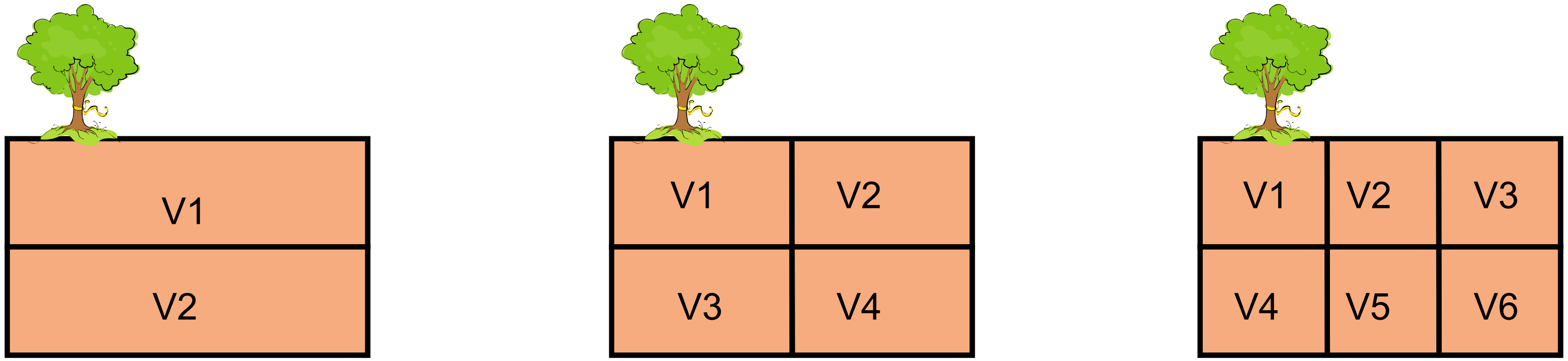
When one of the things you don't know is the number of things you don't know



N=2 N=4 N=6

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THE AUSTRALIAN NATIONAL UNIVERSITY

