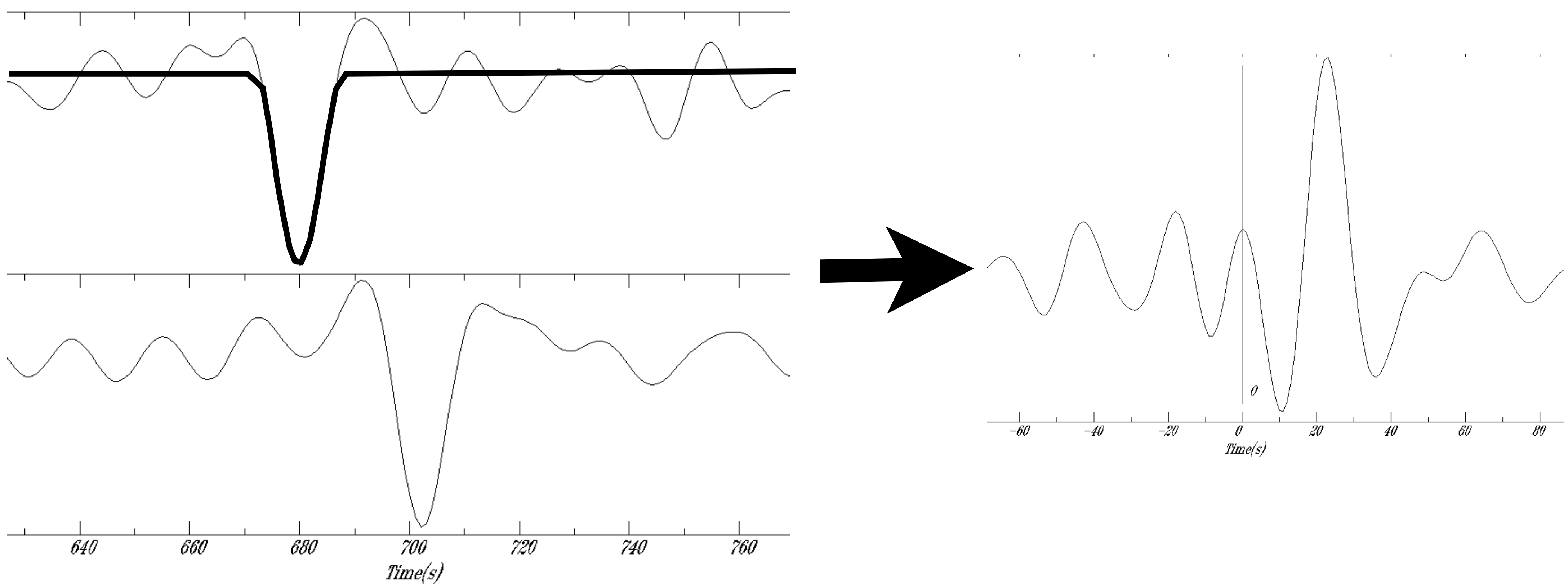
$$\frac{1}{2}\left(1 + \mathrm{Erf}\,\frac{t'}{(2\tau t)^{\frac{1}{2}}}\right) \quad . \qquad . \qquad (19)$$

cross-correlation



$$C_{uv}(t) = \int u(\tau)v(\tau - t)\ \mathrm{d}\tau$$

cross-correlation



$$C_{uv}(t) = \int u(\tau)v(\tau - t)\ \mathrm{d}\tau$$

But what arrival time are we measuring in this way?

Common definitions

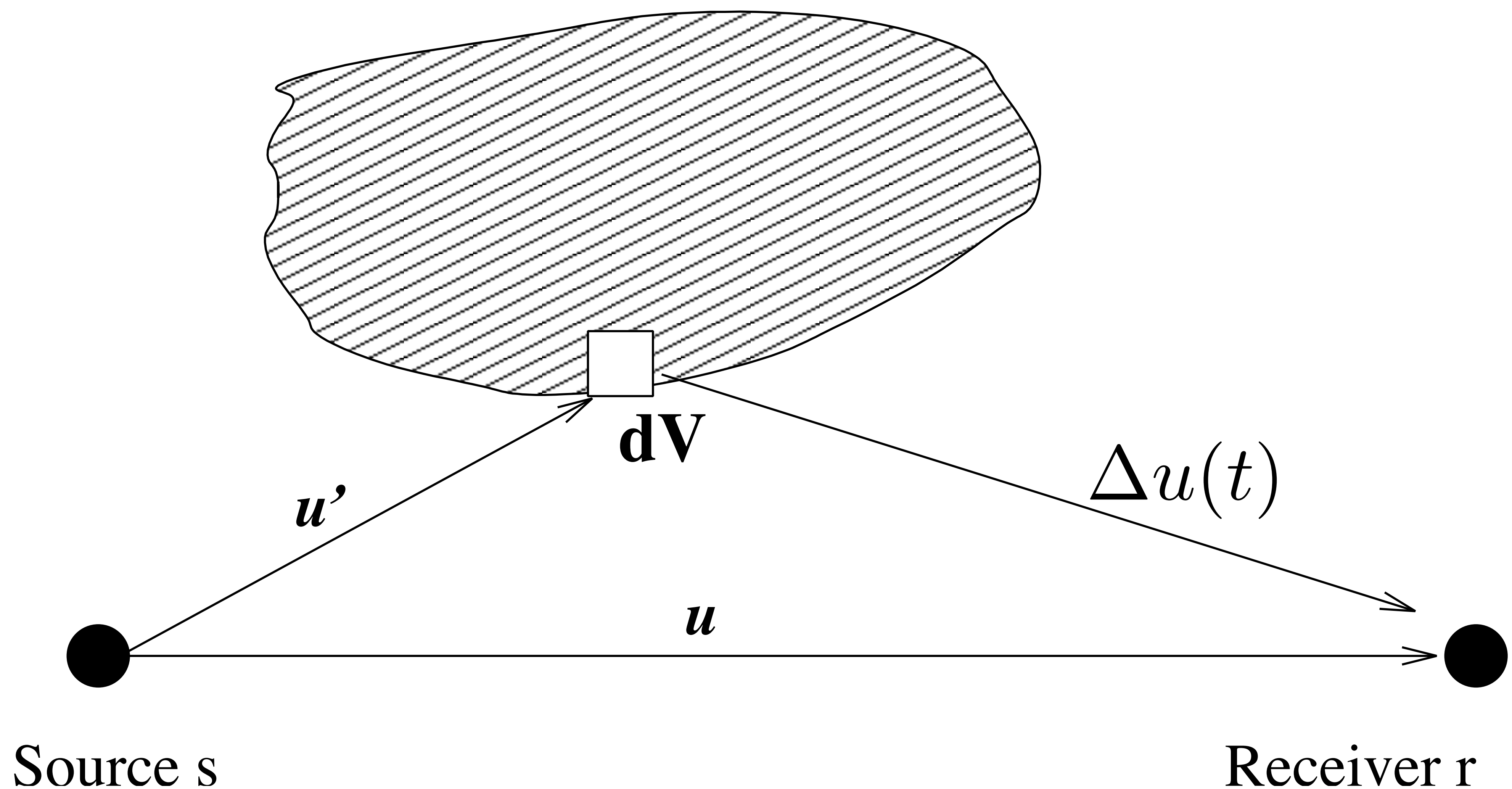
- Signal velocity
  ⇄time of earliest (observable) nonzero signal
- Phase velocity
  ⇄time of crest of a monochromatic wave
- Group velocity
  ⇄time of crest of the envelope of wave
- Energy velocity
  ⇄time of crest of the (kinetic) energy signal

None of these corresponds to x-correlations....

