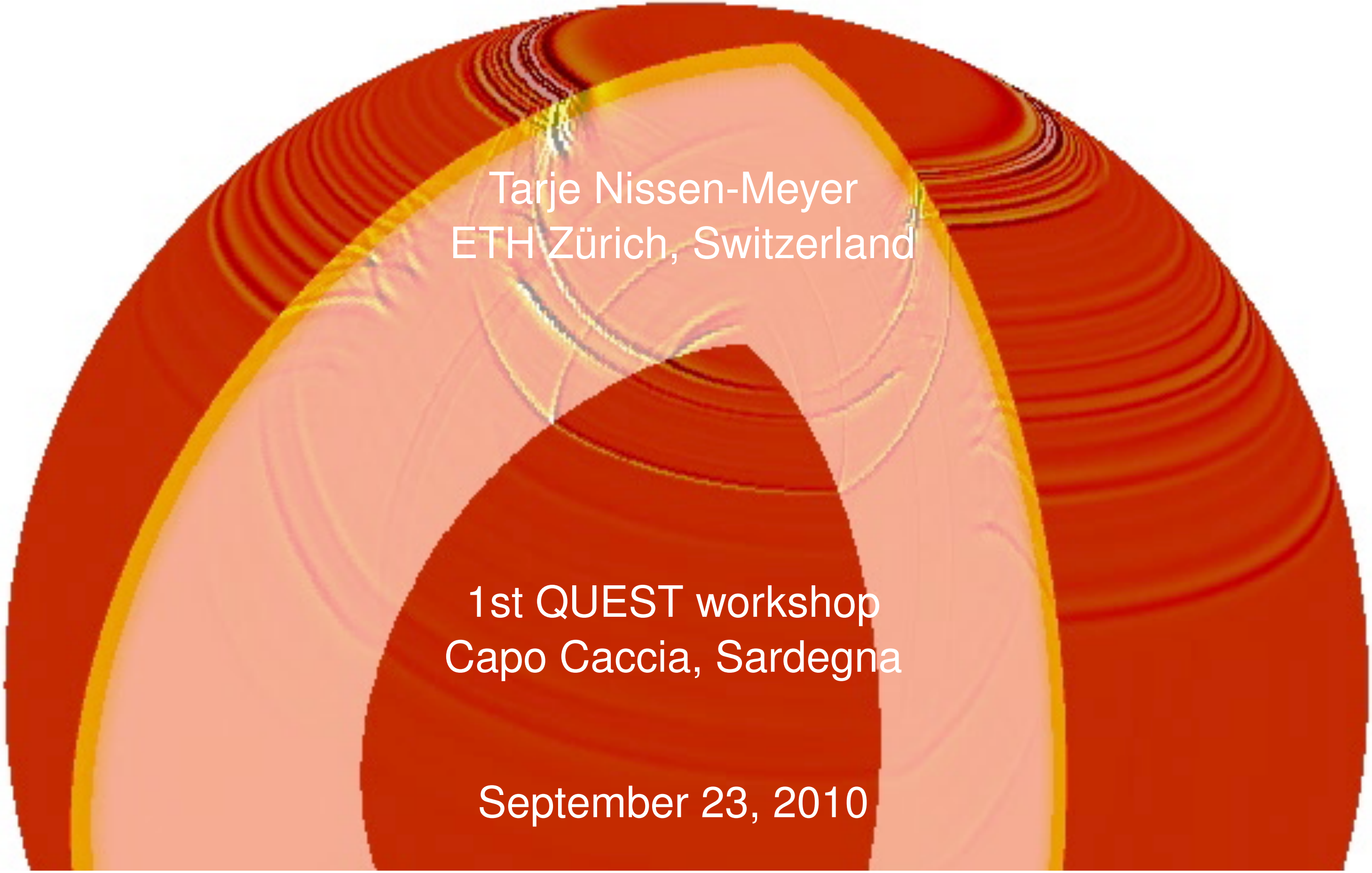
1D model, 2D domain, 3D waves: Axisymmetric spectral-element method



Tarje Nissen-Meyer
ETH Zürich, Switzerland

1st QUEST workshop
Capo Caccia, Sardegna

September 23, 2010

Computational grand challenges

Problem	$f[Hz]$	$\Delta[\lambda^{-1}]$	DOF	RAM$[GB]$
hydrofracture monitoring	150	150	5×10^7	10
exploration seismology	30	300	2×10^9	300
seismic hazard	3	100	4×10^7	6
global body waves	0.15	300	2×10^9	300
multiple-orbit surface waves	0.005	150	4×10^8	70

Need: Accurate simulations for >100 wavelengths at all frequencies across the seismic spectrum

Performance-based design

Given an error tolerance, find scheme to minimize CPU time & memory

Example: Major-arc Rayleigh wave (R2)

- Epicentral distances up to $330°$,
- Dominant period $\approx\ 70 - 175$ seconds,
- Average phase velocity 4 km/s,
- $\Rightarrow$ propagation distances 5-130 wavelengths,
- Observational uncertainties: 3-20 % of the period,
- Synthetics one order of magnitude more accurate.

$\Rightarrow$ Error tolerance: $\epsilon = 10^{-3}$ at 130 wavelengths distance.

Task: Find scheme that meets these criteria with least cost

2. A forward problem:

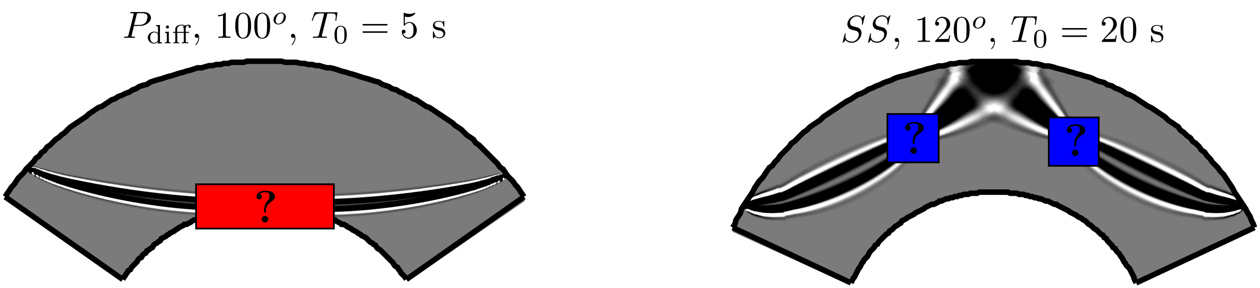
Solving (an)isotropic

(an)elasto-acousto-dynamics

at high resolution at the global scale

"Exact" Fréchet derivatives?

How about non-geometric phenomena such as
diffracted or **caustics?**



(Nissen-Meyer et al., 2007)

"Exact seismic sensitivity":

- Inclusion of full-wave effects ? ,
- Covering all frequencies of high-quality broadband data.

$\Rightarrow$ Full-wave solution necessary

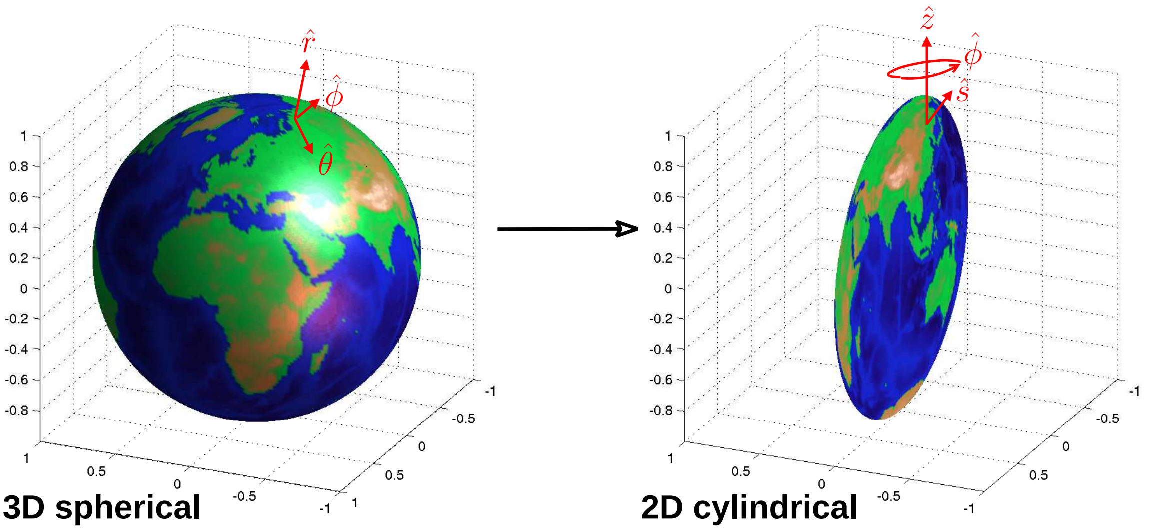
2D Earth

Analytical multipole source radiation (spherical symmetry):

$$\mathbf{u}(r,\theta,\phi) = \left[\hat{\boldsymbol{\theta}} u_\theta + \hat{\mathbf{r}} u_r\right](r,\theta)\cos m\phi - \hat{\boldsymbol{\phi}} u_\phi(r,\theta)\sin m\phi$$