Thomas Bodin &
Malcolm Sambridge

Outline

1. The problem of model parameterization

2. Trans-dimensional Inverse Methods
   - Non-linear regression example

3. Application in Seismology
   - Receiver functions
   - Seismic tomography
   - Joint inversion

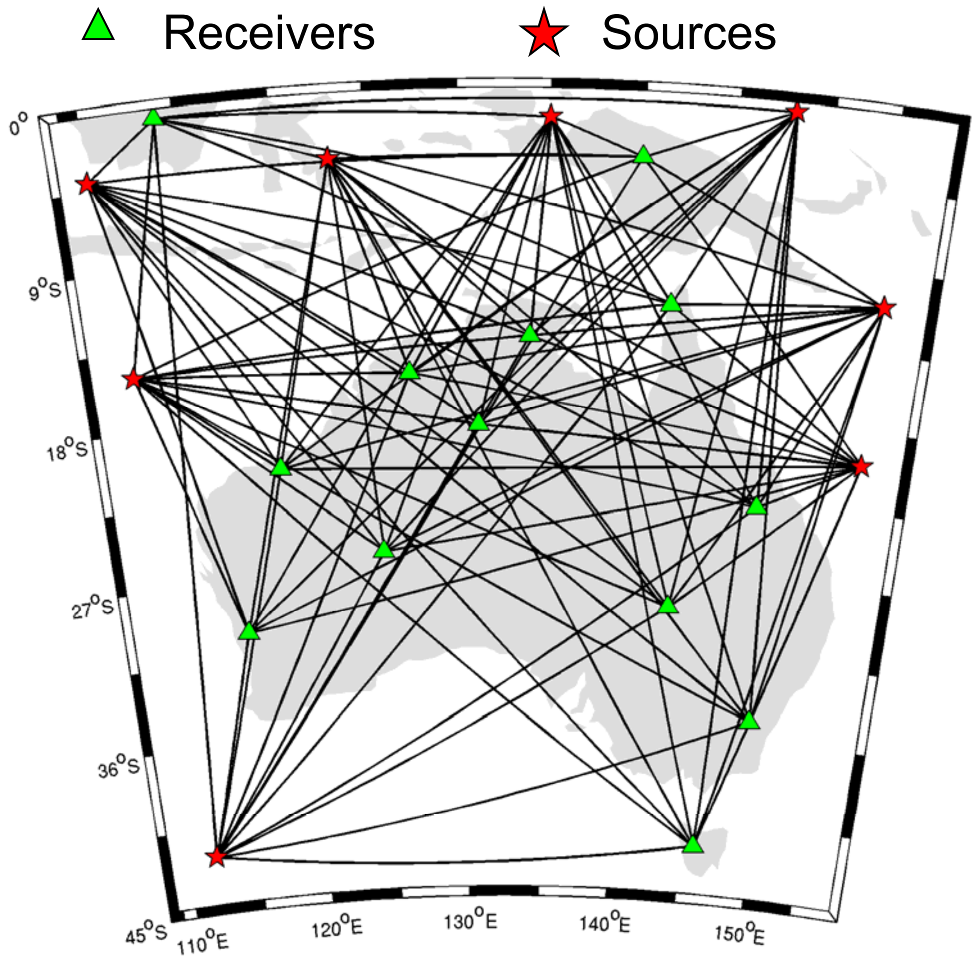
2D Seismic surface wave Tomography

We want

A map of surface wave velocity

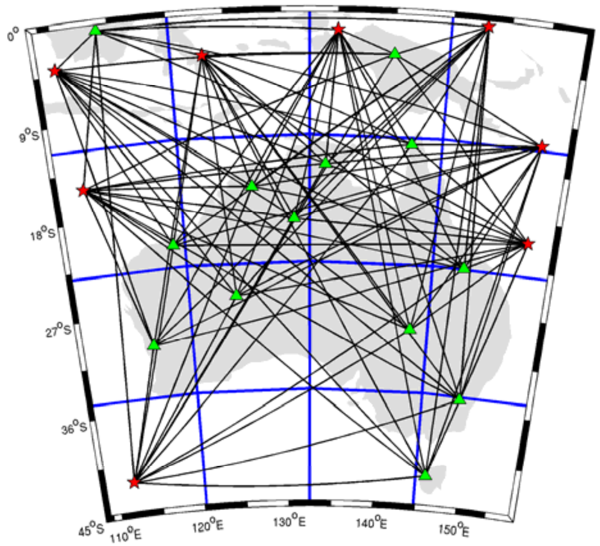
We have

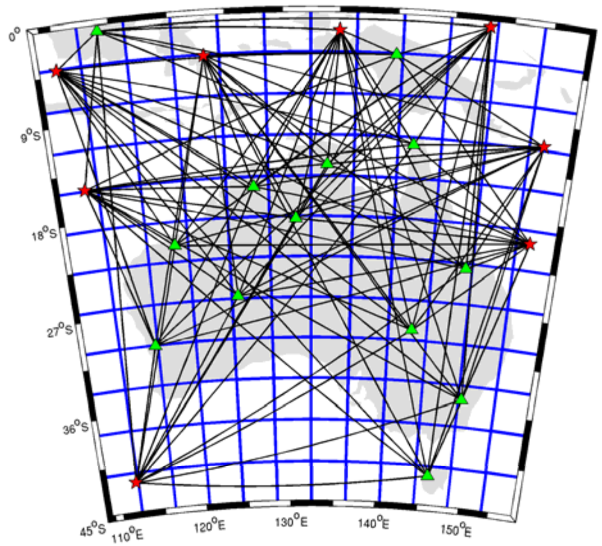
Average velocity along seismic rays



Regular Parameterization

Coarse grid | Fine grid

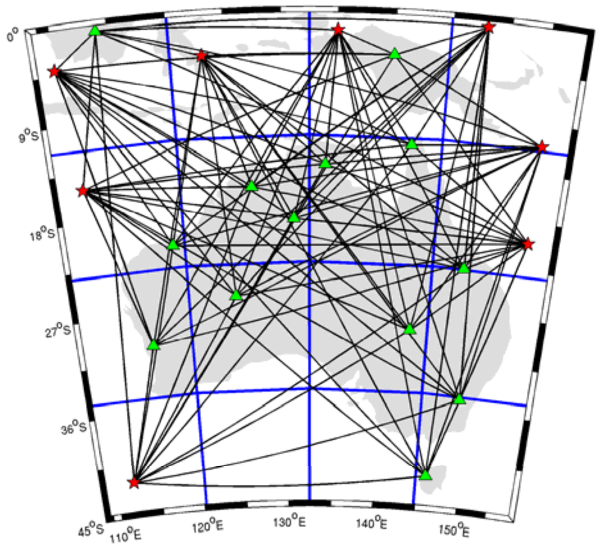




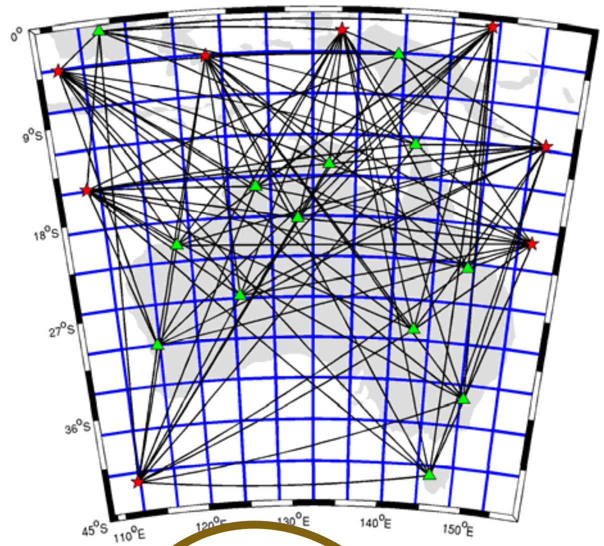
	Coarse grid	Fine grid
Resolution	Bad	Good
Constraint on the model	Good	Bad