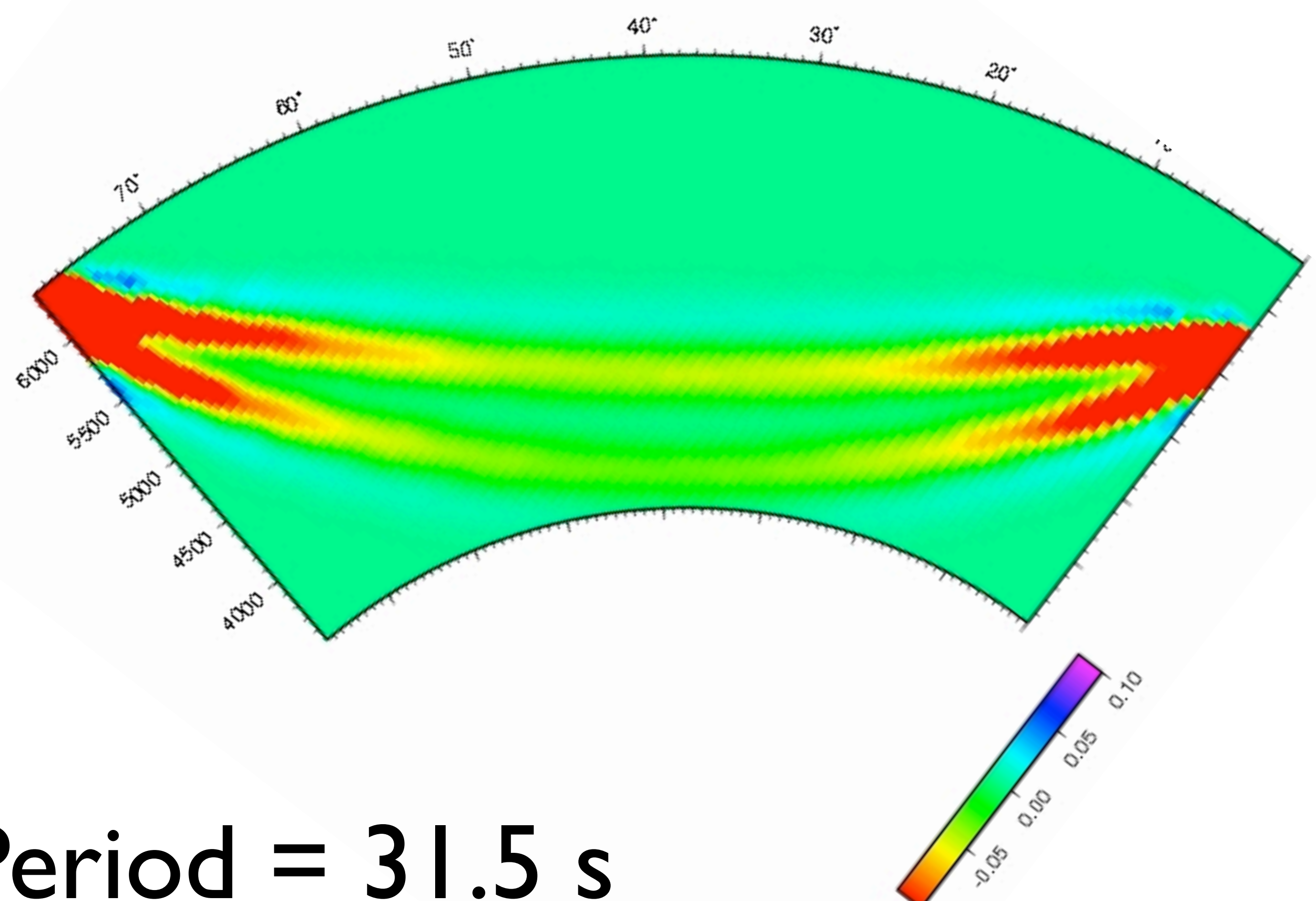
Problem: not just one path but single (forward) scattering

Multiple scattering = ill posed inverse problem

Born theory = first order scattering

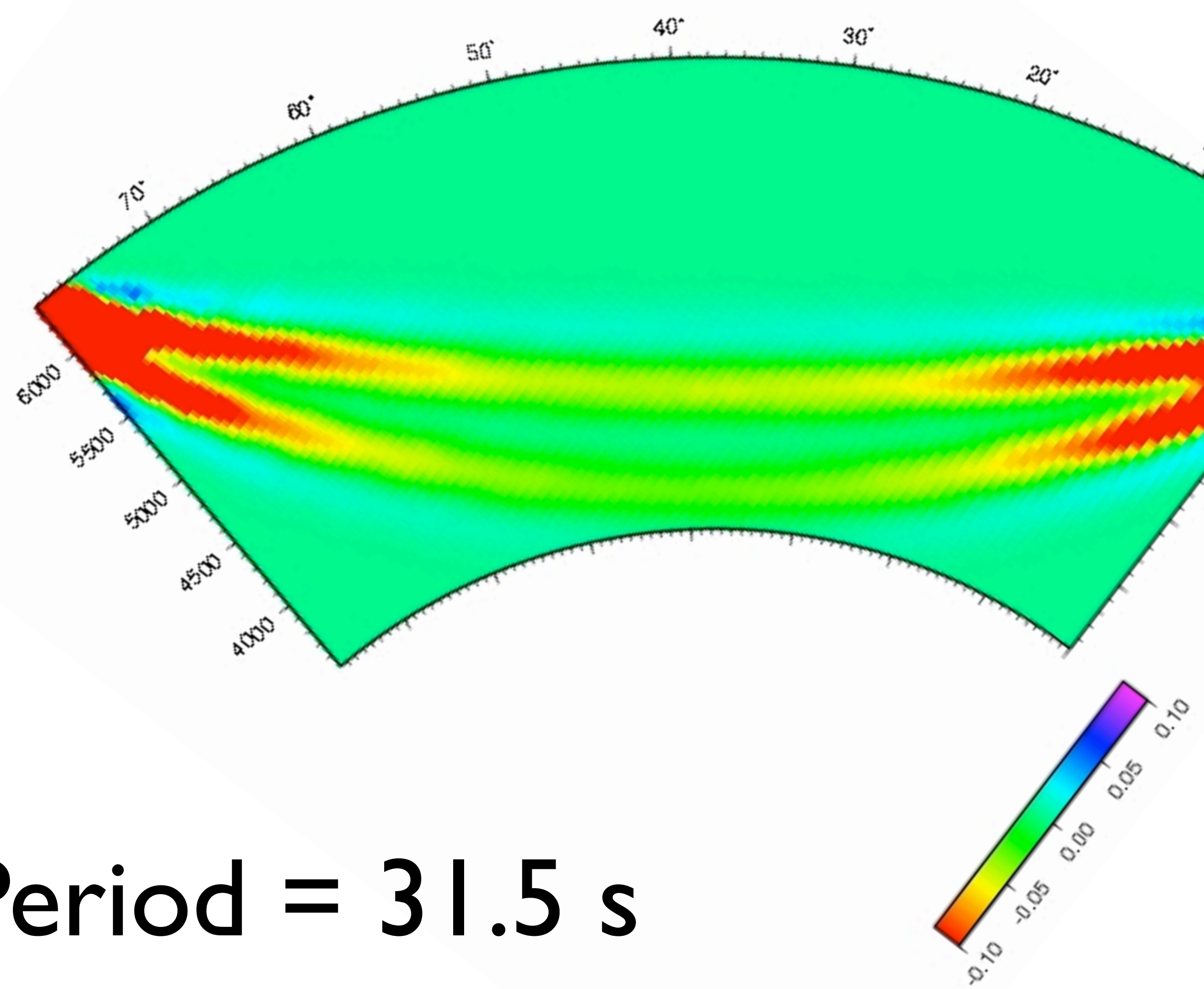


'Banana-doughnut' kernels



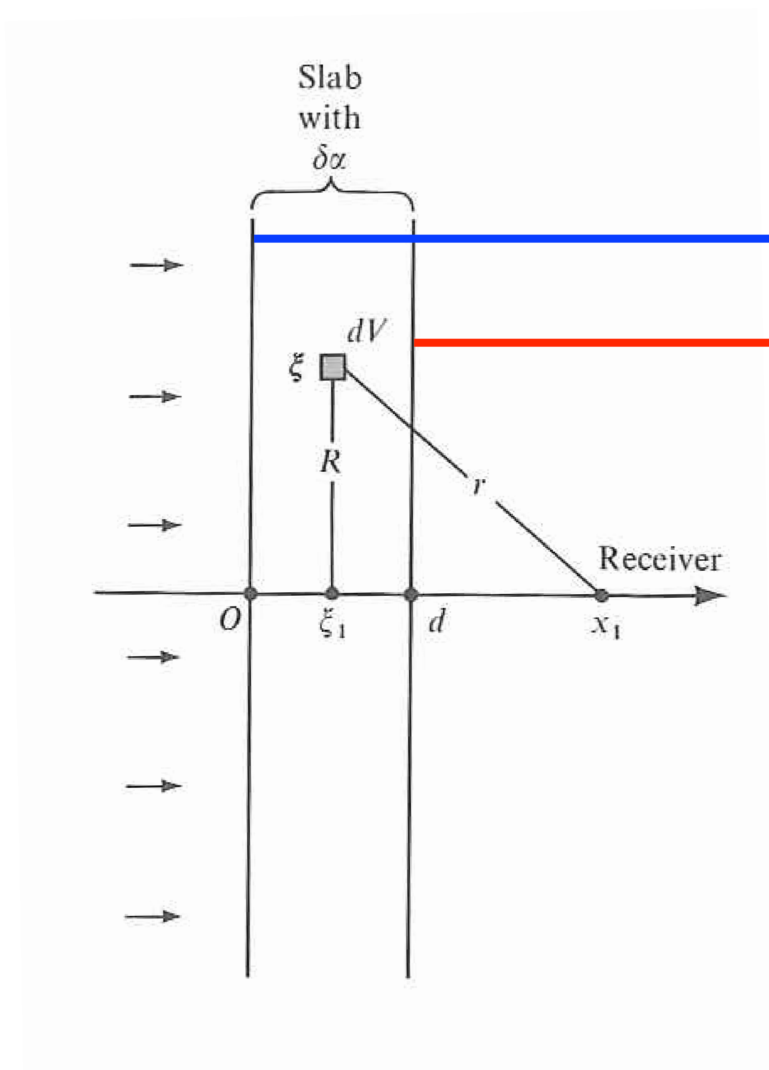
Period = 31.5 s

'Banana-doughnut' kernels



Period = 31.5 s

small δu and ray theory



Aki & Richards (1980 edition)

