Accurate prediction of ground motion using an hp-adaptive discontinuous Galerkin finite-element method (DG-FEM)

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1st QUEST Workshop
Capo Caccia, Sardinia, 2010 September 19-25

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Introduction

We investigate the potentials of DG-FEM for 3D seismic modeling

Advantages

- Common to FEM
  - Arbitrary mesh geometry (fit of complex topographies)
  - Adaptive mesh to physical properties (h-adaptivity)
  - Local method suitable for parallel computing
- Specific to DG-FEM
  - Interpolation order mixing (p-adaptivity)
  - Discontinuous wavefield can be supported (fluid/solid interface)

Drawbacks

- Computational cost, but...

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DG-FEM is competitive with other methods when complex topographies or extreme velocity contrasts are considered

Introduction

Characteristics of our approach

- 3D velocity-stress formulation in the time-domain
- Unstructured tetrahedral mesh (more flexible than hexahedral mesh)
- Nodal form of DG-FEM
- Constant physical properties per element
- Favour use of low interpolation orders for fine discretisation
- CPML absorbing boundary condition
- Intensive use of interpolation order mixing
  - Adapt the order according to elements size and medium properties
  - Lower order in the CPMLs to reduce the computational cost
- Centered flux (non-dissipative)

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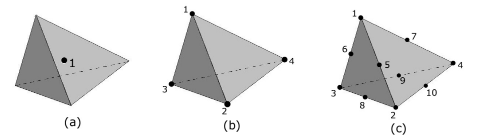
DG-FEM - Spatial and time discretizations

Spatial discretization

- Discretize with non-overlapping and conformal tetrahedra
- Nodal form of DG-FEM (Hesthaven and Warburton, 2008)
- Approximate solution with Lagrangian polynomial basis functions and equidistant nodes



(a) P_0 element with unique DOF. (b) P_1 element element with 4 DOF. (c) P_2 element with 10 DOF.

Time discretization

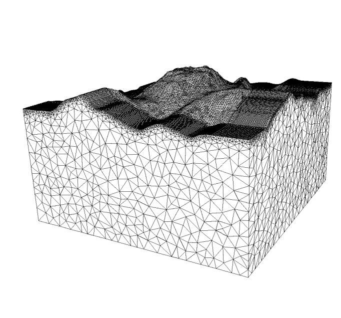
- Second order explicit leap-frog scheme
- Stability condition for the DG-FEM (Käser et al., 2008) gives $\Delta t < \frac{1}{2d_i+1} \cdot min_i \frac{2r_i}{V_{P_i}}$
  - r_i is the radius of the sphere inscribed in the element i
  - V_{P_i} the P-wave velocity
  - d_i the interpolation order of the cell
- **The minimum required time step is imposed to all elements**

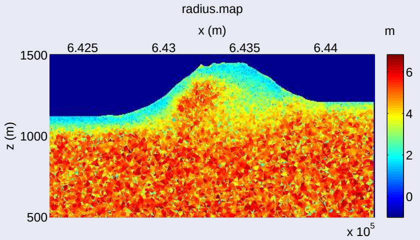
DG-FEM - Tetrahedral meshing

Benefits of tetrahedral meshing

- Based on the Delaunay triangulation principle (Delaunay, 1934)
- Great flexibility in terms of design (complex shape) and refinement (local adaptivity)
- Efficient tetrahedral meshers are available (we use TETGEN)

Example of a complex mesh, the volcano 'La Soufrière'





Insphere radius

DG-FEM - Boundary conditions

Free surface

- Explicit condition by changing locally the flux expression
- Introduce virtual cells which are exactly symmetric to the cells located on the free surface
- Inside these cells, impose identical velocity but opposite stress wavefield