Regular Parameterization

Coarse grid



Fine grid

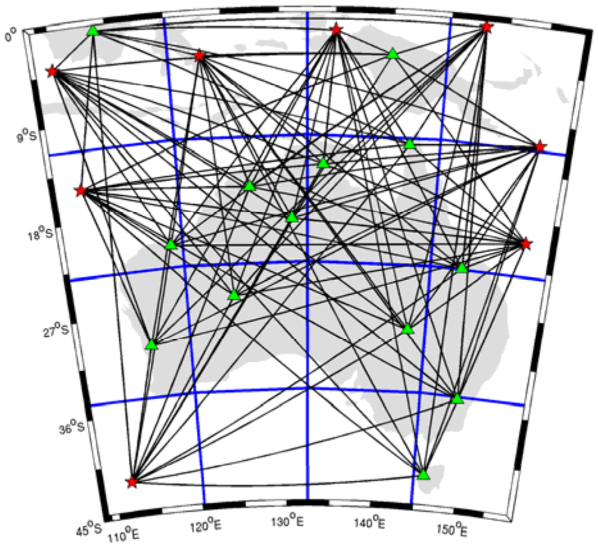


	Coarse grid	Fine grid
Resolution	Bad	Good
Constraint on the model	Good	Bad

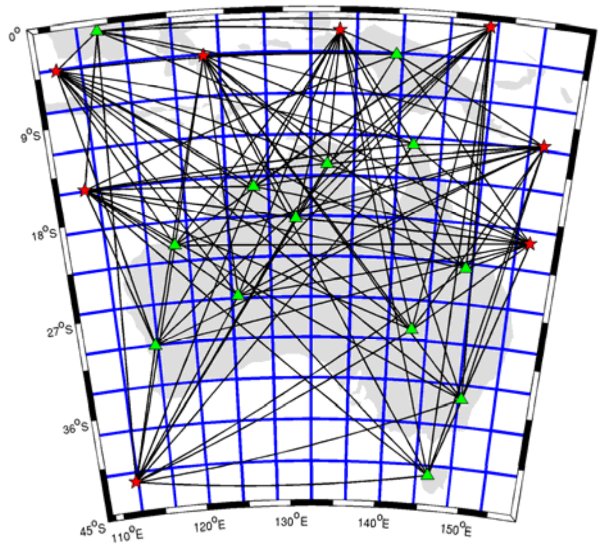
➡ Define arbitrarily more constraints on the model