($m = 0, 1, 2$: Monopole, dipole, quadrupole radiation)

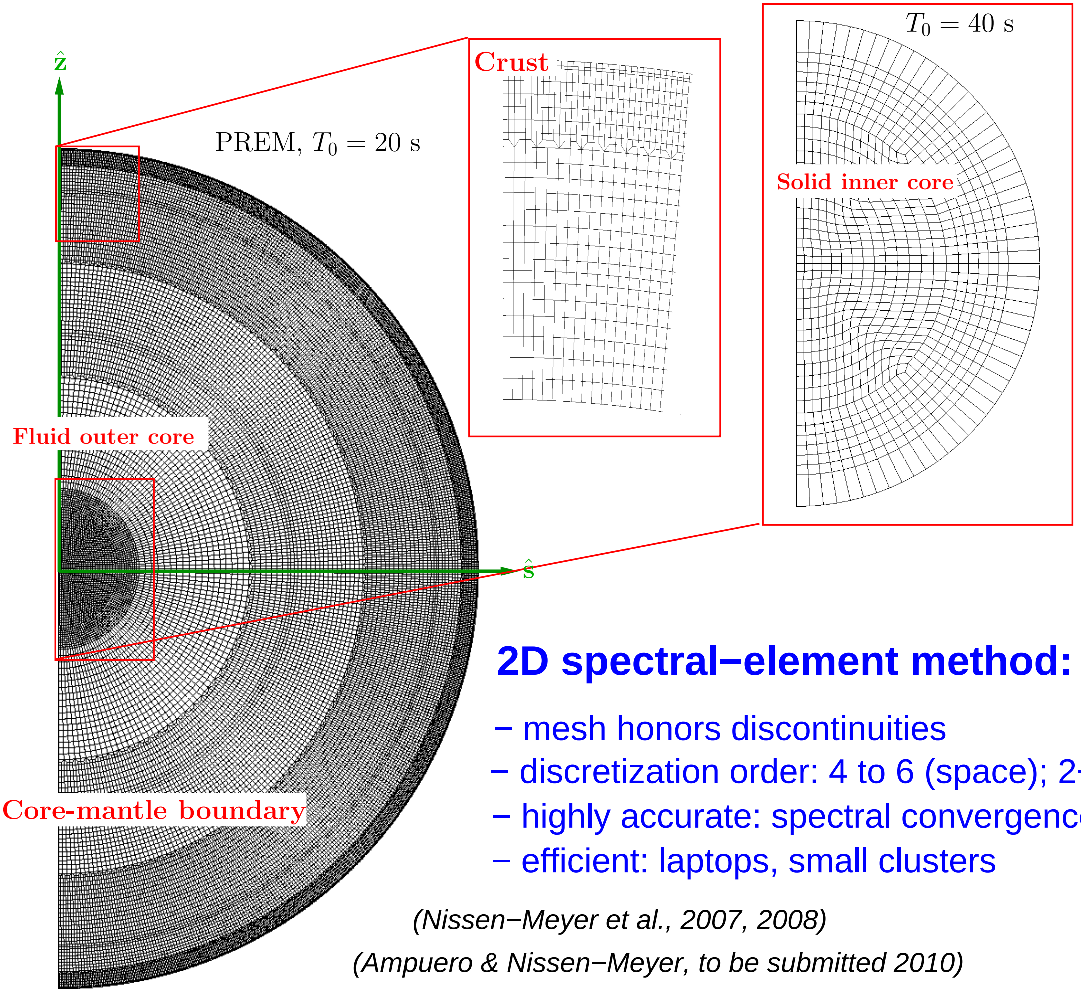
Collapse the azimuth: $\int_0^{2\pi} \cos^2 2\phi \, d\phi = \pi$



3D spherical **2D cylindrical**

$\Rightarrow$ 3-D integral form upon a **2-D computational domain**

1D model, 2D domain, 3D waves



2D spectral−element method:

- mesh honors discontinuities
- discretization order: 4 to 6 (space); 2−4 (time)
- highly accurate: spectral convergence
- efficient: laptops, small clusters

(Nissen−Meyer et al., 2007, 2008)

(Ampuero & Nissen−Meyer, to be submitted 2010)

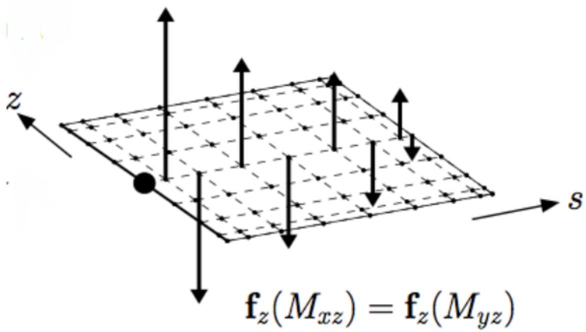
Space discretization

Generally:

Analytical mapping, Gauss-Lobatto-Legendre basis

Axis treatment:

- s^{-1}ingularities $\Rightarrow$ G-L-Jacobi basis, l'Hospital's rule
- Essential axial boundary conditions $\Rightarrow$ explicit masking



Source:

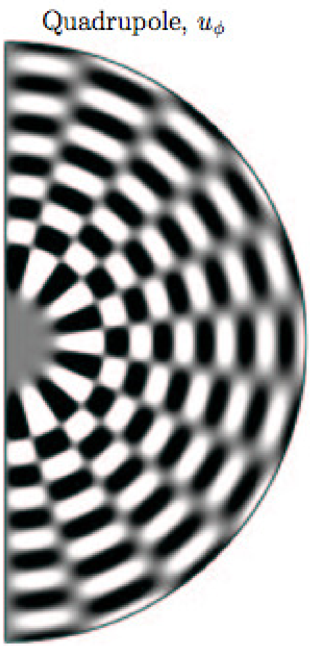
- Located along the axis
- Moment tensor: decomposed into 4 separate solutions
- Receiver components: decomposed into 2 solutions

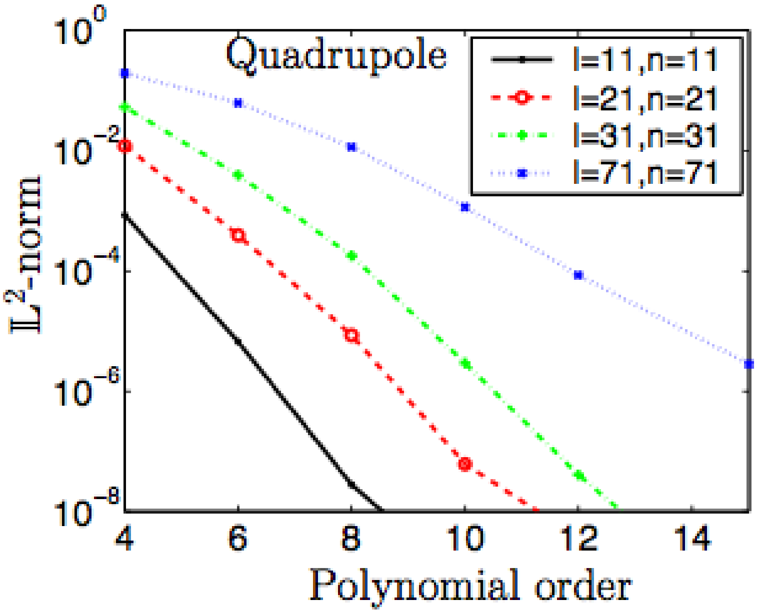
Spectral convergence

The frequency-domain elastostatic weak wave equation,

$$\mathbf{Ku} = {}_n\omega_l^2 \mathbf{Mu},$$

with eigenfrequency ${}_n\omega_l$ of degree l and overtone n, is satisfied by **toroidal eigenfunctions** ${}_n\mathbf{u}_l$.