Absorbing condition

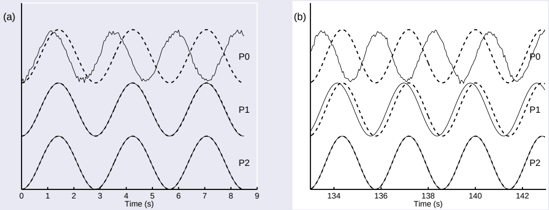
- Convolutional Perfectly Matched Layer (CPML) (Komatitsch and Martin, 2007)
- Unsplit formulation with memory variables
- Improve absorption of waves at grazing incidence
- In the CPML, the damping function is defined in the frequency domain as follows

$$s_\theta = \kappa_\theta + \frac{d_\theta}{\alpha_\theta + i\omega} \qquad \forall \theta \in \{x, y, z\}$$

  with the angular frequency ω and $\kappa_\theta \geq 1$ and $\alpha_\theta \geq 0$
- If $\kappa_\theta = 1$ and $\alpha_\theta = 0$, one get the classical PML formulation (Berenger, 1994)

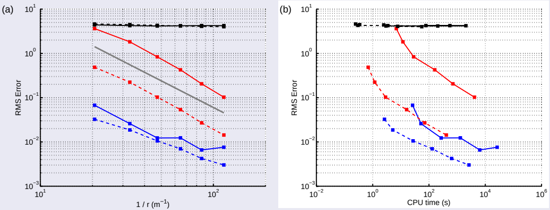
DG-FEM - Convergence study

Eigen mode in a cube



Initial conditions + free surfaces = continual monochromatic signals

Convergence rate

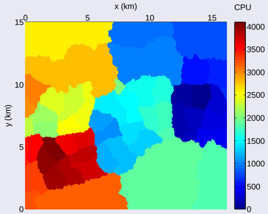


Second order convergence for P_1 (red) or P_2 (blue), but no convergence for P_0 (black)

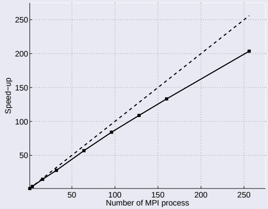
DG-FEM - Computing aspects

Parallelism

- Domain decomposition strategy, one subdomain = one CPU
- MPI communication between subdomains
- Efficient load balancing with mesh partitioning (METIS)
- **Average parallelism efficiency of 80 %**



EUROSEISTEST mesh partitioning



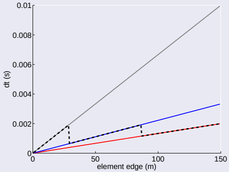
Possible bottle-neck

- Time step is common for all subdomains
- Dramatic effects of badly shaped elements
- **Mitigate these negative effects with p-adaptivity**

DG-FEM - hp-adaptivity

Interpolation order mixing : an efficient mean to mitigate effects of badly shaped tetrahedra

Downgrade order of badly shaped element = increase time step



Time step versus the element size for different interpolation orders with $V_P = 6000$ m/s
Grey curve : P_0 ; blue curve : P_1 ; red curve : P_2 ; dashed line : interpolation order mixing

Our 2-step refinement approach

- 1^{st} step : Iterative **mesh refinement** (h-adaptivity)
  - based on medium properties and discretization target (3 cells / λ with P_2)
  - repeated until required discretisation is reached
- 2^{nd} step : **adapt the interpolation order** with an **a priori error estimate** (p-adaptivity)

$$\left.\begin{array}{ll} P_2 & \text{if } \lambda/8 < \text{cell size} \\ P_1 & \text{if } \lambda/24 < \text{cell size} \leq \lambda/8 \\ P_0 & \text{if cell size} \leq \lambda/24 \end{array}\right\} \text{heuristic criteria}$$

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EUROSEISTEST - Model description

EUROSEISTEST Verification and Validation project

- Organized by CEA, LGIT, University of Thessaloniki and Institute Laue Langevin
- 10 modeling teams (FDM, FEM, SEM, DEM, DG-FEM and PSM)