Regular Parameterization

Coarse grid



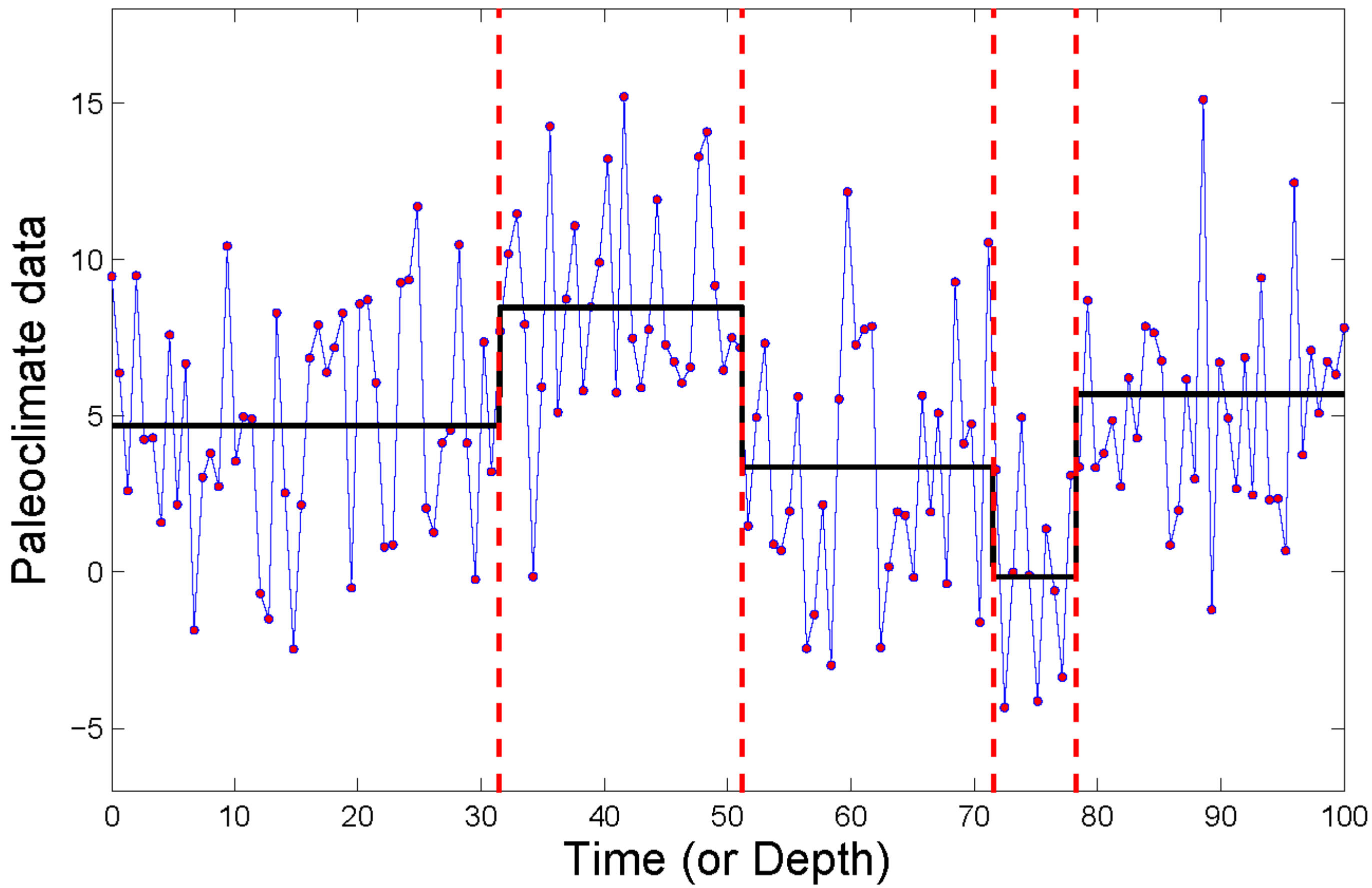
Fine grid



3 Problems

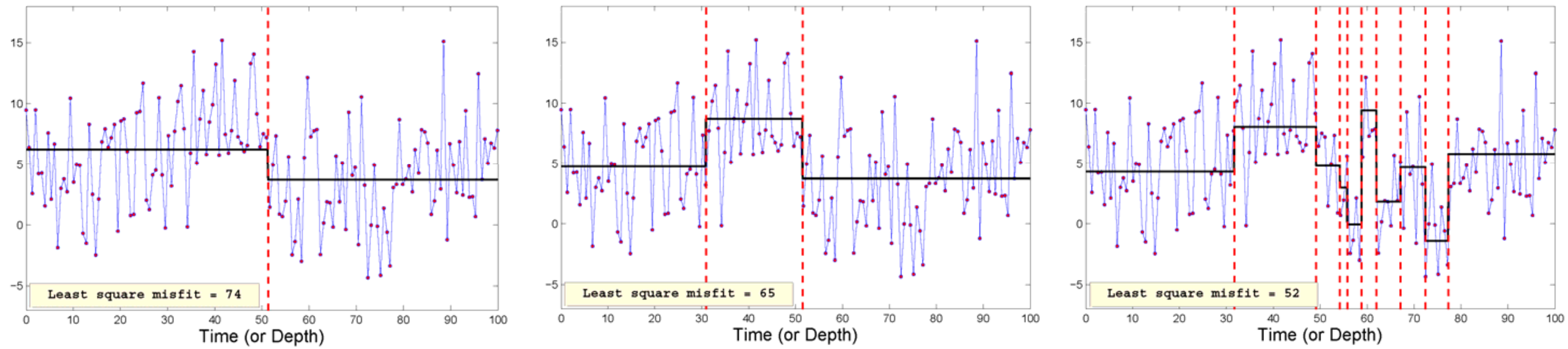
- Number of cells
- Level of damping or smoothing
- Regularization is global

Change Point Modelling of Paleoclimate Data



Difficulty of Optimization Schemes

$$\Phi(m) = \sum_i \left(\frac{d_i - g(m)_i}{\sigma} \right)^2$$



Worse —— Data fit ——> Better

Simple <—— Number of Change points —— Complex

Importance of data noise for choosing the solution

Bayesian Inference

Optimization:

Fix the number of cells minimize the data misfit

$$\Phi(m) = \sum_i \left(\frac{d_i - g(m)_i}{\sigma} \right)^2$$

Trans-dimensional Bayesian formulation:

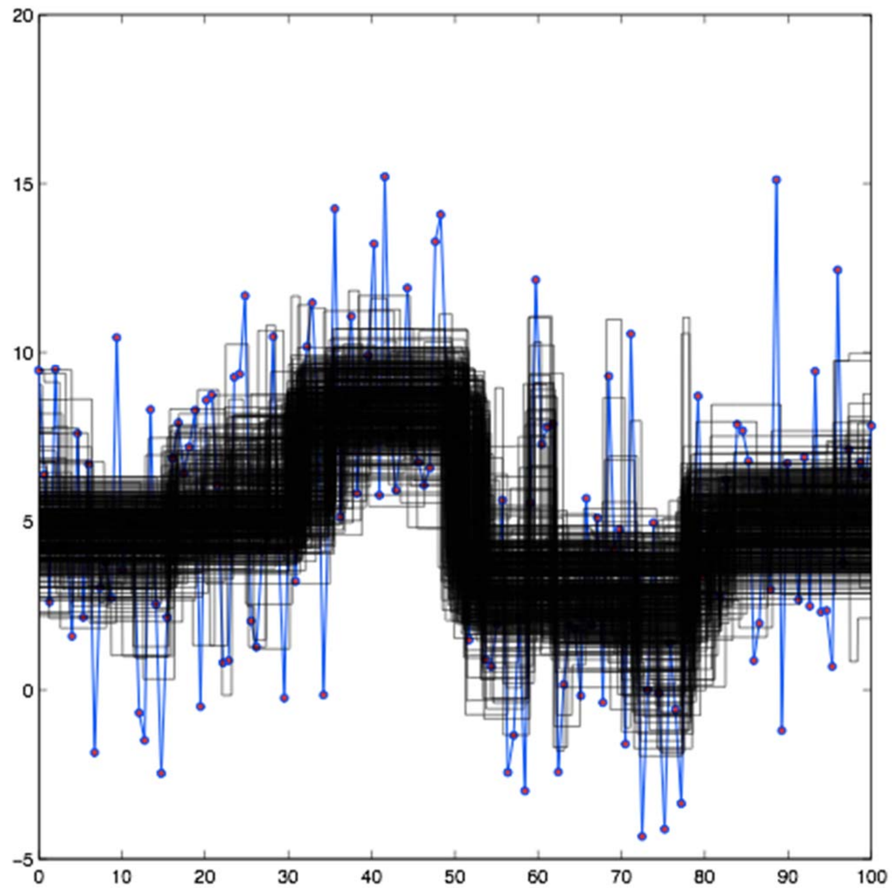
The number of cells is variable and the solution is a probability distribution