positive δu

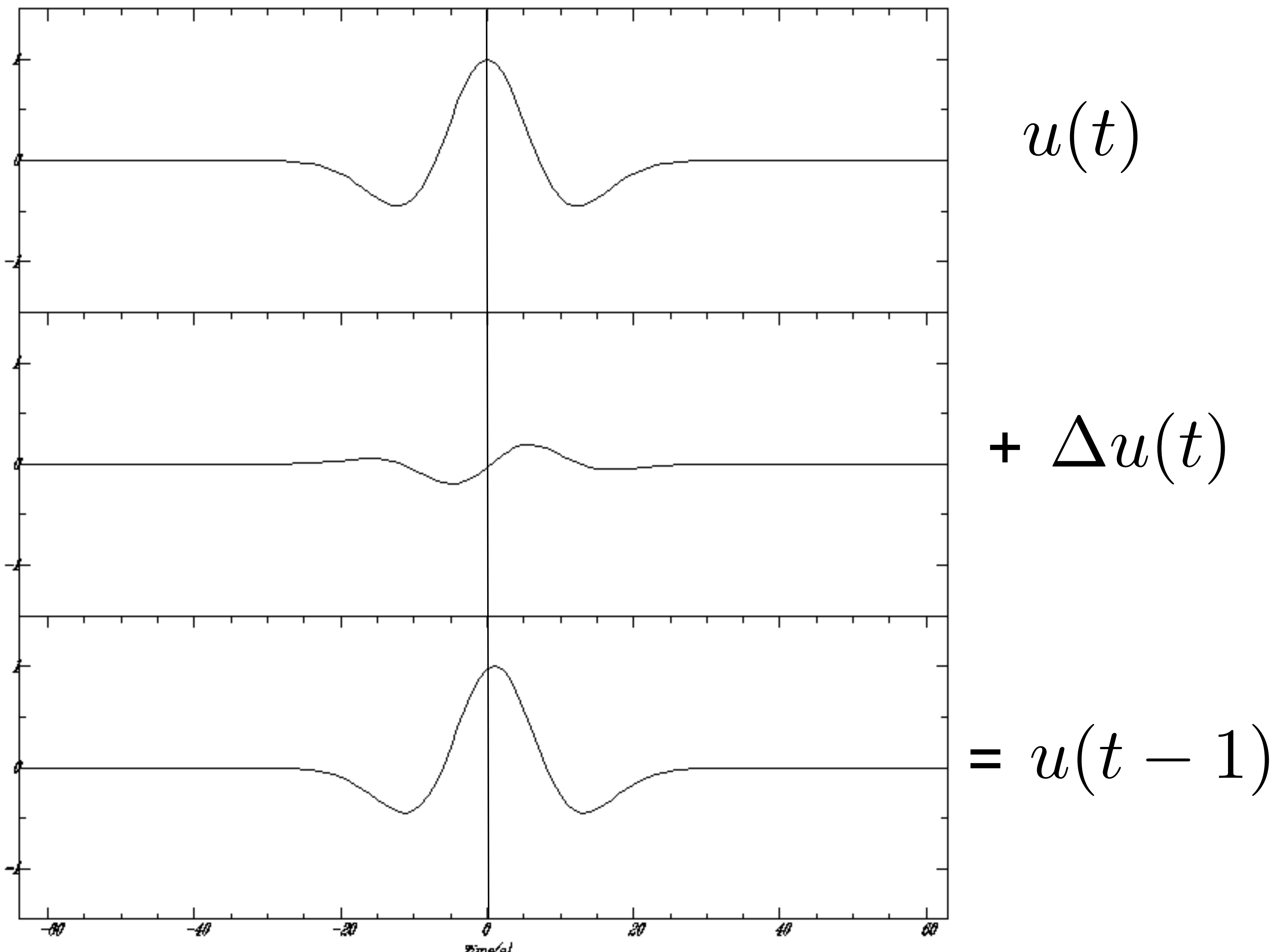
negative δu

} (du/dt) Δt

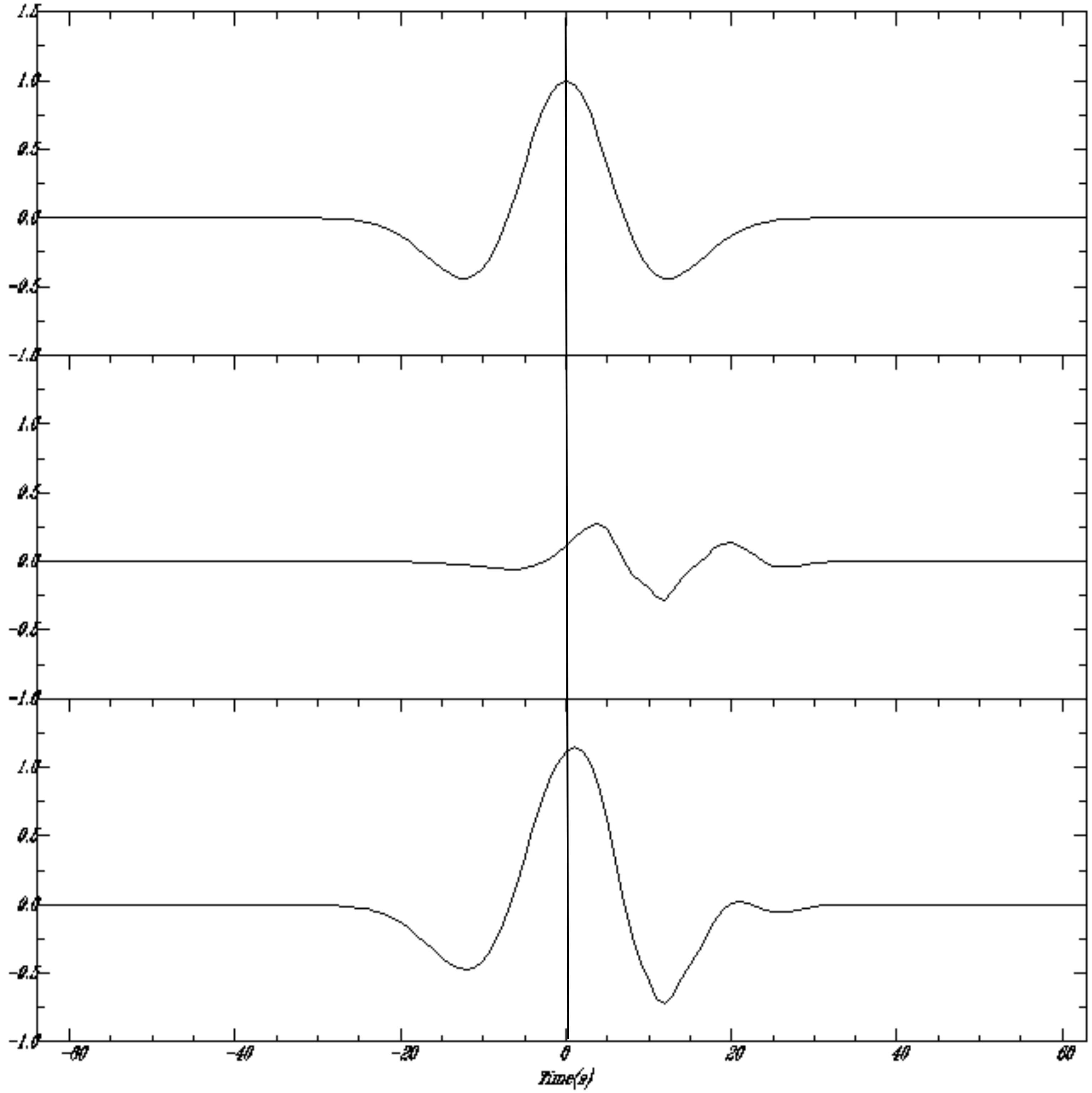
$$u(t + \Delta t) = u(t) + (du/dt)\,\Delta t$$

total δu

nondispersive delays



dispersive delays



$u(t)$

+ $\Delta u(t)$

= $u(t) + \Delta u(t)$

The sensitivity of a cross-correlation delay

$$\delta T = -\frac{\delta\dot{\gamma}(0)}{\ddot{\gamma}(0)} = -\frac{\int_{-\infty}^{\infty} \dot{u}(t')\delta u(t')\mathrm{d}t'}{\int_{-\infty}^{\infty} \ddot{u}(t')u(t')\mathrm{d}t'} \,.$$