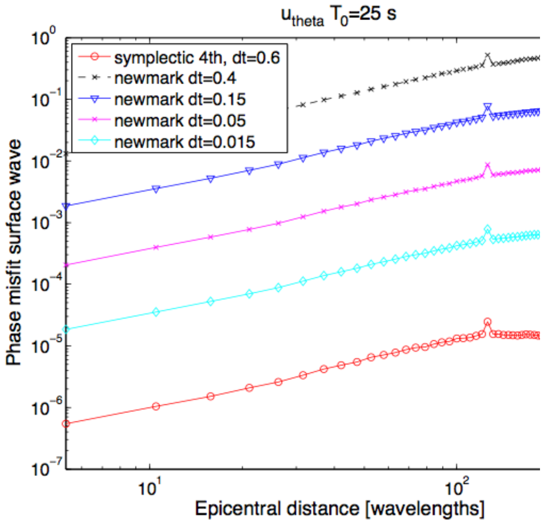


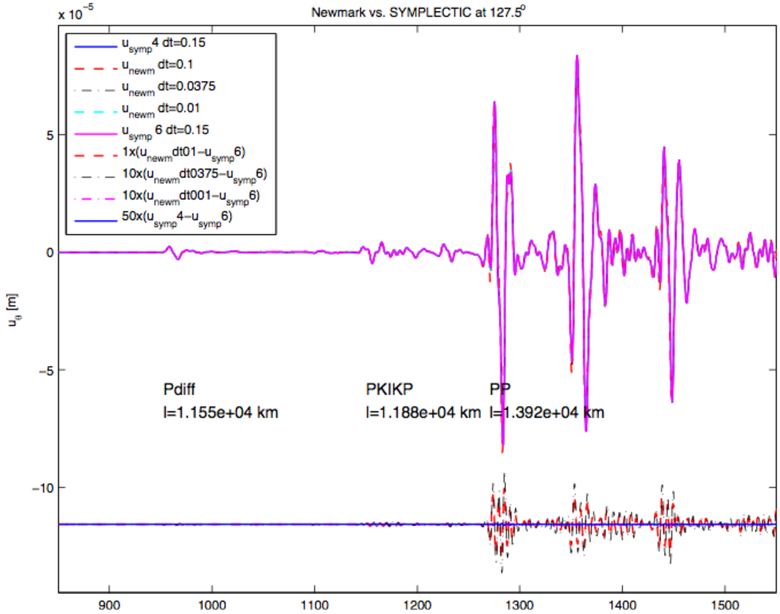
Time discretization

Temporal ODE system of the discretized weak form:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{F}(t)$$

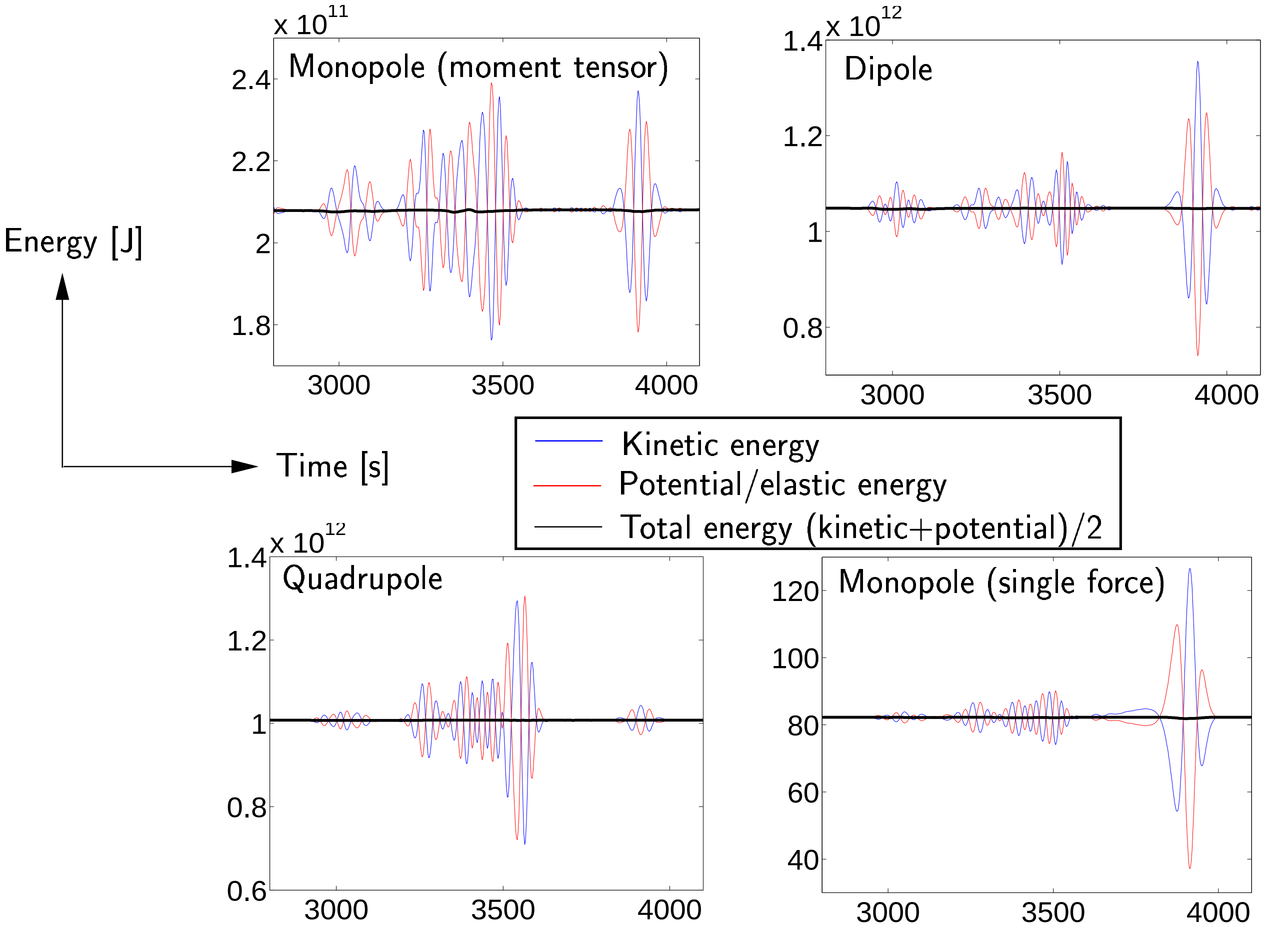
$\mathcal{O}^4$ symplectic scheme: 4-fold force evaluation per Δt