Model characteristics

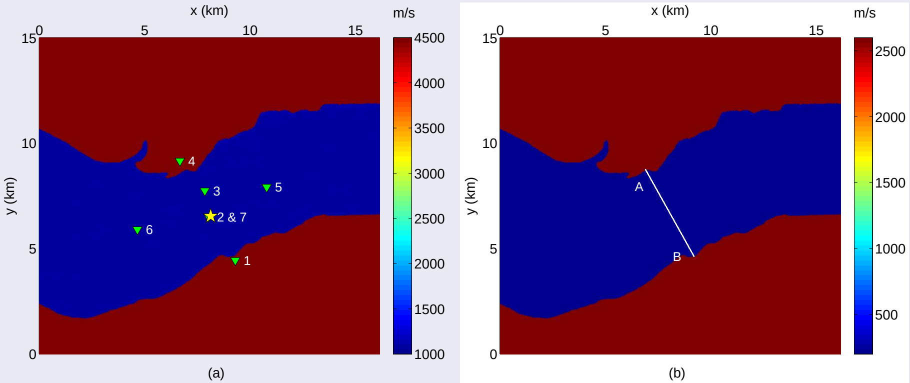
- Sedimentary basin 30 km E-NE of Thessaloniki (Northern Greece)
- Low velocity in basin and high velocity bedrock

	P-wave velocity	S-wave velocity	Ratio V_P / V_S	Max. depth
Basin	from 1000 to 3027 m/s	from 200 to 848 m/s	from 5.00 to 3.57	411 m
Bedrock	from 4500 to 6144 m/s	from 2600 to 3444 m/s	from 1.73 to 1.78	8 km

- **Ratio Max. V_S / Min. V_S = 17**
- High Poisson ratio in the basin
- **Thin structures**
- Double-couple source with max. frequency of 4 Hz ($M_w = 1.3$)

EUROSEISTEST - Model description

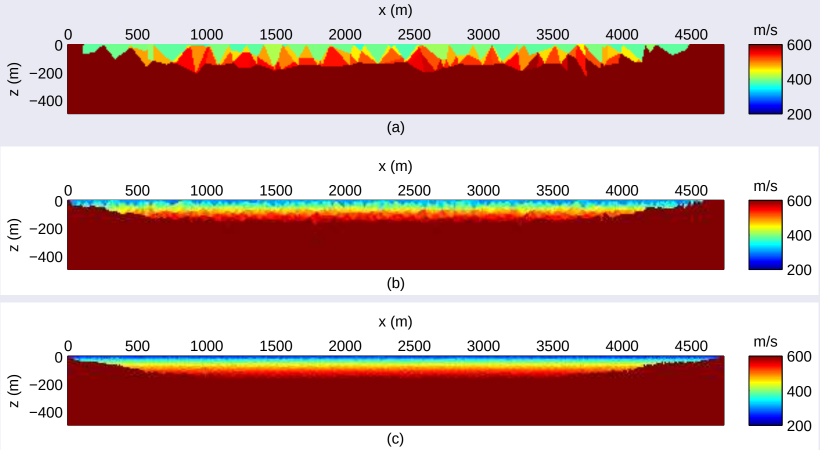
Velocity model



(a) View of the mesh in the plan xy at $z = 0$ m showing the P-wave velocity associated to each cell in the EUROSEISTEST model. The receivers are represented with numbered green triangles and the source epicenter with a yellow star. (b) Same with S-wave velocity associated to each cell. The direction of the cross-section AB is indicated with a white segment.