The sensitivity of a cross-correlation delay

Compute with Born (SEM, ray theory)

$$\delta T = -\frac{\delta\dot{\gamma}(0)}{\ddot{\gamma}(0)} = -\frac{\int_{-\infty}^{\infty} \dot{u}(t')\,\boxed{\delta u(t')}\,\mathrm{d}t'}{\int_{-\infty}^{\infty} \ddot{u}(t')u(t')\mathrm{d}t'}\;.$$

adjoint computation of δu

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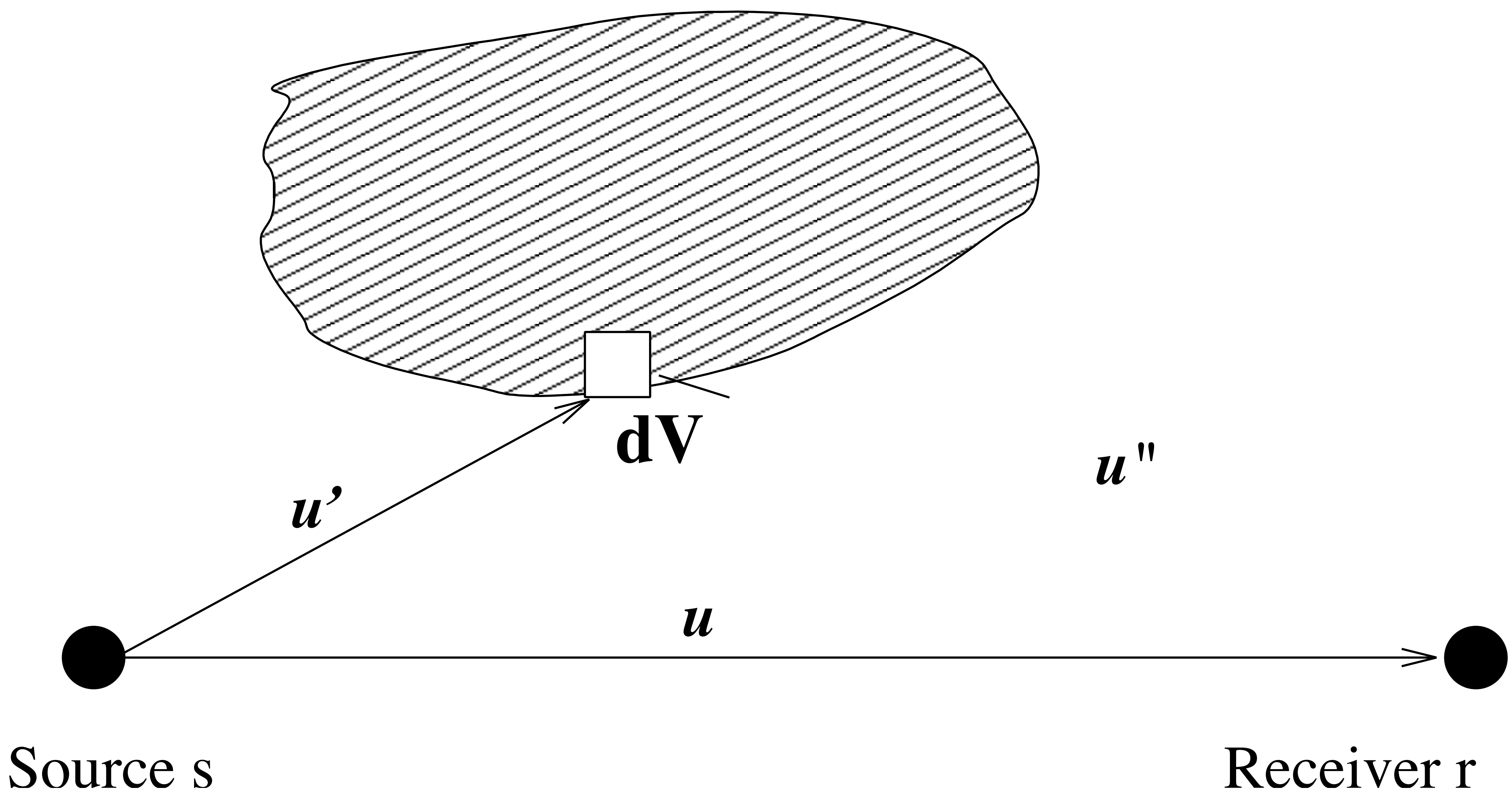

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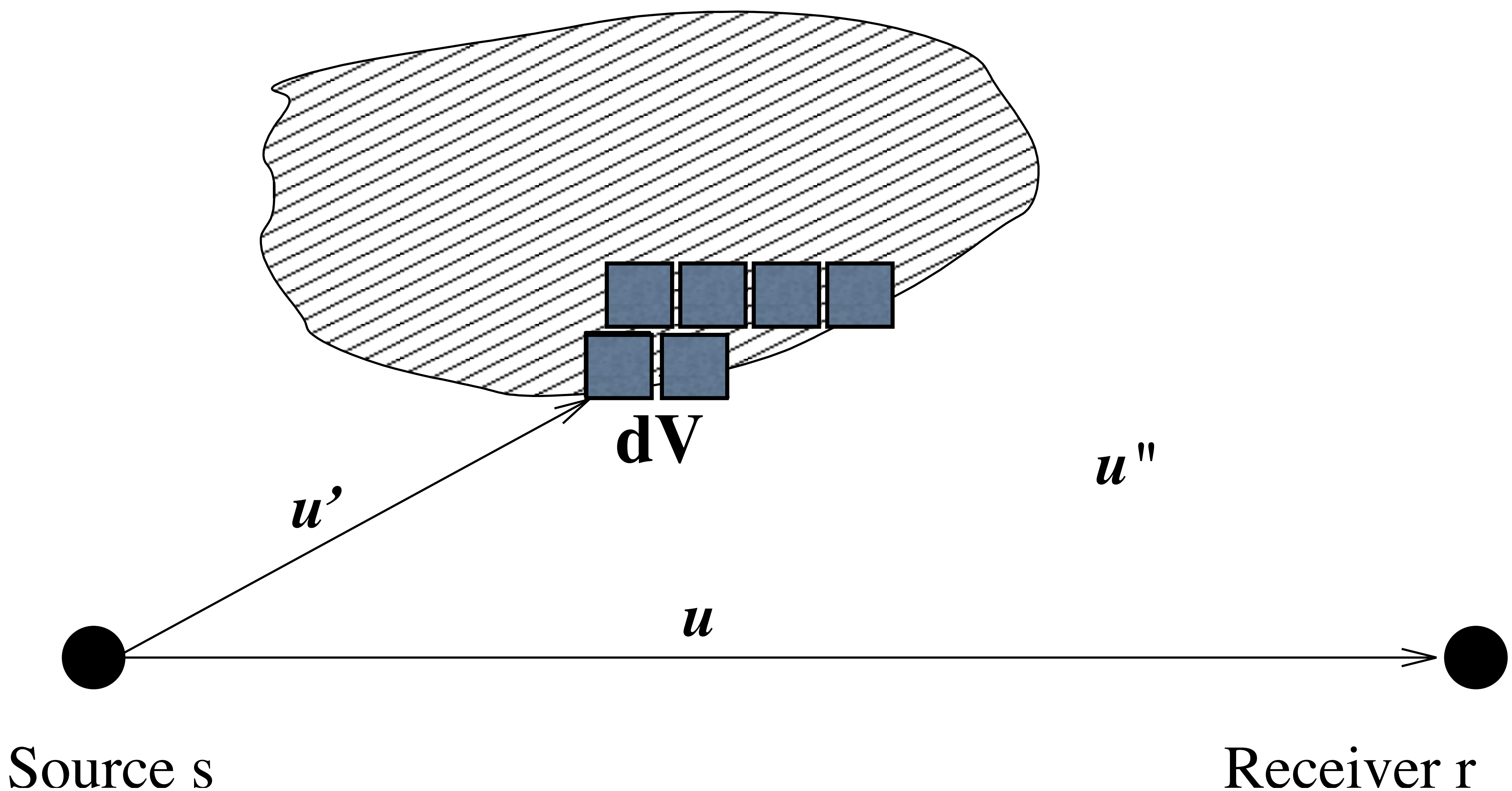