⇒ Symplectic scheme more cost-effective

Total energy conservation

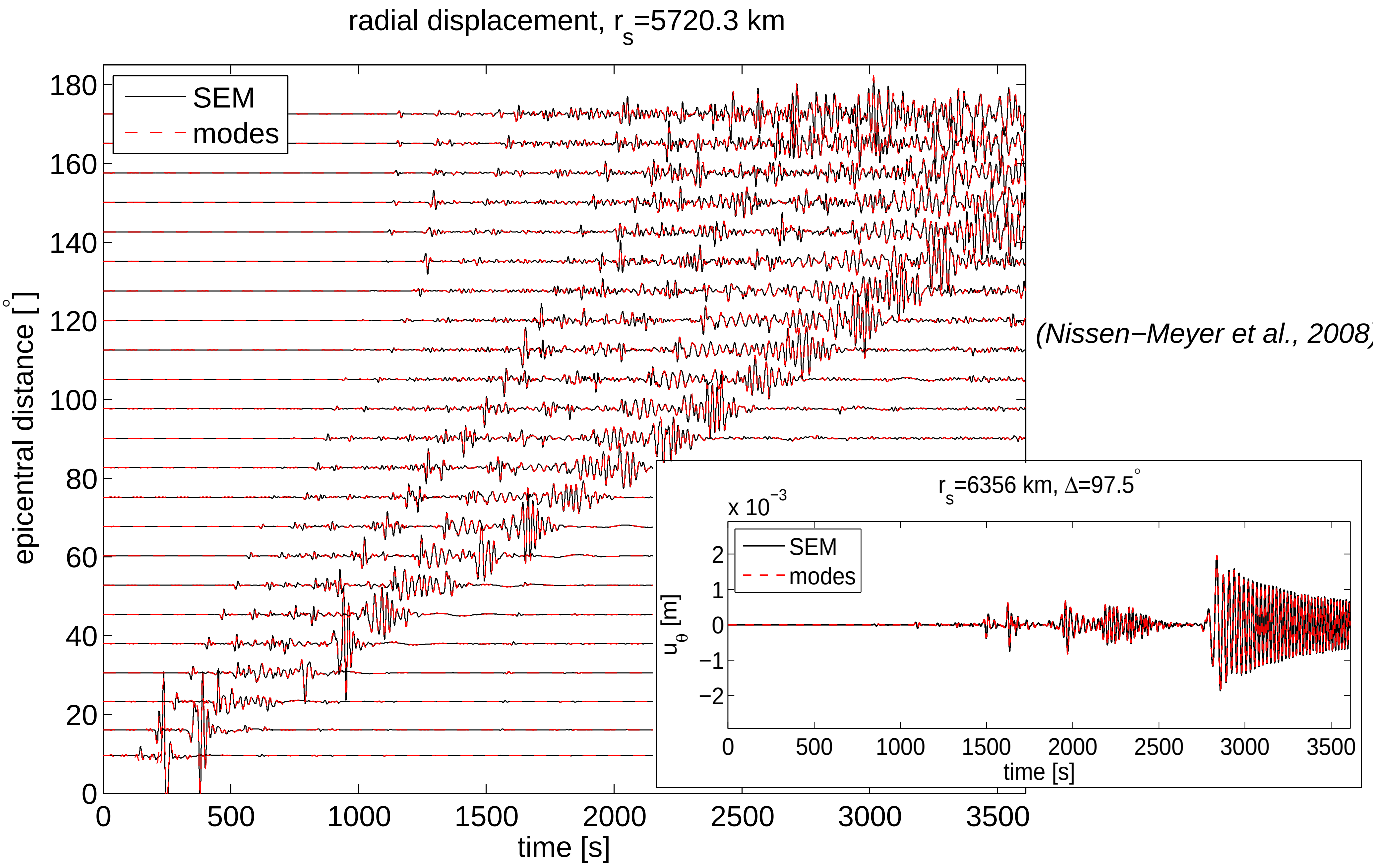


Debugged: ✓

3. Applications & Projects:

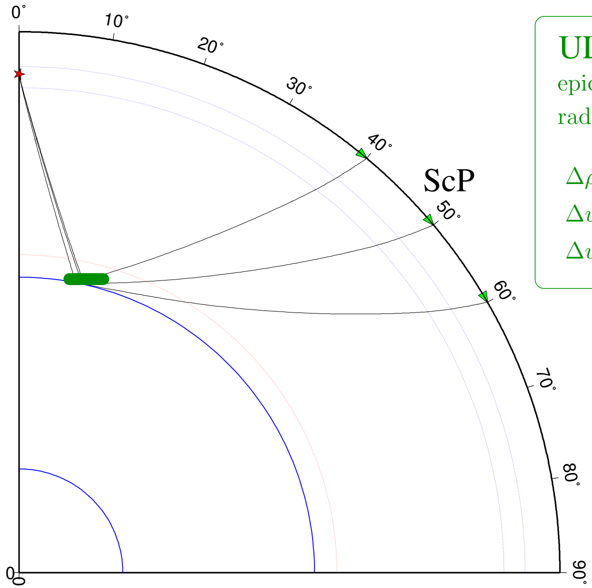
P_{diff}, v_p/v_s, heterogeneous models

I. Forward synthetics



Mesh resolution: $T_0 = 9\,\mathrm{s}$, 721500 grid points, 35000 time steps.

II. Heterogeneities & hi-res waveform



ULVZ

epicentral distance: 9 – 15°

radial extent: 15km atop CMB

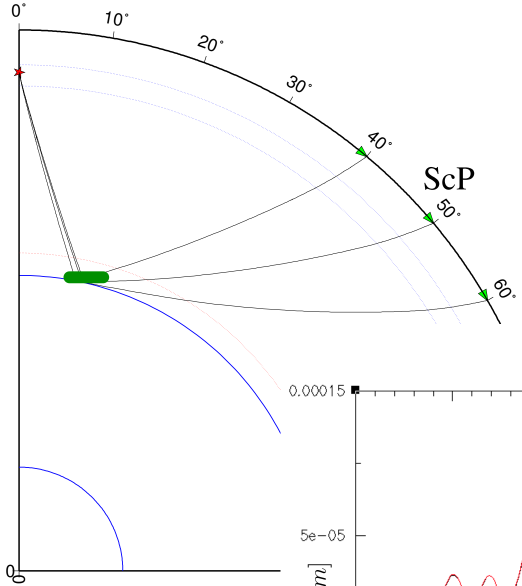
$\Delta\rho = +10\%$

$\Delta v_p = -15\%$

$\Delta v_s = -45\%$

(with Jensen & Thorne)

ULVZ's: ScP



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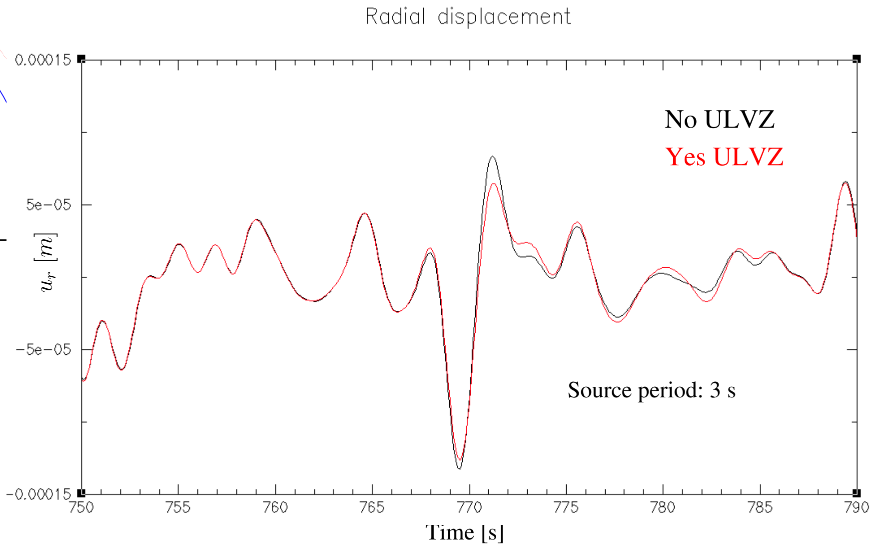

(with Jensen & Thorne)

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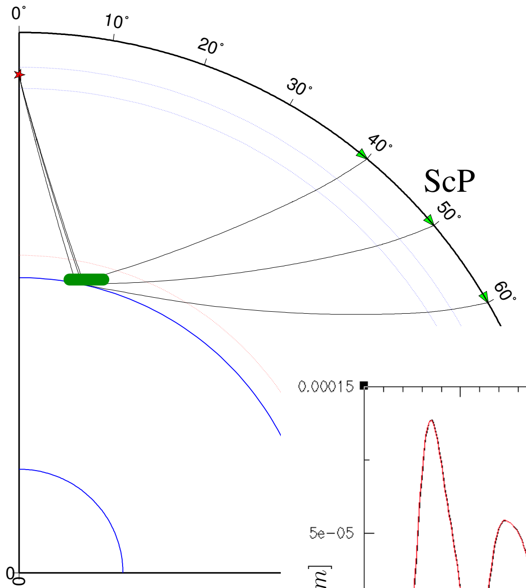

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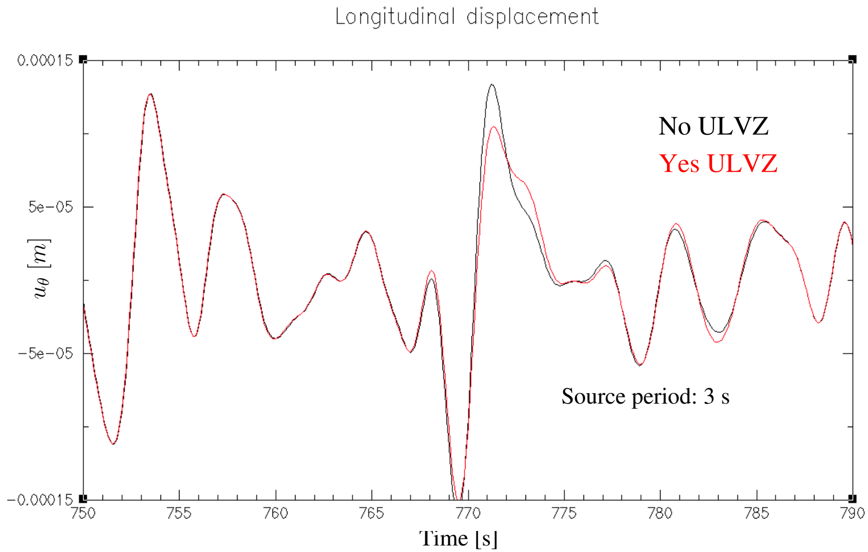


(with Jensen & Thorne)

$G(\mathbf{x}_s, \mathbf{x}_r; t)$ applications: ✓

3b. Seismic sensitivity:

Forward methods: $\mathcal{F} : m_0 \rightarrow d_0$

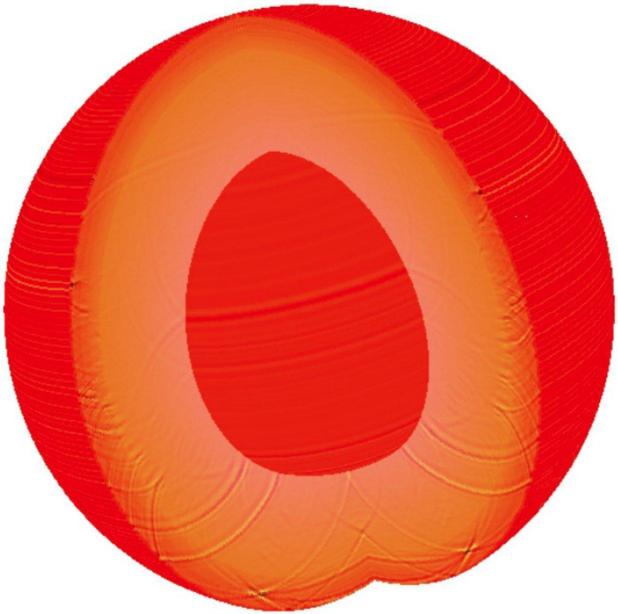
Inverse methods: $\mathcal{F}^{-1} : d \rightarrow m$