EUROSEISTEST - Mesh building

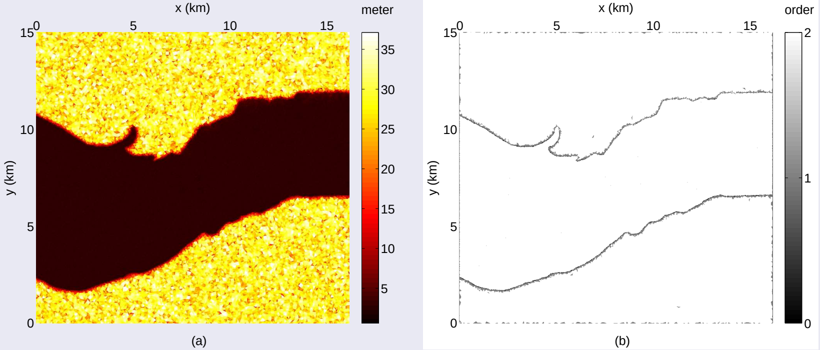
Iterative mesh refinement (h-adaptivity)



(a) Cross section *AB* of the mesh at the first iteration of the *h*-refinement showing the S-wave velocity associated to each cell in the EUROSEISTEST model. (b) Same as (a) at the second iteration of the *h*-refinement. (c) Same as (a) at the sixth and last iteration of the *h*-refinement.

EUROSEISTEST - Mesh building

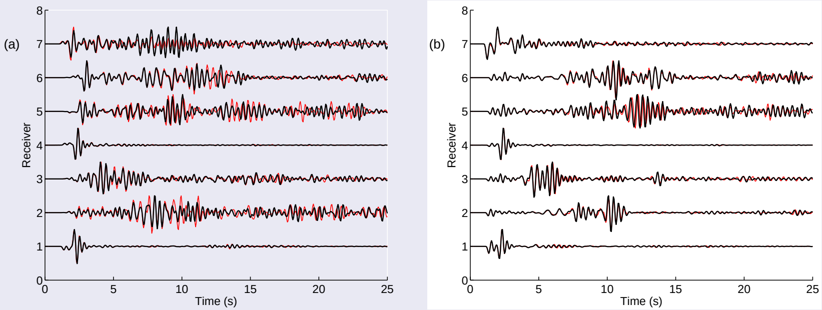
Interpolation order mixing (p-adaptivity)



(a) View of the mesh in the plan *xy* at z = 0 m showing the size of the elements (insphere radius) in the EUROSEISTEST model. (b) Same with interpolation order associated to each cell. P_2 elements are represented in white, P_1 in grey and P_0 in black.

- The basin represents 1 % of the model and contains more than 70 % elements in the mesh
- 65.51 % P_2, 34.36 % P_1 (with 31.51 % in CPML) and 0.13 % P_0 elements
- **Interpolation order mixing plays an important role at the basin edges**

Comparison between DG-FEM and SEM



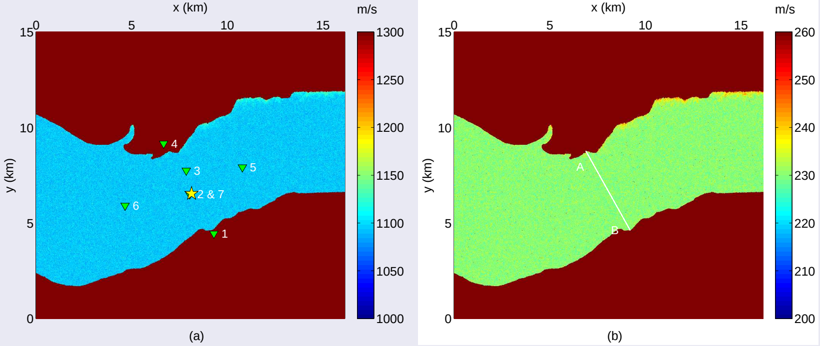
(a) Seismograms of v_x computed with DG-FEM (black line) and SEM (red line). (b) Same as (a) with v_z.

- Perfect match between both solutions for receivers in bedrock (1 and 4)
- For other receivers
  - for v_z very good agreement
  - for v_x good for short times but misfits are increasing with time

EUROSEISTEST - Numerical results

Model discretization

When looking closely at the free surface with a reduced velocity scale...