The solution is an ensemble of models with variable dimensions

Ensemble Inference

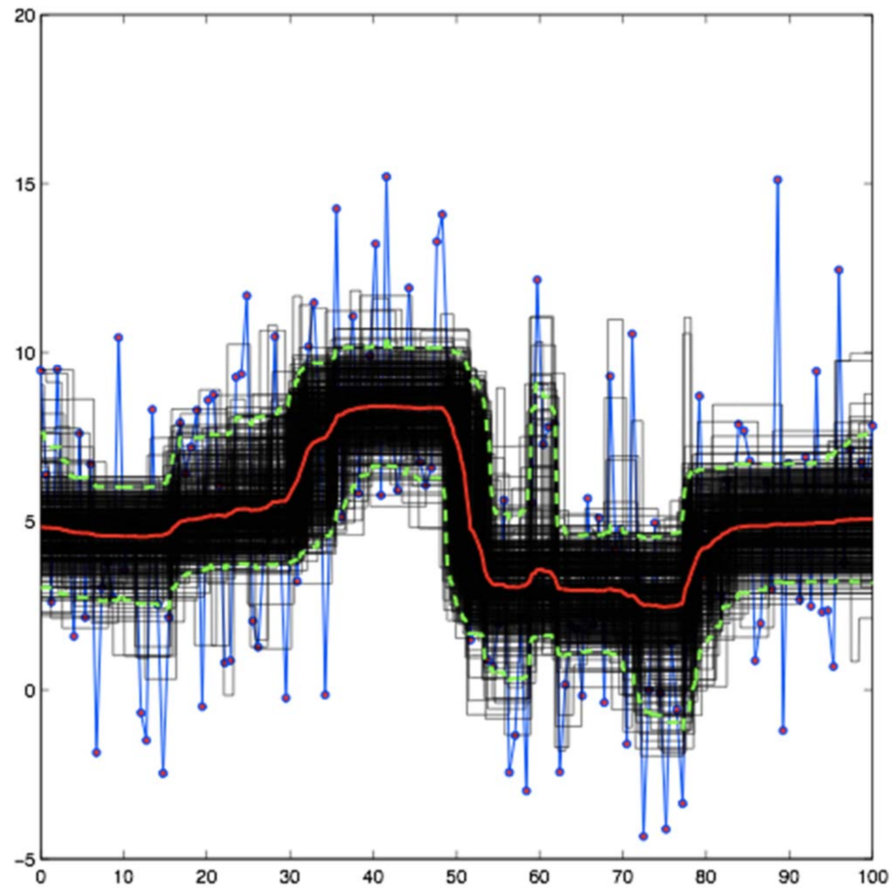
Trans-dimensional Markov chain

Solution is a large ensemble of models with varying parameterization.