Fréchet derivatives

Spatio-temporal sensitivity kernels

$$\mathcal{K}_\kappa\left(\mathbf{x},t\right) = -\int_0^t \left[\boldsymbol{\nabla}\cdot\overrightarrow{\mathbf{u}}\left(\mathbf{x},\tau\right)\right]\left[\boldsymbol{\nabla}\cdot\overleftarrow{\mathbf{u}}\left(\mathbf{x},t-\tau\right)\right]\,d\tau$$

Forward strain trace



Time: 1250 seconds

Epicentral distance: 120 degrees

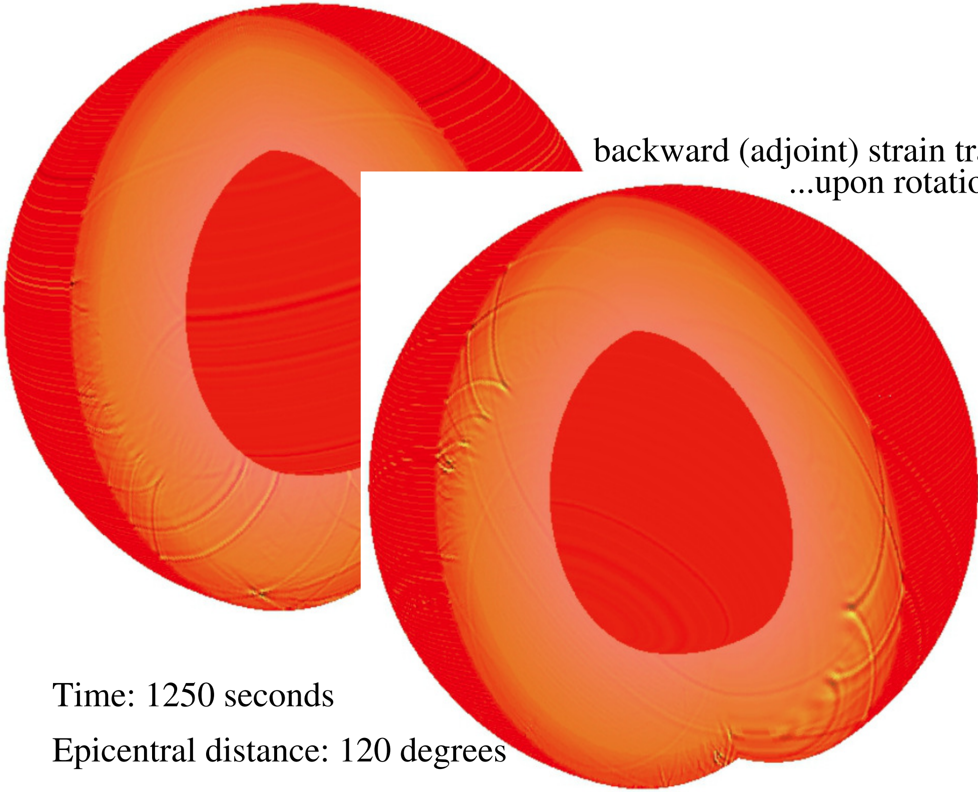
Spatio-temporal sensitivity kernels

$$\mathcal{K}_\kappa\,(\mathbf{x},t) = -\int_0^t \left[\boldsymbol{\nabla}\cdot\overrightarrow{\mathbf{u}}(\mathbf{x},\tau)\right]\left[\boldsymbol{\nabla}\cdot\overleftarrow{\mathbf{u}}(\mathbf{x},t-\tau)\right]\;d\tau$$

Forward strain trace

backward (adjoint) strain trace

...upon rotation

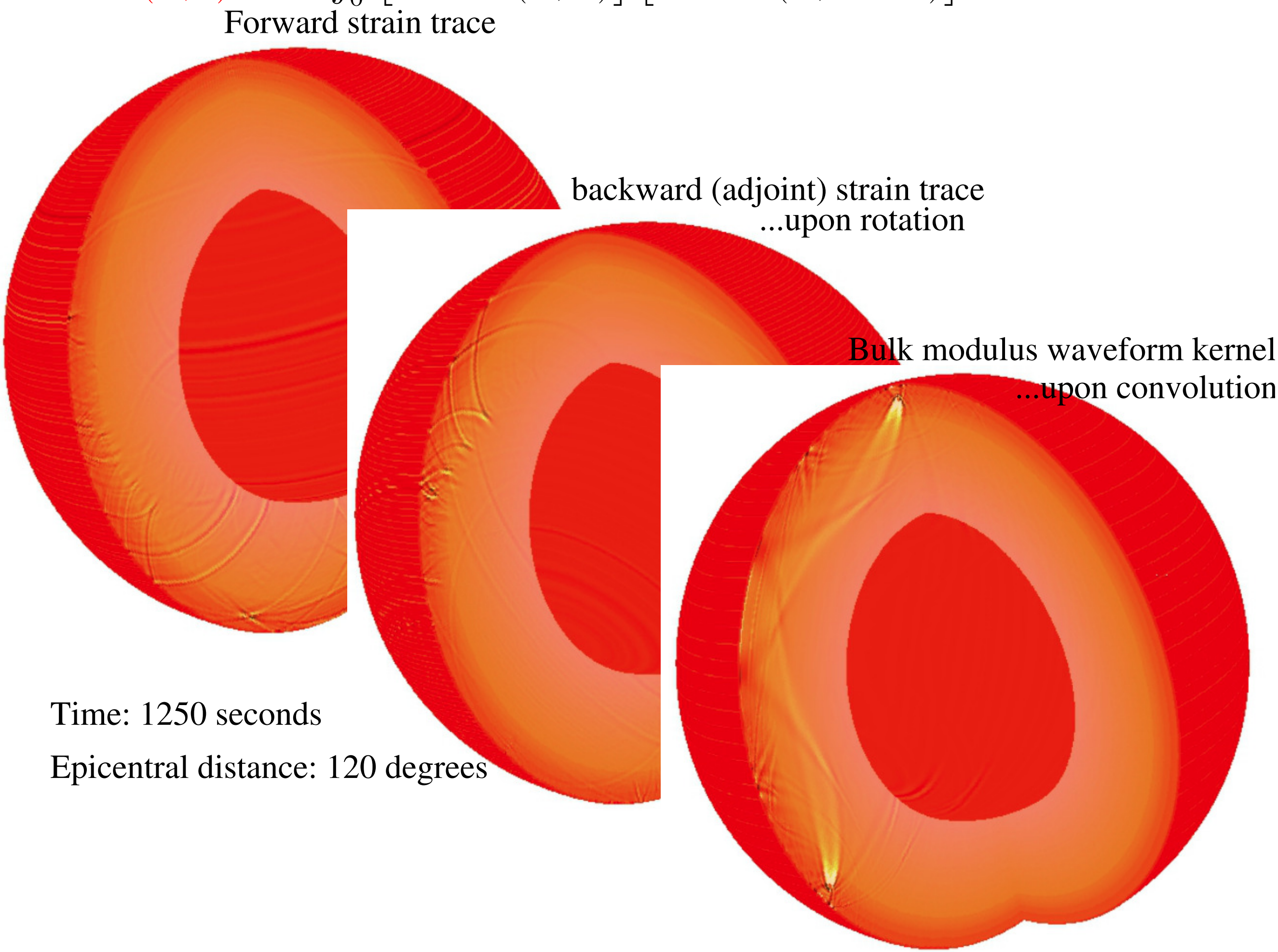


Time: 1250 seconds

Epicentral distance: 120 degrees

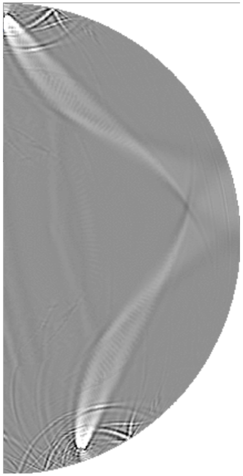
Spatio-temporal sensitivity kernels

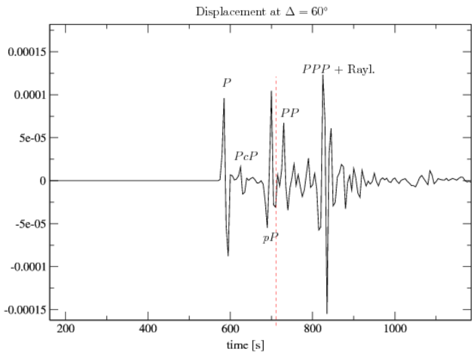
$$\mathcal{K}_{\kappa}\left(\mathbf{x},t\right) = -\int_0^t \left[\boldsymbol{\nabla}\cdot\overrightarrow{\mathbf{u}}(\mathbf{x},\tau)\right]\left[\boldsymbol{\nabla}\cdot\overleftarrow{\mathbf{u}}(\mathbf{x},t-\tau)\right]\, d\tau$$