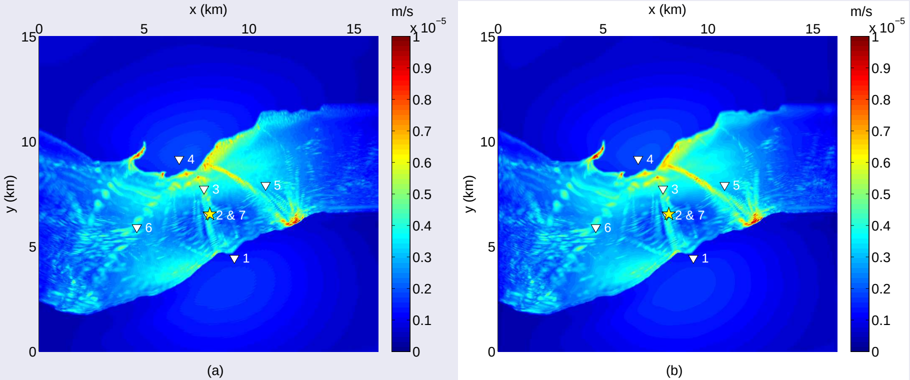


(a) View of the mesh in the plan xy at z = 0 m showing the P-wave velocity. (b) Same with S-wave velocity.

Due to constant properties per element, velocities at the free surface are higher than real values (V_P = 1000 m/s and V_S = 200 m/s)
$\Rightarrow$ It may be the origins of the misfits at long times

EUROSEISTEST - Numerical results

Comparison between DG-FEM and SEM



(a) Peak Ground Velocity (PGV) map computed for the EUROSEISTEST modeling with DG-FEM. The receivers are represented with numbered white triangles and the source epicenter with a yellow star. (b) Same as (a) computed with SEM.

Excellent agreement of the PGV maps between DG-FEM and SEM

EUROSEISTEST - Numerical results

Comparison between DG-FEM and SEM

If time permits,
let's see a little movie of the ground motion...

EUROSEISTEST - Numerical results

Comparison between DG-FEM and SEM

Mesh statistics

Method	Order	l_{min}	l_{max}	Nb elements	Nb DOF	Nb unknowns
DG-FEM	$P_2/P_1/P_0$	2.5 m	399.8 m	16.3×10^6	131.6×10^6	1.31×10^9
SEM	P_4	20.0 m	908.0 m	1.4×10^6	91.7×10^6	0.27×10^9

Computation times

Method	Nb time steps	Nb procs	CPU time	Mem.	CPU type
DG-FEM	122 565	144	52h	26 GB	IBM E5420 2.5 Ghz
SEM	75 000	144	7h	25 GB	IBM E5420 2.5 Ghz

- Ratio DG-FEM CPU time / SEM CPU time is 7 (for similar accuracy)
  - Number of unknowns is 4.8 times higher with DG-FEM
  - Number of time steps is 1.6 times higher with DG-FEM
- **In more complex media, DG-FEM should be more competitive...**

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Conclusions and perspectives

Conclusions

- DG-FEM has great potentials for seismic modeling in highly heterogeneous media
- Designed an effective hp-adaptive scheme (CPU time reduced by one order of magnitude)
- Approach has been validated with the EUROSEISTEST Verification and Validation Project

Perspectives

- Take into account anisotropy
- Allow for physical properties variation inside elements
  - Explore higher orders in space
  - Explore higher orders in time
- Implement visco-elastic rheologies (Käser et al., 2007)
- Wave propagation in fractured media
- Dynamic rupture (BenJemaa et al., 2007, 2009; de la Puente et al., 2009)
- Application of the method to Full Waveform Inversion (FWI)

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Käser, M., Hermann, V., and de la Puente, J. (2008). Quantitative accuracy analysis of the discontinuous Galerkin method for seismic wave propagation. *Geophysical Journal International*, 173(2) :990–999.

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