Ensemble Inference

Some useful statistical information can be extracted from the ensemble solution

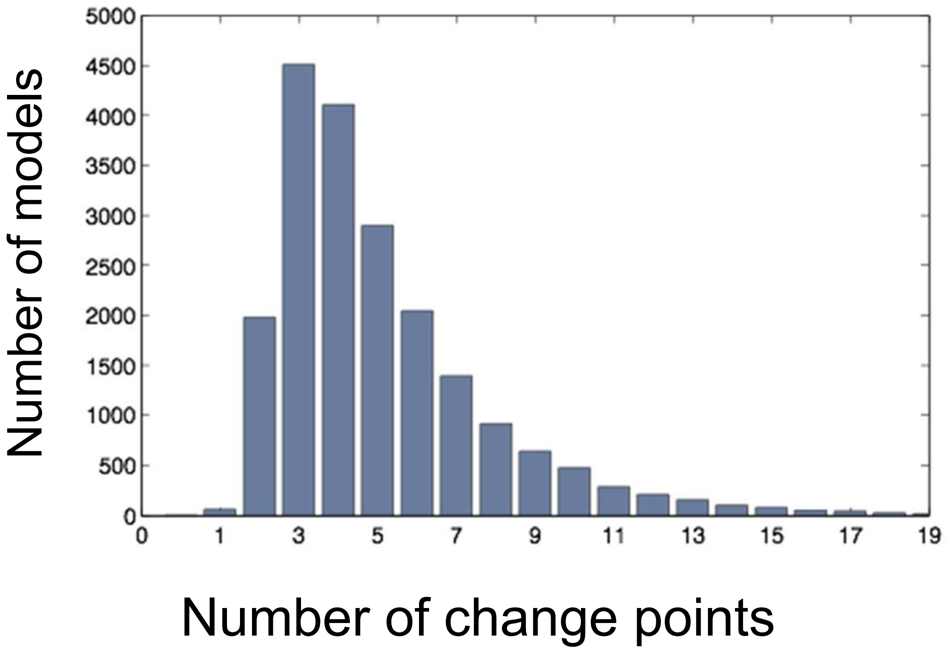


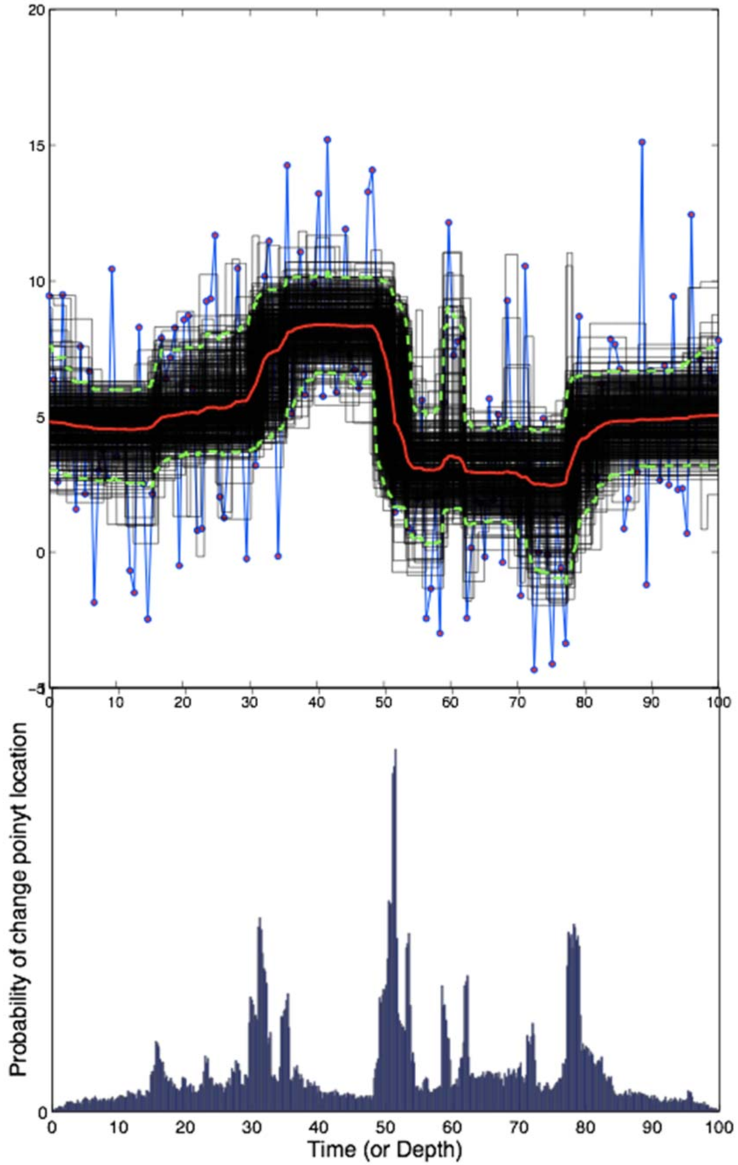
Mean

95% credible interval

Ensemble Inference

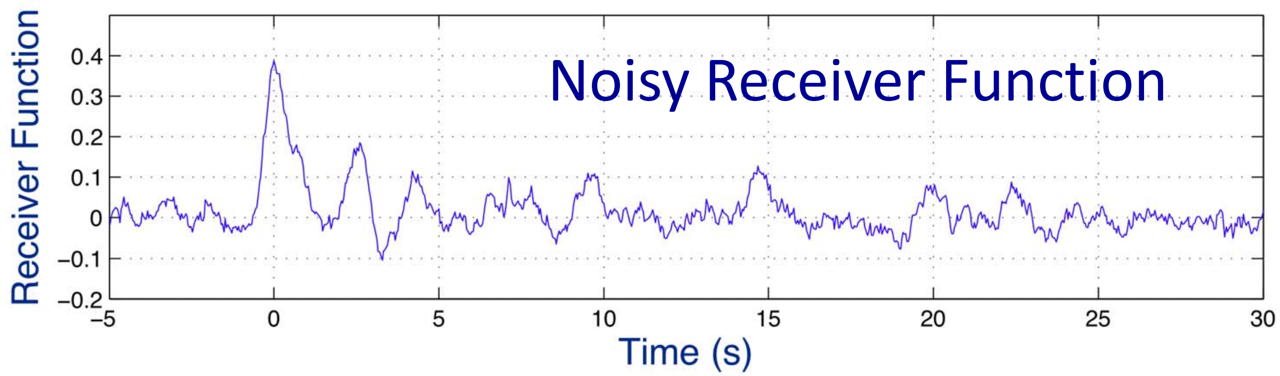
Some useful statistical information can be extracted from the ensemble solution