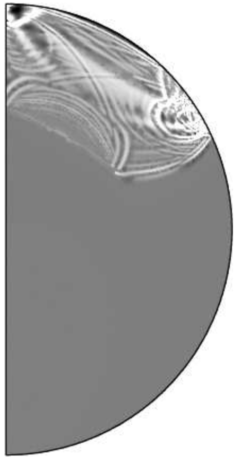
$K_1(\mathbf{x}, t)$: Seismogram & structure

165°, core phases, surface multiples





60°, core reflections



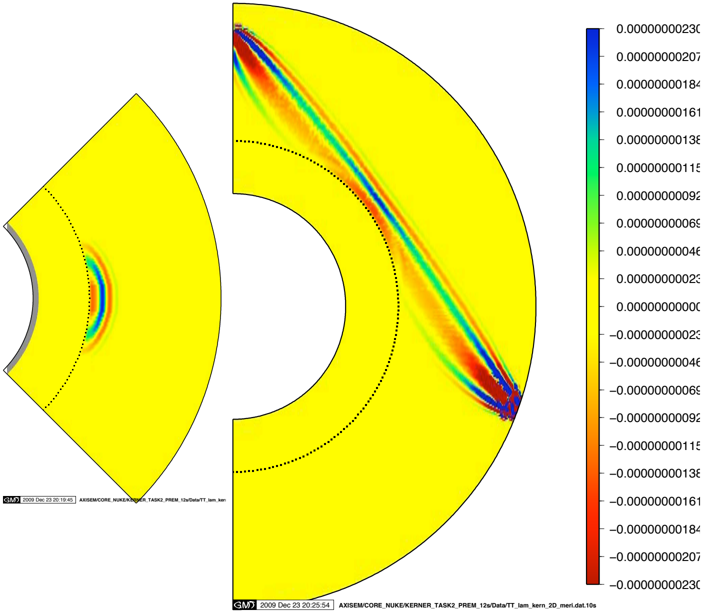
(Nissen-Meyer & Fournier, to be submitted)

..... and the **Hessian**!