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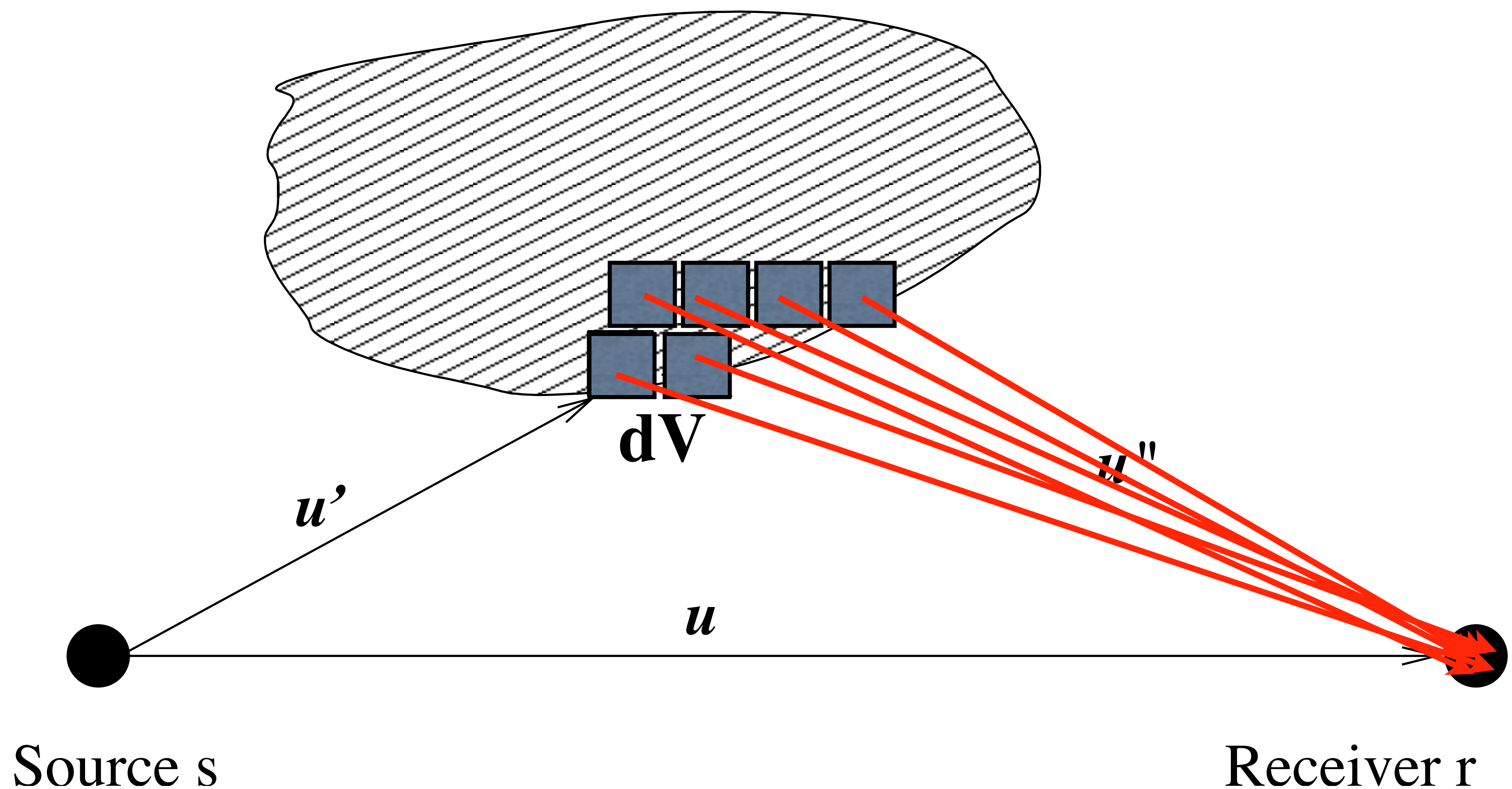

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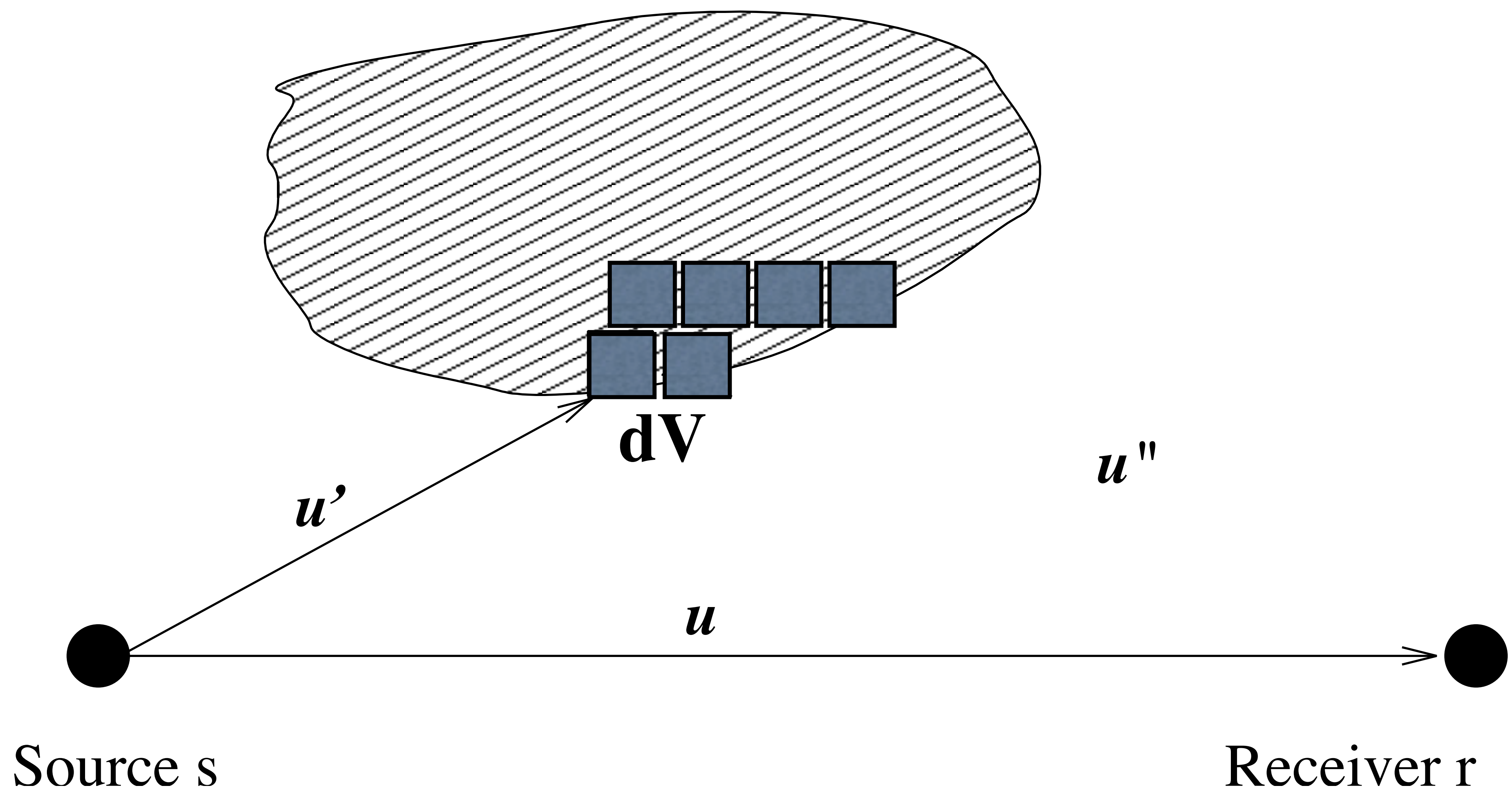