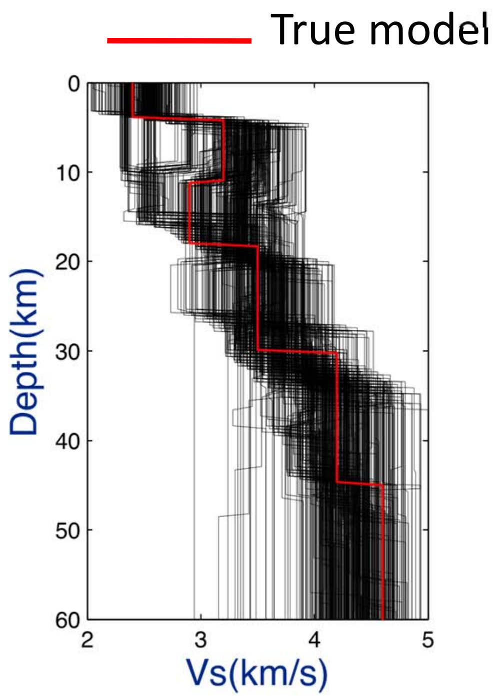




Inversion of Receiver Function

Synthetic experiment

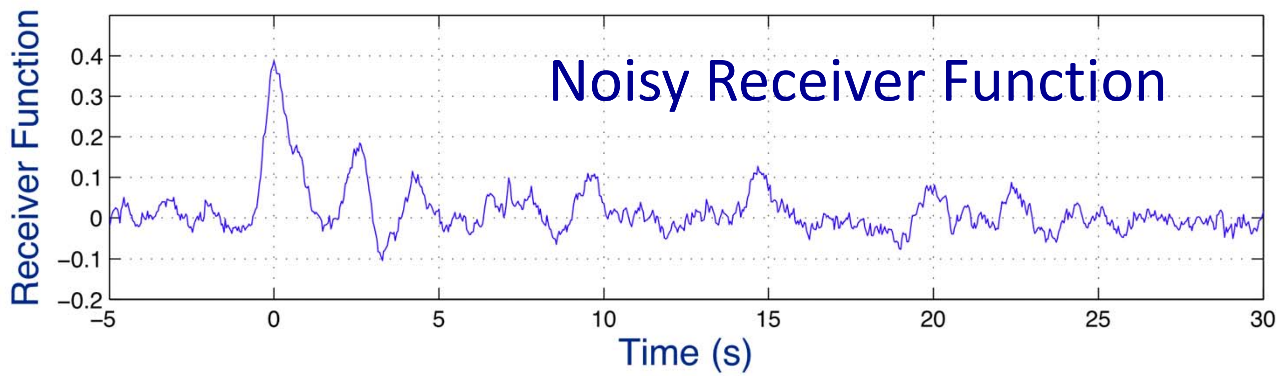


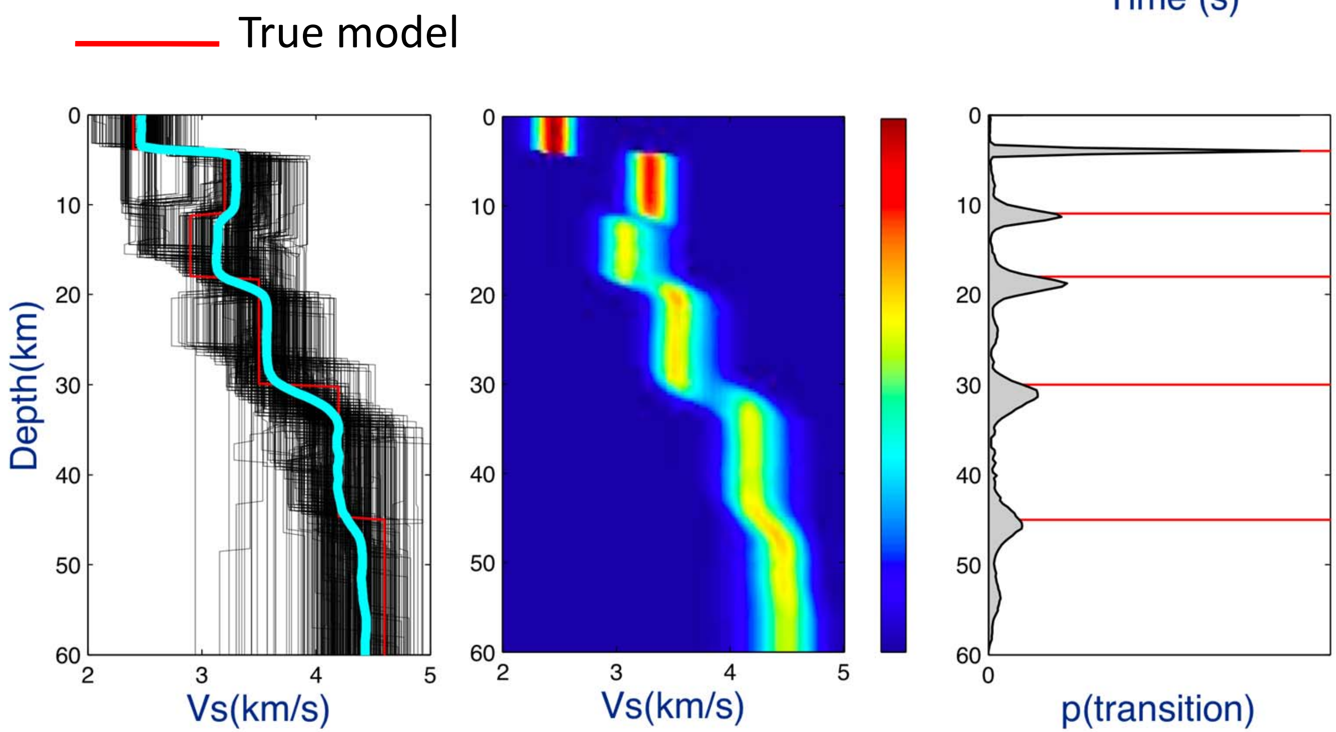


Solution is a large ensemble of models distributed according to the target distribution