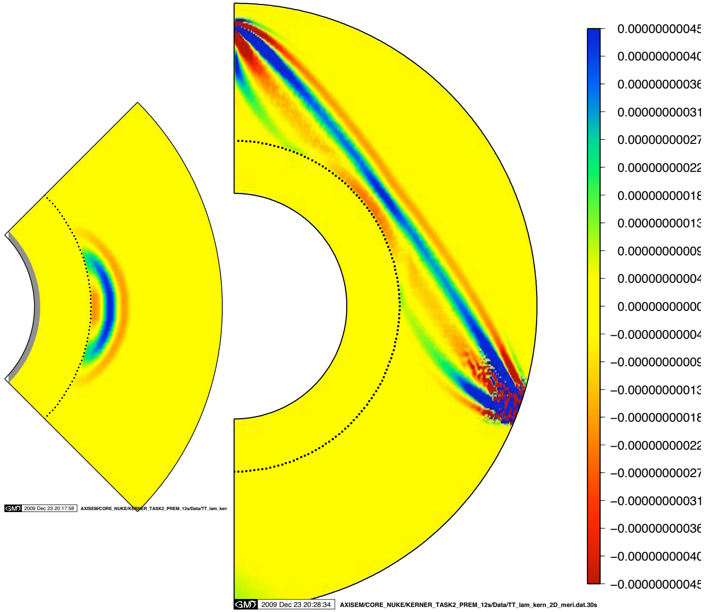
$K_2(\mathbf{x}, t)$: Born in ω space

Pdiff, bulk modulus, 110deg

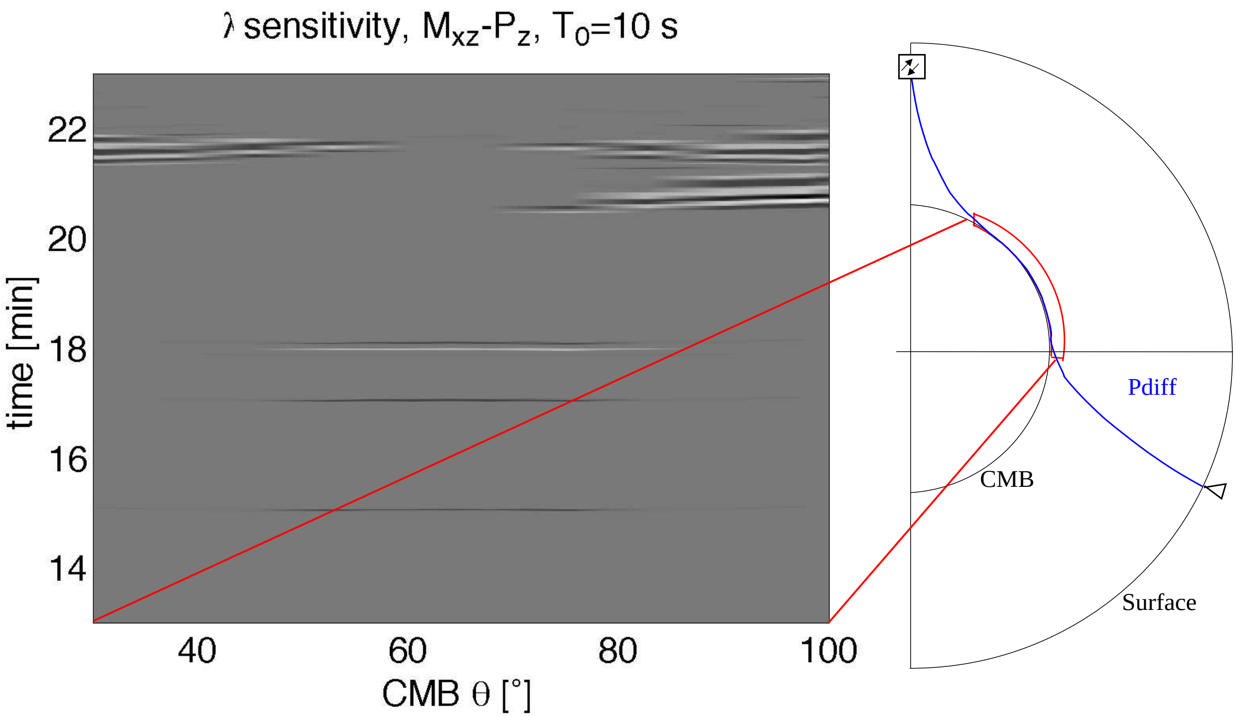
T=10sec



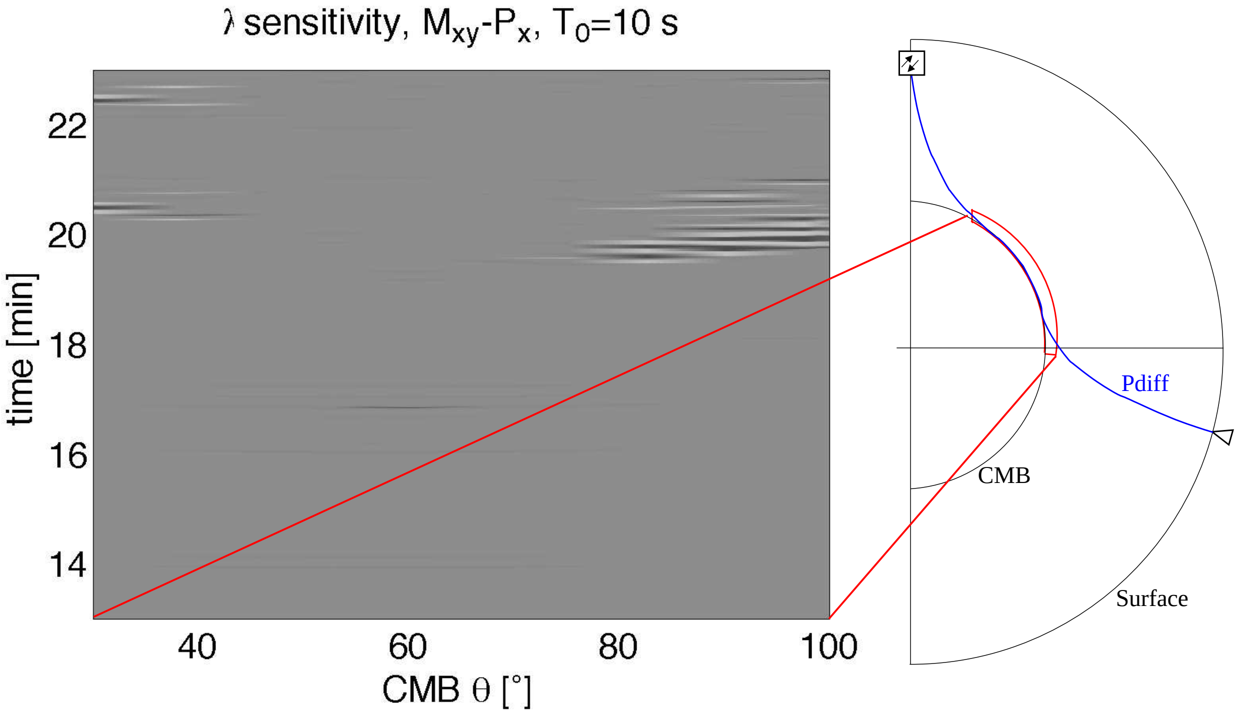
T=30sec



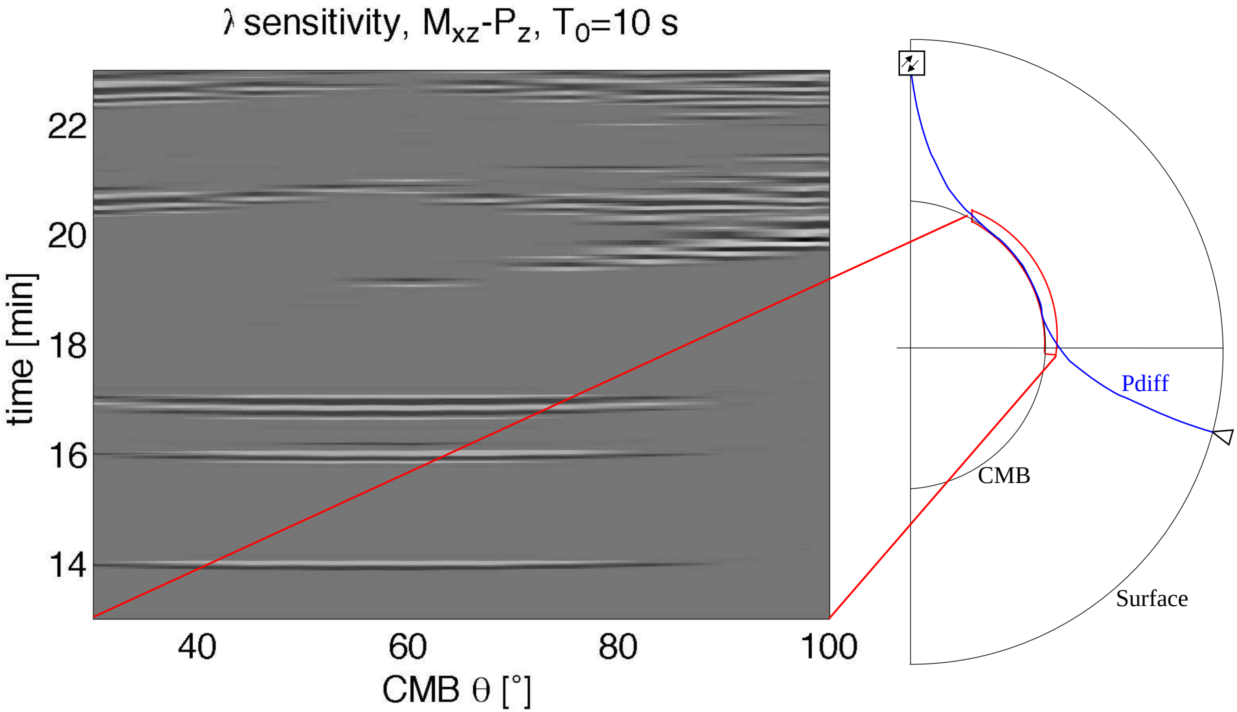
$K_3(\mathbf{x}, t)$: is sensitive: $\Delta = 127°$



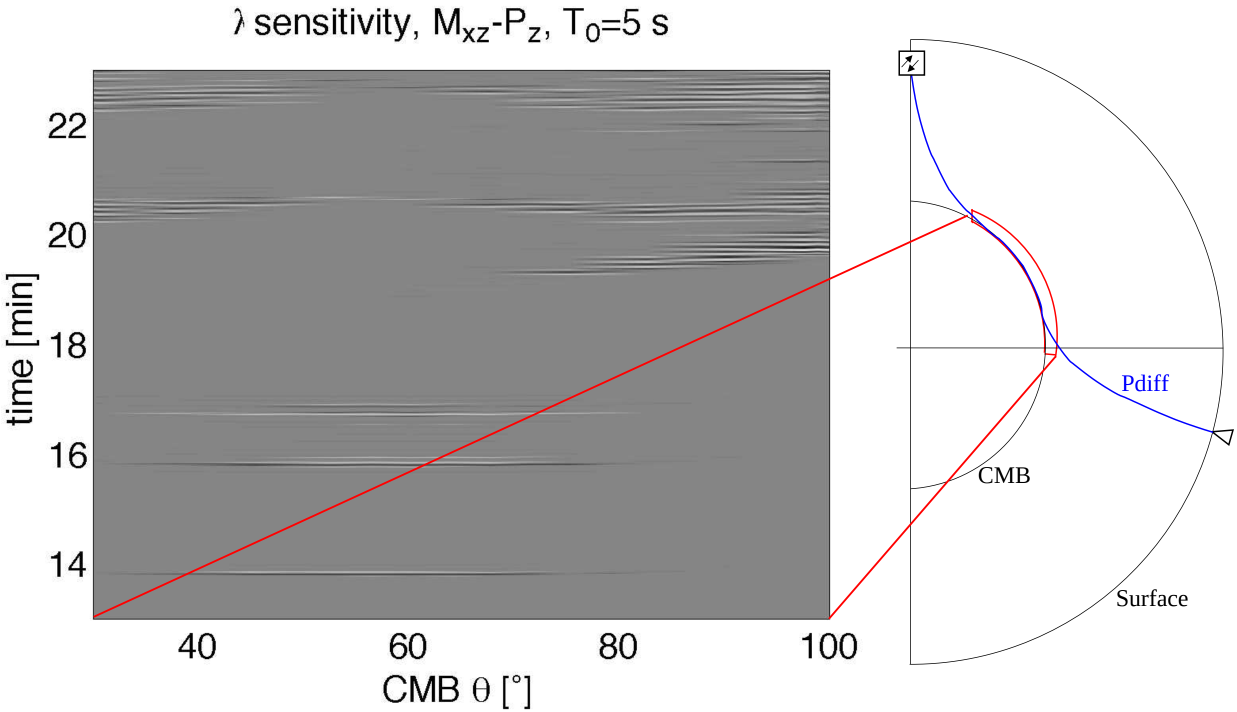
$K_3(\mathbf{x}, t)$: is sensitive: $\Delta = 112°$



