

Equations and Scaling

Inversion 0000

What Geodynamicists need from Seismologists

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Paleoworlds

Part I: Introduction



- Achievements
 - many of them
 - global
 - high resolution
 - comp. efficient
- Challenges
 - link to data





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Geodynamic Models

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(Bunge)



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Recent Earth history (Late Mesozoic/Cenozoic Record):

horizontal motion plate histories

(Dietmar. Mueller, Earthbyte Project University of Sydney, 2011)



Recent Earth history (Late Mesozoic/Cenozoic Record):

vertical motion epi-orogeny

94 Myr:





Recent Earth history (Late Mesozoic/Cenozoic Record):

vertical motion epi-orogeny

94 Myr: western interior seaways in the Americas, subsidence in the Tethys realm





Recent Earth history (Late Mesozoic/Cenozoic Record):

vertical motion

epi-orogeny

66 Myr:





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Recent Earth history (Late Mesozoic/Cenozoic Record):

vertical motion epi-orogeny

66 Myr: *interior seaways, subsidence in the Tethys realm, mid-eastern oil*





Recent Earth history (Late Mesozoic/Cenozoic Record):

vertical motion epi-orogeny

14 Myr:





Recent Earth history (Late Mesozoic/Cenozoic Record):

vertical motion epi-orogeny

14 Myr: shallow seas in the Tethys and eastern Europe





Recent Earth history (Late Mesozoic/Cenozoic Record):



(Blakey, Paleoworlds)



Recent Earth history (Late Mesozoic/Cenozoic Record):



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Part II: Equations and Scaling



Conservation of Mass

Focus on longer time scales, and need to suppress acoustic waves. Done through the so called *anelastic approximation* $(\partial \rho / \partial t = 0)$, which yields mass conservation in the form:

$$abla \cdot (
ho \mathbf{v}) = \mathbf{0}$$



Conservation of Momentum

Focus on highly viscous, so called stokes flow.

acceleration = internal friction and driving





Simplifications for Stokes Flow

Focus on highly viscous, so called stokes flow.

$$\underbrace{\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right)}_{\text{small}} = -\nabla p + \eta \nabla^2 \mathbf{v} + \mathbf{f}$$

- Accelerations 20 magnitudes smaller, can be omitted
- Instaneous equilibrium of driving/resisting forces
- Elliptic equation, Boundary conditions are part of global equilibrium This is the reason we model global flow.



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- Boundary condition:
 - current plate motion
- Output:
 - present-day flow
- Lesson:
 - no simple flow geometry
 - no stable piles





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(Torsvik, 2010)



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Lassak et al., 2009





B) Thermochemical Piles



C) Dhuma Chustana



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Conservation of Energy

Temp changes = advection, conduction and heat sources





Conservation of Energy

Advection dominates diffusion because the mantle is a good insulator.





The Peclet Number

 $Pe = rac{UL}{\kappa}.$

An estimate of the advection dominance in the mantle is given through the Peclet number, which is of order 10^4 . This indicates that advective processes in the mantle dominate thermal diffusion by four orders of magnitude, outside of thermal boundary layers.

Consequence: Thermal gradients are sharp, thermal boundary layers accommodate large temperature changes, and hence the lateral temperature variations arising from the boundary layers are high. (one needs to cool the core.)



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⁽Trampert, 2004)



Geodynamic Forward Models

- Schuberth et al., 2008a,b
- Schaber et al., 2009
- Goal:
 - quantitative comparisson with seismic models by going through the convection process and mapping to elastic variation
 - testing compositional mantle models with dynamically plausible temperatures



Histogram Geodynamic Forward Model



Model M2 Visc. 100/1/100 Q_{CMB} 35%



Histograms Tomographic Models





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Seismic Filtering





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Gedankenstütze



 High resolution (of order 10⁹ grid points) geodynamic forward models of mantle heterogeneity can be constructed for comparisson to seismic models



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Part III: Geodynamic Inversions



The Adjoint equations of mantle convection

$$\nabla \cdot \phi = 0$$
$$\nabla \cdot (\eta \nabla \phi) + \tau \nabla \theta = \nabla \chi$$
$$-\frac{\partial \tau}{\partial t} - \nabla \cdot (\tau \mathbf{v}) + R \hat{\mathbf{e}}_r \cdot \phi = \nabla^2 \tau + \delta(\mathbf{x}, t - t_1) [\theta(\mathbf{x}, t_0) - \theta_I(\mathbf{x})]$$

Solve a set of unintuitive *adjoint* equations

- terminal condition on temperature
- adjoint diffusion operator stable vs. time-reversal
- iterative procedure: computationally expensive, but now feasible
 ⇒ optimise for suitable flow histories (backwards in time)



An example of our estimate for sub-icelandic mantle, 100 Myrs ago.