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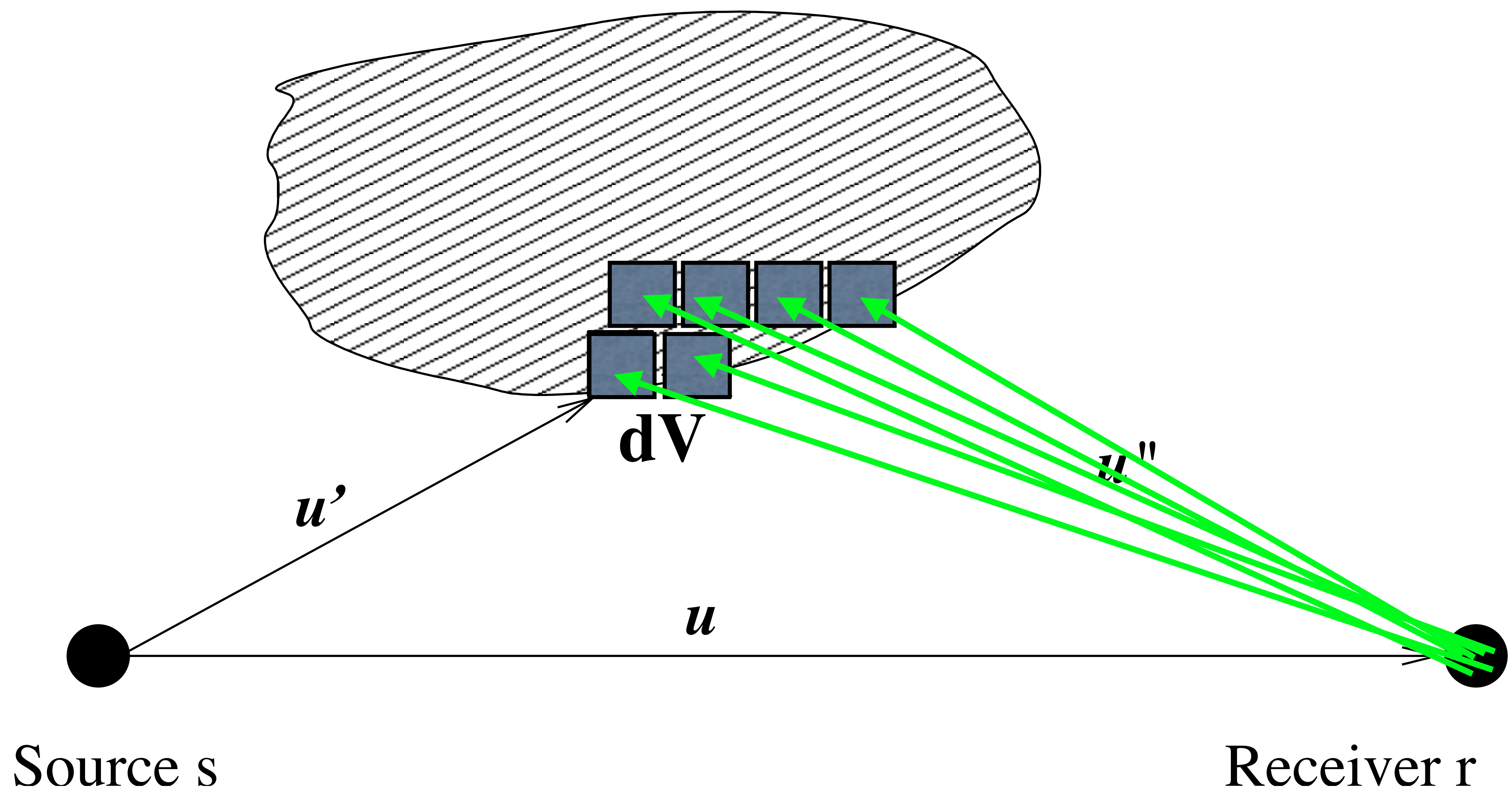
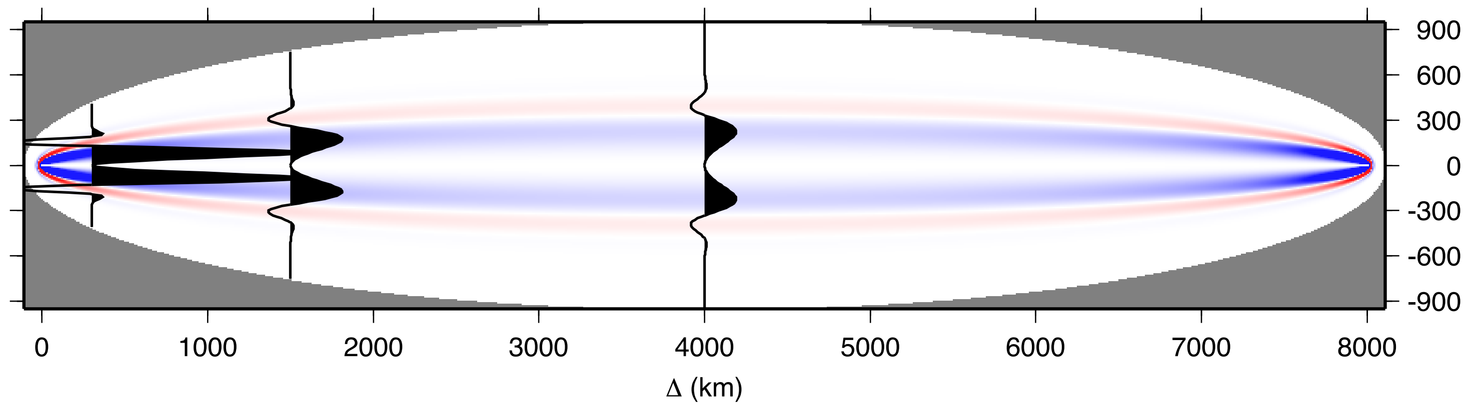


$$\delta T = \int \underbrace{K_{\mathrm{P}} \left(\frac{\delta V_{\mathrm{P}}}{V_{\mathrm{P}}} \right)} \; \mathrm{d}^3 \boldsymbol{r}_{\mathrm{x}}$$

$$-\frac{\int_{-\infty}^{\infty} \dot{u}(t')\delta u(t')\mathrm{d}t'}{\int_{-\infty}^{\infty} \ddot{u}(t')u(t')\mathrm{d}t'} \; .$$

cross-correlation maximum



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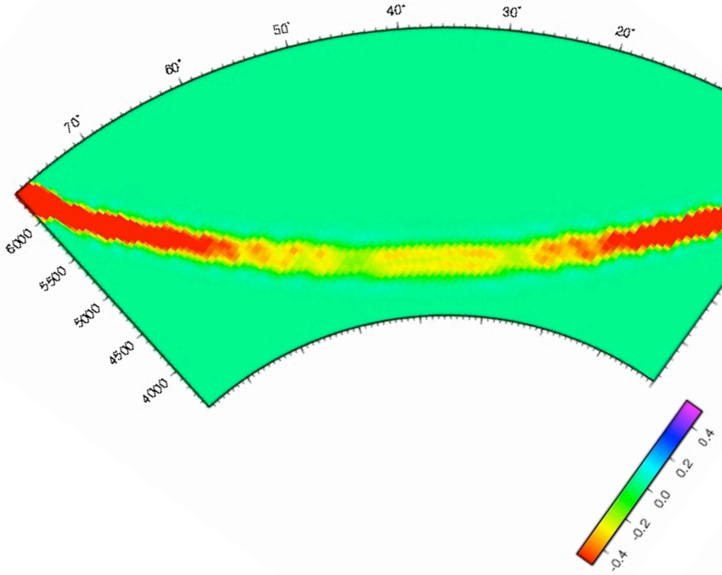
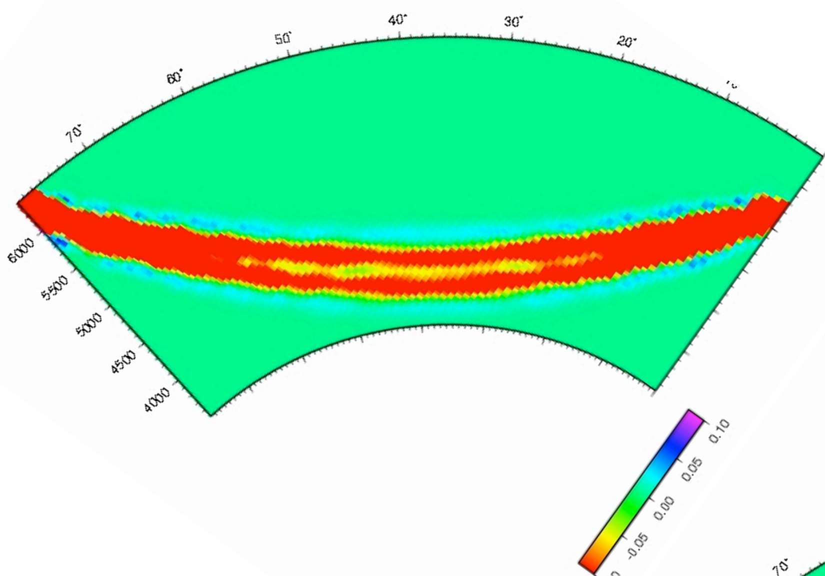
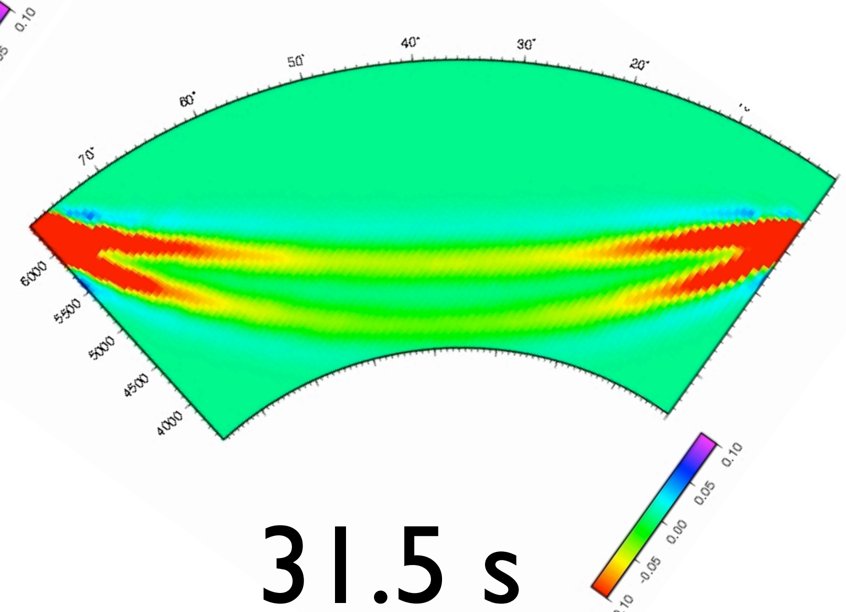
frequency dependence of the sensitivity