$K_3(\mathbf{x}, t)$: is sensitive: $\Delta = 112^\circ$



$K_3(\mathbf{x}, t)$: is sensitive: $\Delta = 112°$

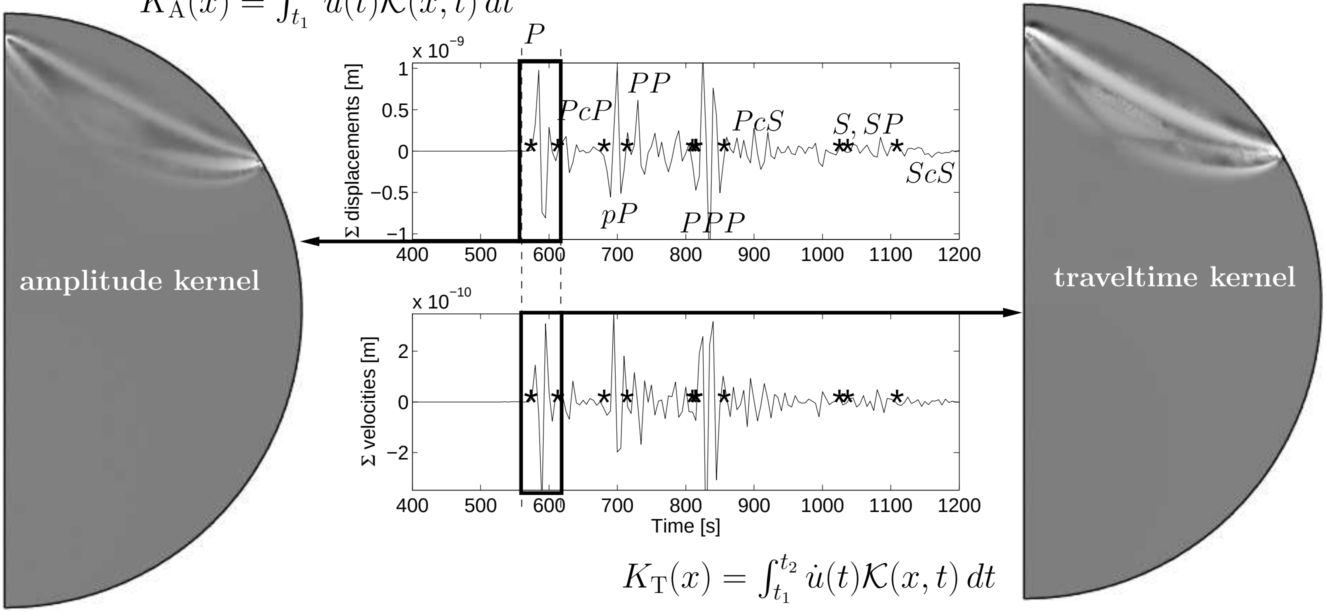


$K_4(\mathbf{x}, t)$: kernel of measure-kernels

⇒ Select observable time window (t_1, t_2)

⇒ Integrate waveform Fréchet derivative and seismogram:

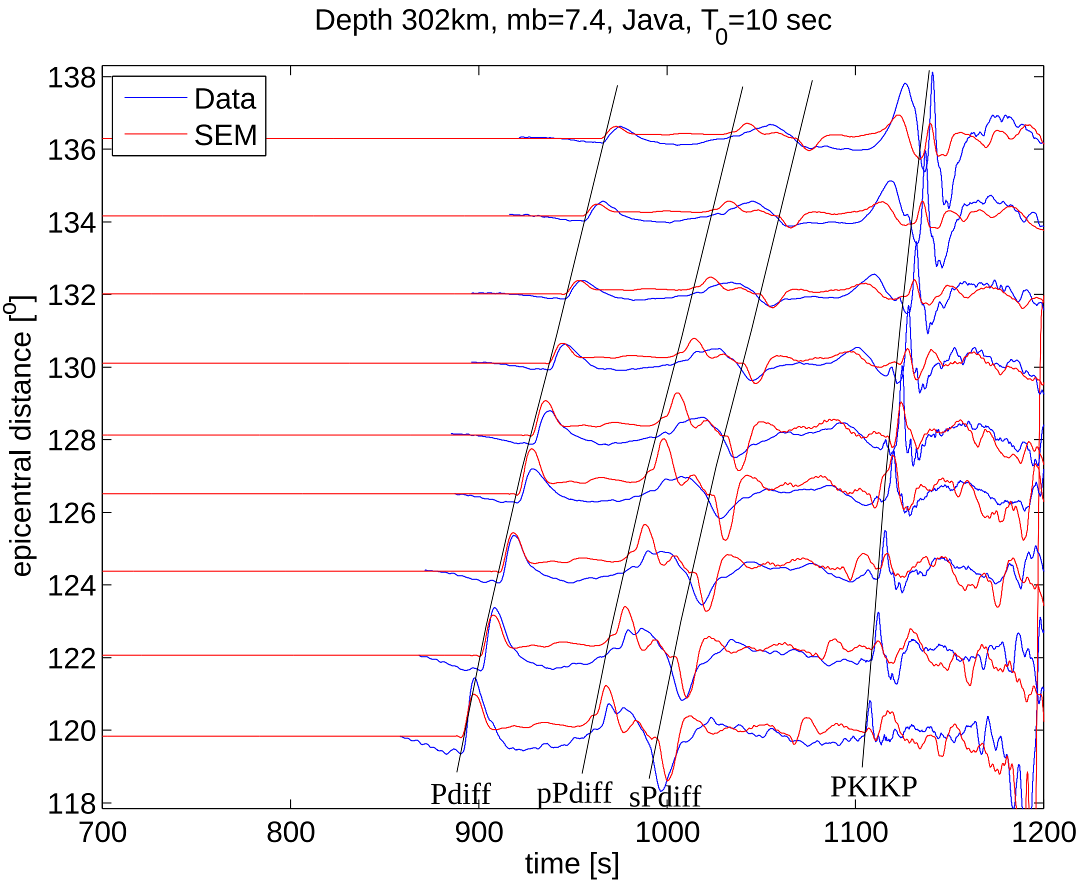
$$K_\mathrm{A}(x) = \int_{t_1}^{t_2} u(t)\mathcal{K}(x,t)\,dt$$



$$K_\mathrm{T}(x) = \int_{t_1}^{t_2} \dot{u}(t)\mathcal{K}(x,t)\,dt$$

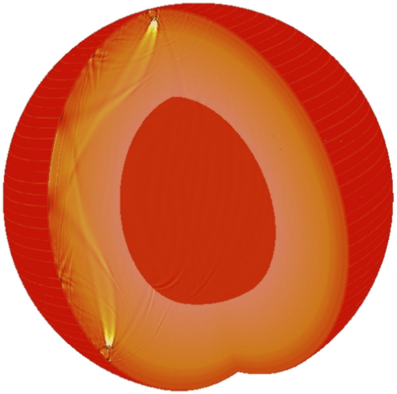
▶ $K(\mathbf{x}, t)$ independent of data selection!

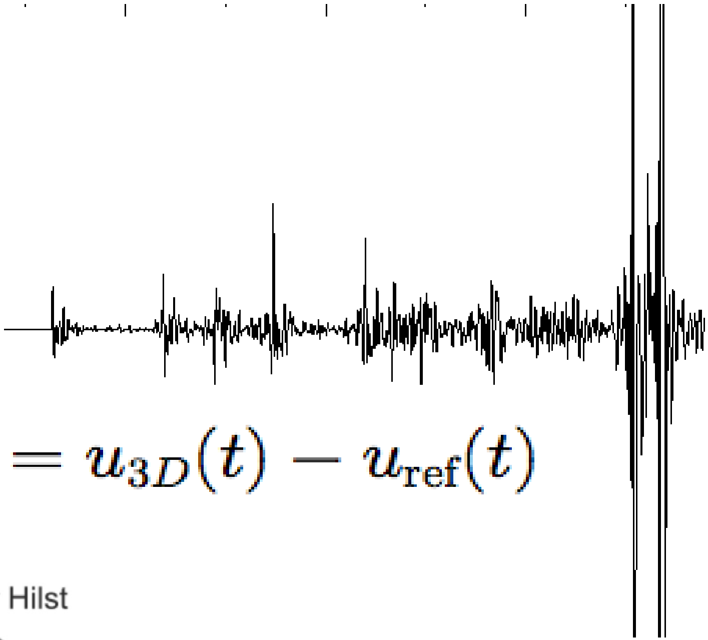
Data & SEM: core diffraction



(Nissen-Meyer & Sigloch, in preparation)

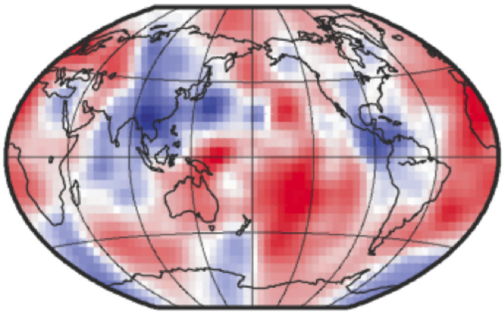
$K_5(\mathbf{x}, t)$: Born modeling





$$\int_V K(\mathbf{x}, t)\Delta m(\mathbf{x})\, d^3\mathbf{x} = u_{3D}(t) - u_{\text{ref}}(t)$$

(d) dVp: Karason & van der Hilst



2 deliverables

Forward solution $G(x_s, x_r; t)$

- 3D wavefields upon 2D SEM in 1D models
- Heterogeneities: High-frequency full-wave modeling

Spatio-temporal kernel-kernel $K(x, t)$

- Seismogram(t) $\sim$ structural sensitivity$(\mathbf{x}, t)$
- Sensitivity of kernels:
  source mechanism, distance, frequency, receiver components, time window, misfit function, ...
- Frequency-domain manipulation: Filtering, convolution
- Pre-data database: Efficient basis for global tomography
- Hessian
- Born again: Synthetics upon tomographic 3D models

Coding

What makes a technique/implementation **popular**?

- favorable cost-error function at various settings
- inclusion of relevant complexity in model and physics
- flexibility to change/add anything (e.g., models)
- code simplicity (readability, good examples)
- availability (open-source, feedback, manual)
- promotion (publications, talks)

How to make it scientifically **relevant**?

- Communicate with data-driven colleagues
- Clearly state the realm of advantageous applications