• Forward simulation:

LUDWIG

- $T \xrightarrow{\text{mineral physics}} V$
- residual forward model vs. tomography (from Grand)
- Adjoint simulation:
 - ► terminal condition
 T ← mineral physics v
 - model update for time 100 Myrs ago







Dynamically supported global topography 40 Myrs ago

 Low lying Tethys and Farallon regions associated with active subduction at the time. (Topography is 500 times exagerated.)



Large temporal Geoid variations implied by our inversions today





Large temporal Geoid variations implied by our inversions **40Myr**







Inversior

Large temporal Geoid variations implied by our inversions difference





Conclusion

- Geophysicists from seismology, mineral physics, and geodynamics together should start exploring time-dependent earth models.
- Large challenges (e.g., seismic resolution, composition, uncertainties in geologic interpretations) are ahead.
- Large payoff in terms of understanding the dynamics of our planet waits in return.

(Special thanks to: B. Schuberth, C. Moder, J. Oeser, M. Mohr, A. Horbach, L. Colli, T. Chust and everyone from the Munich group!)



Thank you!



Continuum: Elastic or Viscous?



Earth is viscoelastic:

- elastic over short times (earthquake waves!)
- viscous over long times (plate tectonics!)



Continuum: Elastic or Viscous?



Earth is viscoelastic:

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Maxwell time = viscous relaxation to 1/e:

$$\tau_{M} = \frac{\eta}{G}$$

 \Rightarrow Lithosphere ($au_{M} = 10,000$ yr) = viscous fluid



Viscous Behavior: Creep

Mechanisms:

- Nabarro-Herring creep (bulk diffusion)
- Coble creep (grain boundaries)
- Dislocation creep



Viscous Behavior: Creep

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Empirical Arrhenius-type equations:

$$k = A \, \mathrm{e}^{-Q_{\mathrm{creep}}/(R \, T)}$$

- (R: gas constant; Q_{creep} : activation energy)
 - T in exponent: sensitive
 - strongest = cold
 - dominant: dislocation creep



Mantle Circulation Models



- Mantle convection: basics well understood
- Needed: comparison with observations (tomography!)
- Problem: unknown initial conditions
 - \Rightarrow assimilation of surface velocities or backwards in time



Prev. TERRA MCMs: Plates from Lithgow-Bertelloni

- Coarse time stepping (10–20 Ma: artifacts like "jumping" slabs)
- Too short timespan (< convective time scale)
- Errors (Tethys!)

 \Rightarrow Need for better plate boundaries + new software





Thank you!



Laboratory Experiments?

- Fast deformation
 ⇒ other creep mechanisms
- Small samples
 ⇒ thermal equilibrium





Assumptions

- Spherical Earth, constant gravity: small error, only vertical direction
- Constant density: error less than 1% + in vertical direction
- Incompressible: $K \ll p$ in lithosphere \Rightarrow volume change < 0.5%
- Anelastic rheology: elasticity small + unknown over geologic times
- Isostatic equilibrium: $au_M \ll$ age of the Earth \Rightarrow equilibrated
- Constant thermal properties, mineralogy: correction via grid (heat)
- Newtonian rheology: approximation, true rheology not known



SHELLS: Only Momentum Equation

Temporal development of faults:

- Behavior of lithosphere depends on its past
- Initial conditions unknown \Rightarrow cannot create present-day state
- Future state unknown \Rightarrow from present-day to future is meaningless

Fault geometry from current forces: continuum deformation and fault formation is highly nonlinear \Rightarrow chaotic

 \Rightarrow Fault geometry as input, no energy equation needed



Comparison with GPS data

- Problem:
 - GPS signal includes elastic deformation
 - SHELLS omits elasticity
- Modifications to the model:
 - Brittle part of fault = locked
 - ► Slip in brittle part = compensated by elastic deformation
 - Comparison of elastic deformation with GPS data



Icosahedral Grid