2.8 s

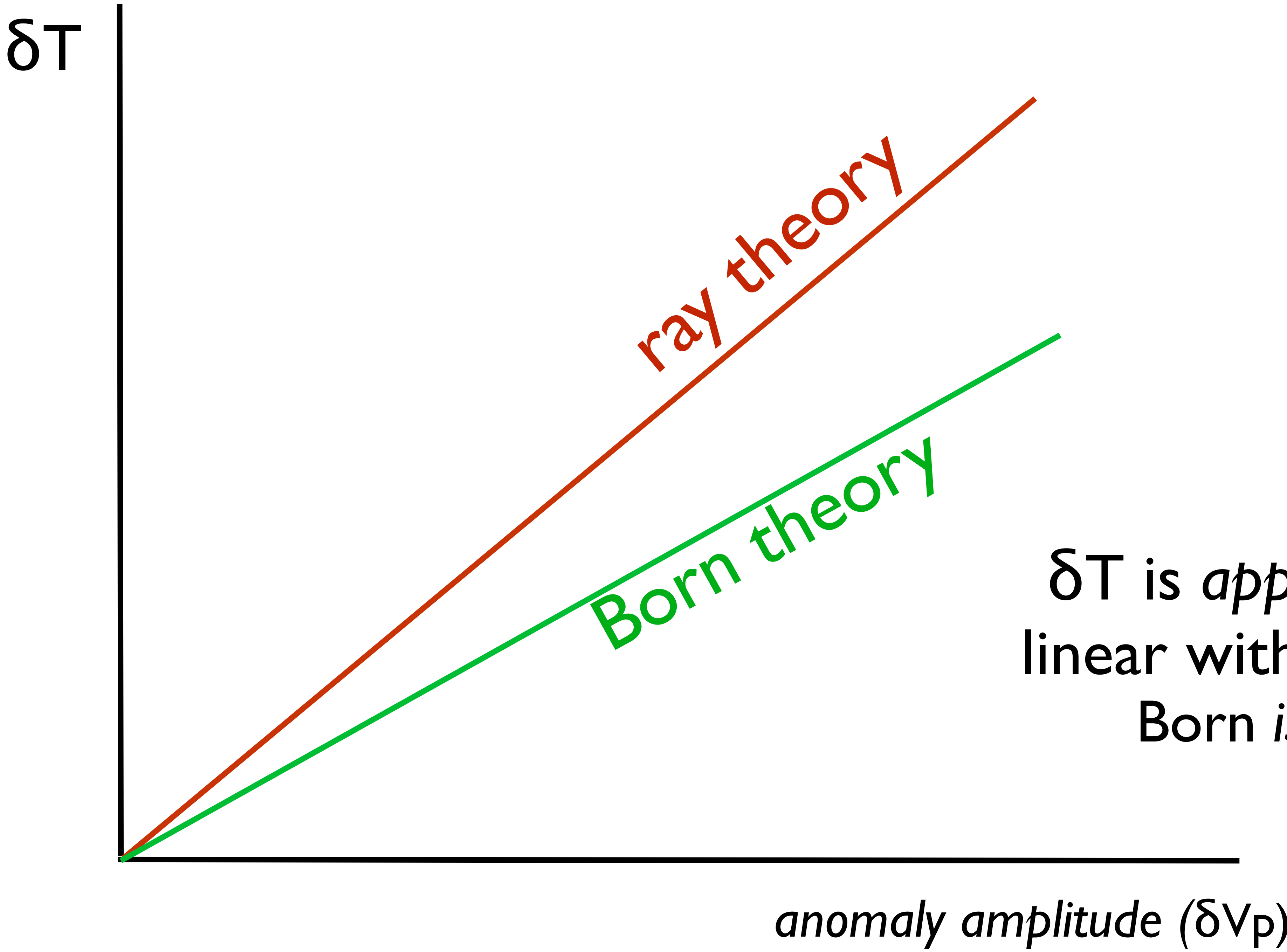


7.2 s



31.5 s

Why δT and not δu ?



δT is *approximately* linear with δVp, while Born *is* linear

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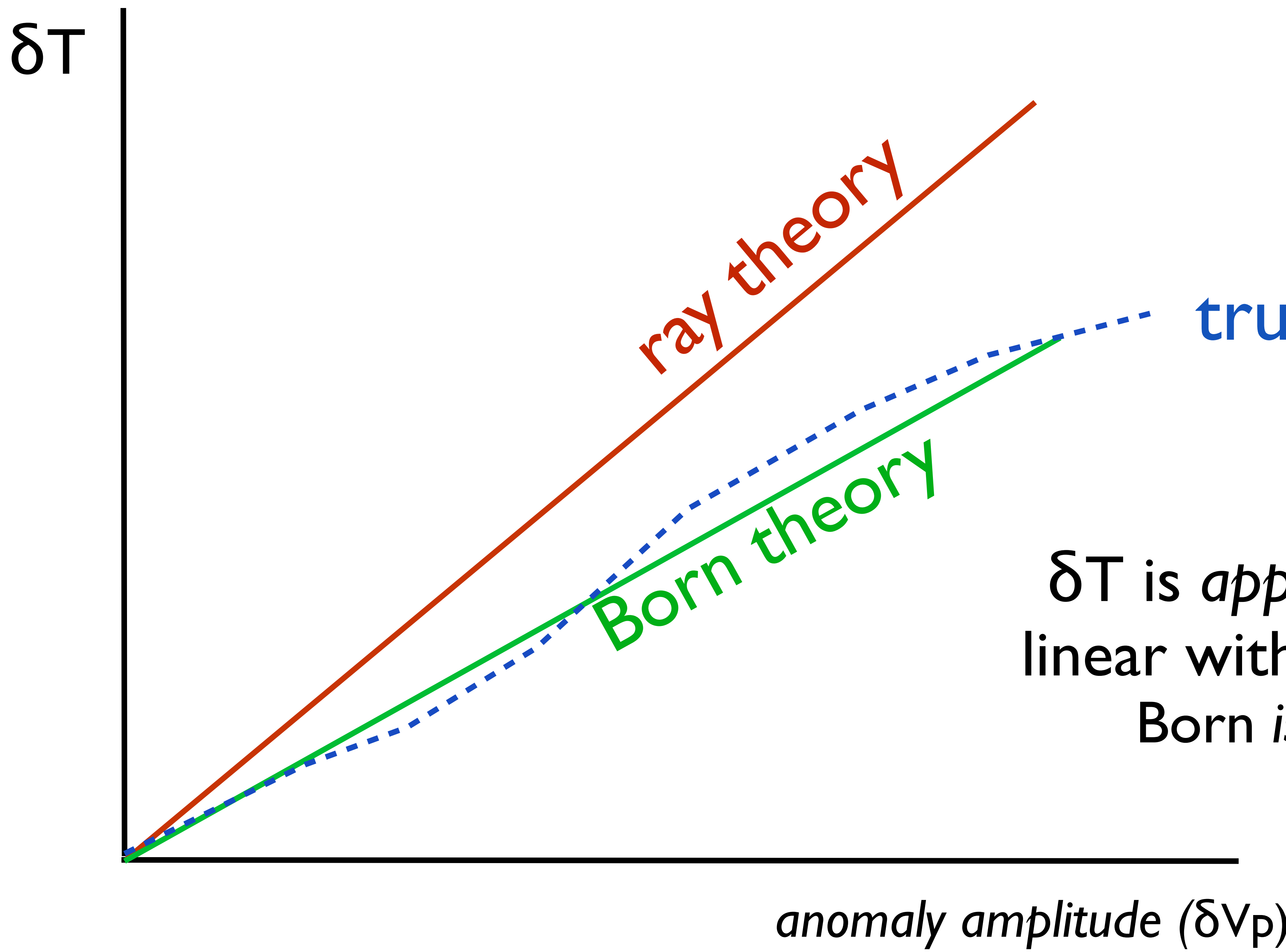

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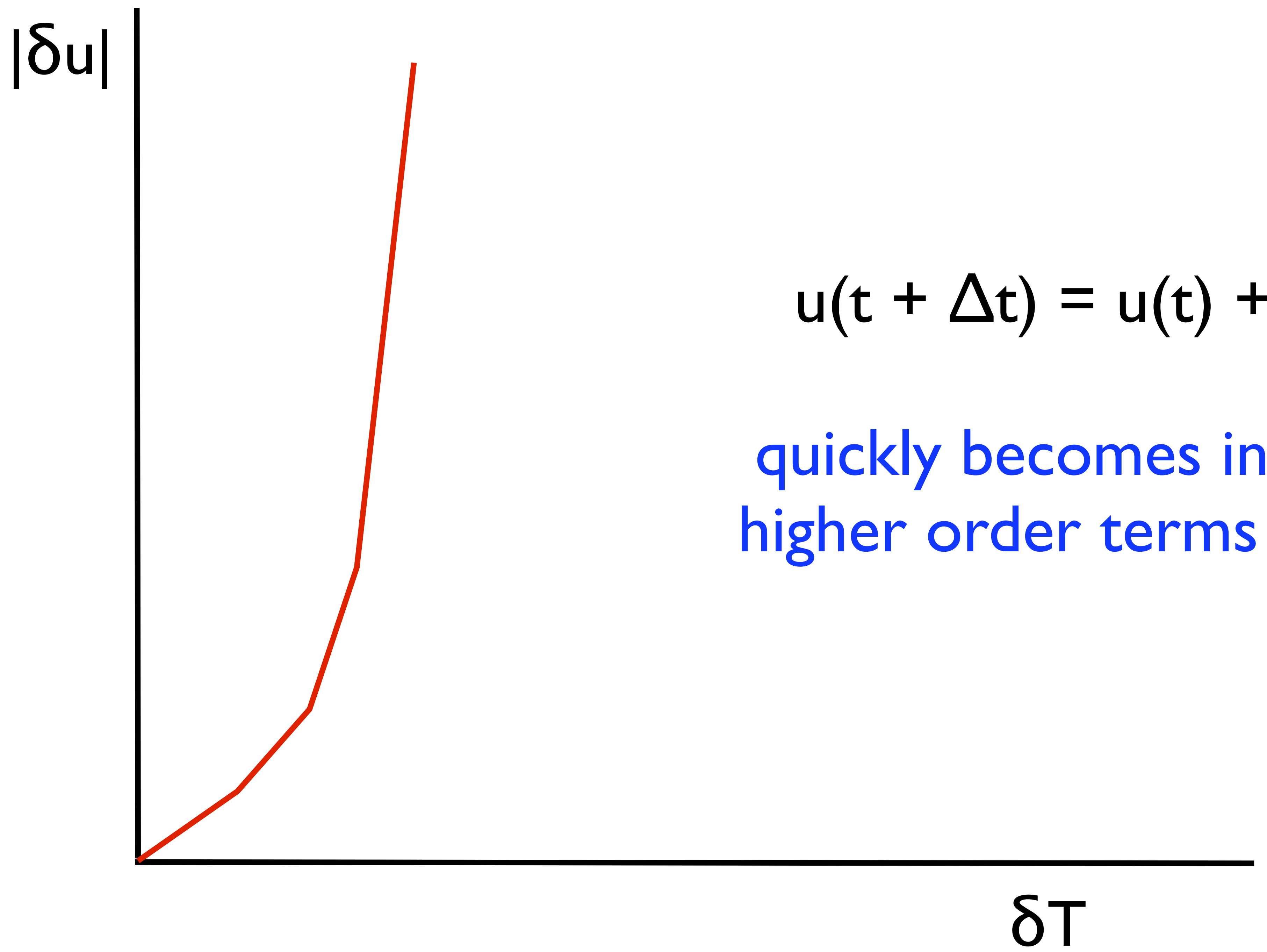

total δu

$$u(t + \Delta t) = u(t) + (du/dt)\,\Delta t$$

quickly becomes invalid because higher order terms are neglected

Why not *group* velocity?

$$\cos[(\omega + \Delta\omega)t - (k + \Delta k)x] + \cos[(\omega - \Delta\omega)t - (k - \Delta k)x]$$

$$= 2\cos(\omega t - kx)\cos(\Delta\omega t - \Delta k x)$$

$$U = \frac{\mathrm{d}\omega}{\mathrm{d}k}$$

Why not *group* velocity?

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Why not *group* velocity?

$$\cos[(\omega + \Delta\omega)t - (\boldsymbol{k} + \Delta\boldsymbol{k}) \cdot \boldsymbol{x}] + \cos[(\omega - \Delta\omega)t - (\boldsymbol{k} - \Delta\boldsymbol{k}) \cdot \boldsymbol{x}]$$

$$= 2\cos(\omega t - \boldsymbol{k} \cdot \boldsymbol{x})\cos(\Delta\omega t - \Delta\boldsymbol{k} \cdot \boldsymbol{x})$$

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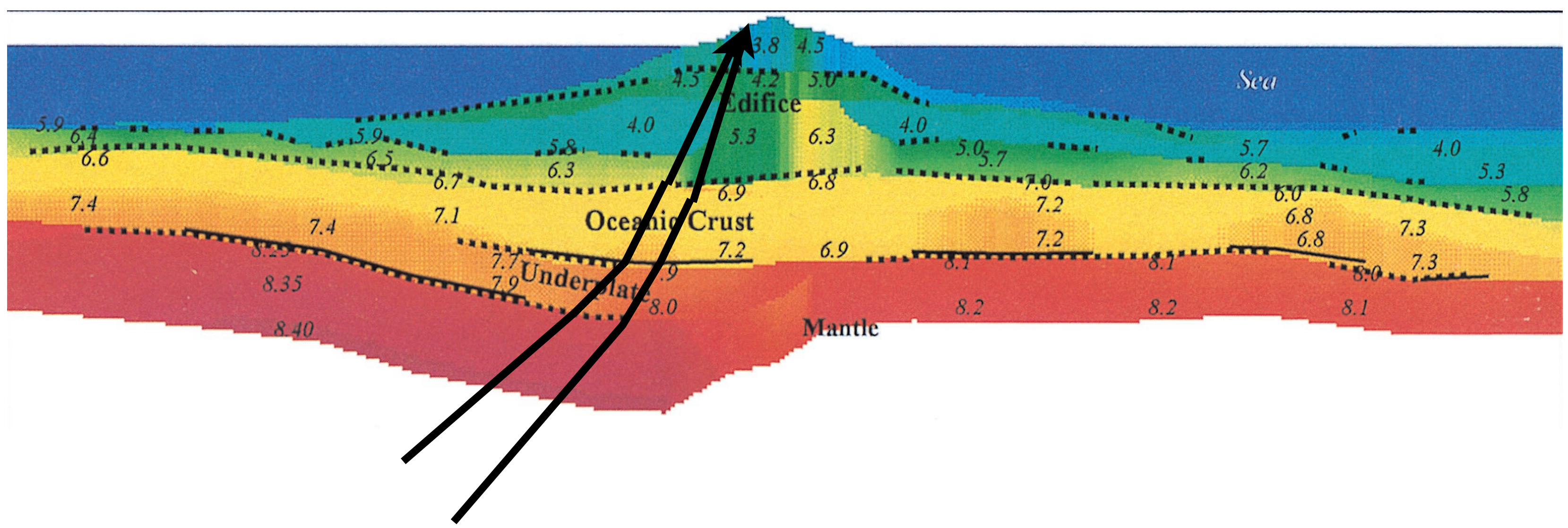
$$= 2\cos(\omega t - \boldsymbol{k} \cdot \boldsymbol{x})\cos(\Delta\omega t - \Delta\boldsymbol{k} \cdot \boldsymbol{x})$$

$$U = \frac{\mathrm{d}\omega}{\mathrm{d}k} \qquad \rightarrow \boldsymbol{U} = \frac{\mathrm{d}\omega}{\mathrm{d}\boldsymbol{k}}$$

The direction of ***U***

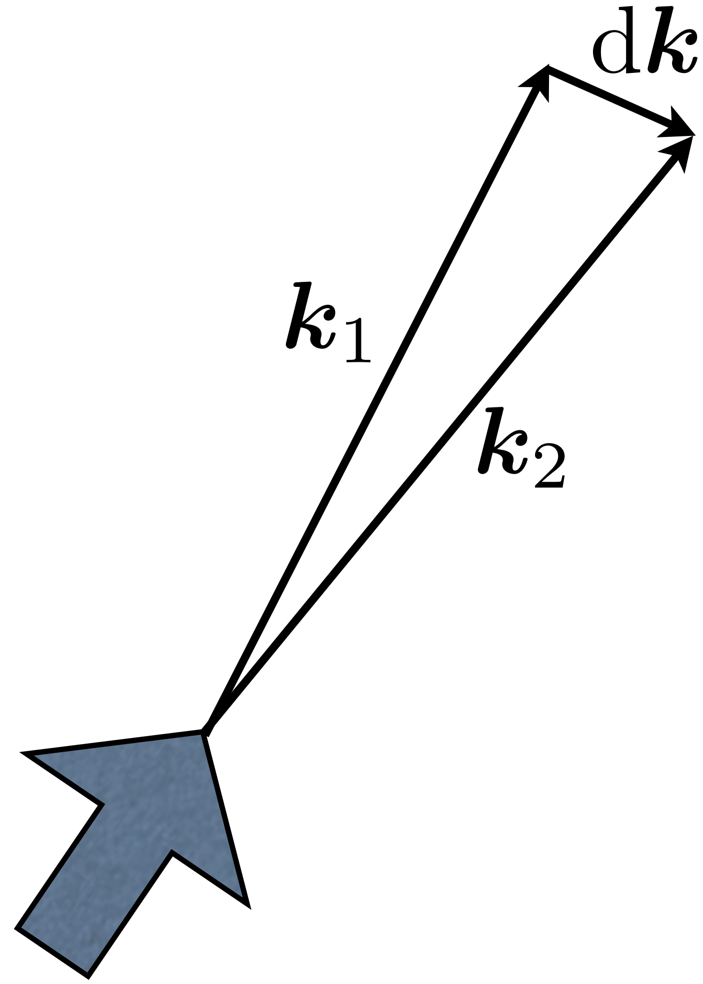


Gallart et al JGR 1999

multipathed P waves

The direction of ***U***

The direction of *U*



multipathed P waves

Conclusions

- Cross-correlations yield a new definition of travel time
- Not to be confused with *group velocity!*
- Which can be handled using (linear) Born theory
- Measuring dispersion yields extra sensitivity (even if absent!)