Inversion of Receiver Function

Synthetic experiment





Different ways to look at the solution

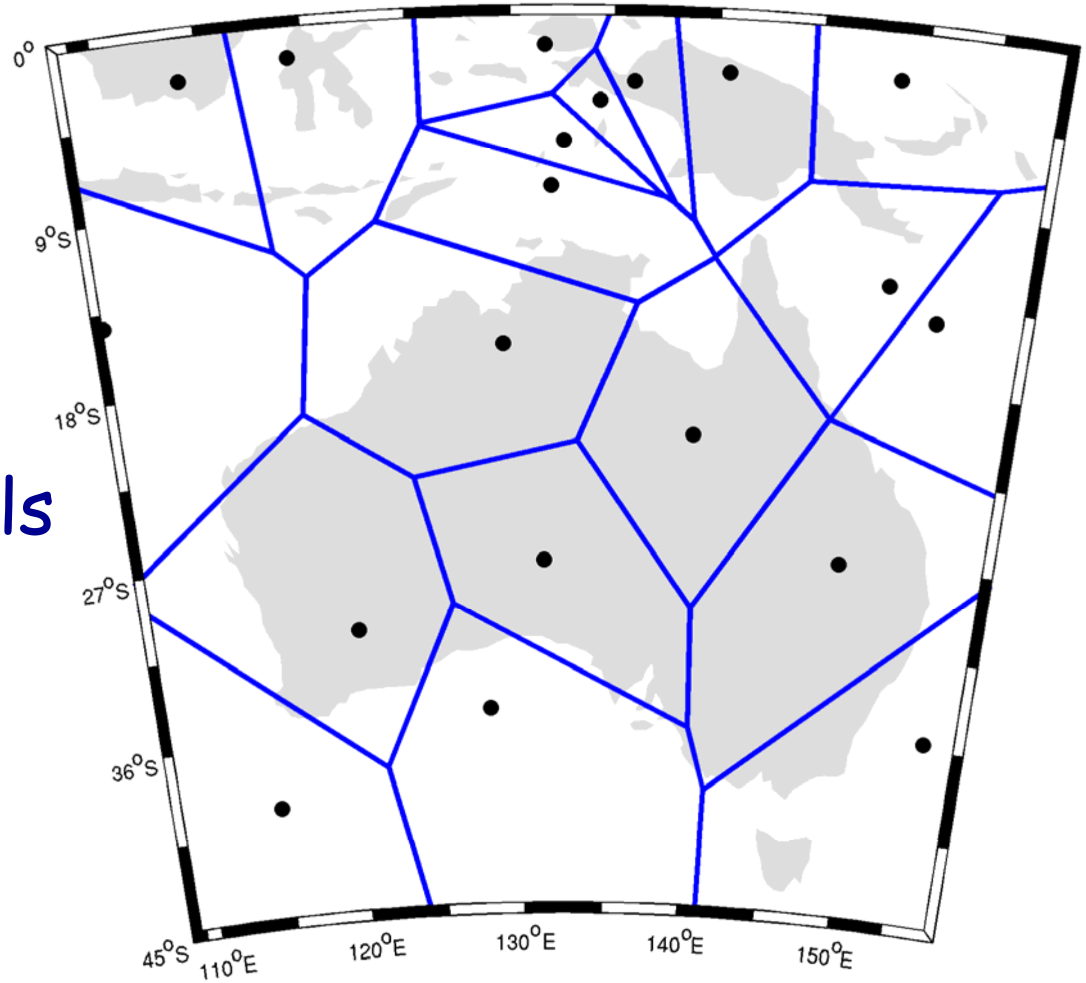
Application to Tomography

Voronoi Cells are only defined by their centres

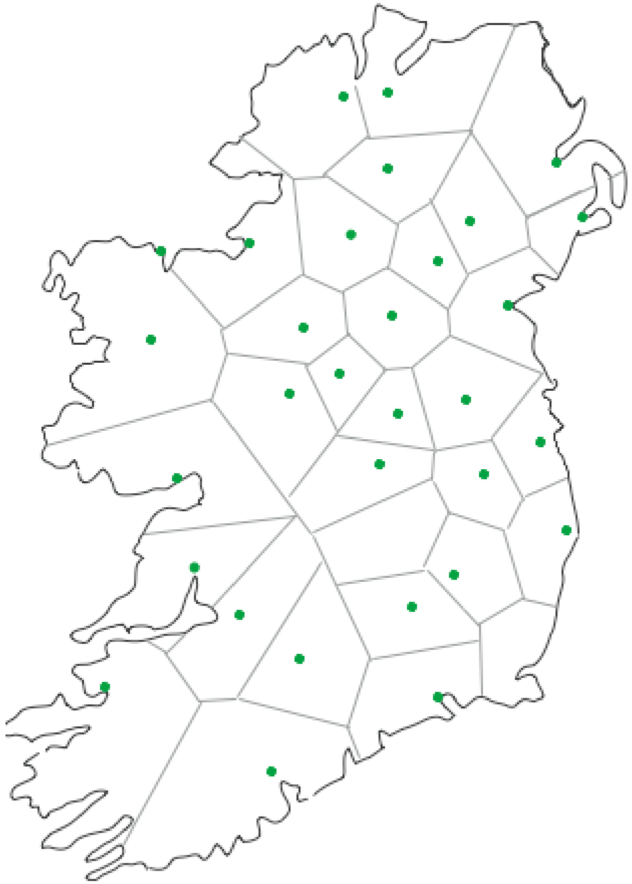
Variable number of cells

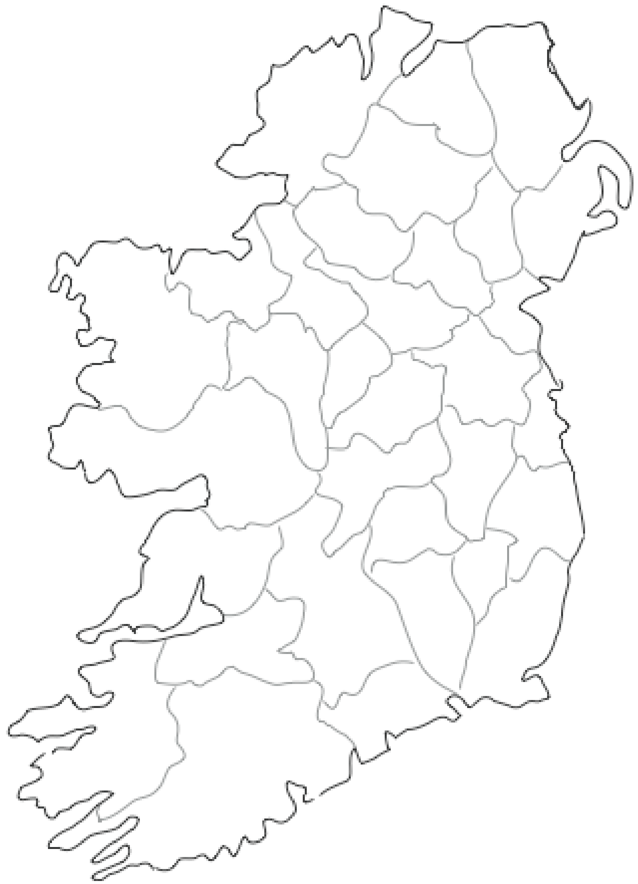
Model is defined by:

- Velocity in each cell
- Position of each cell

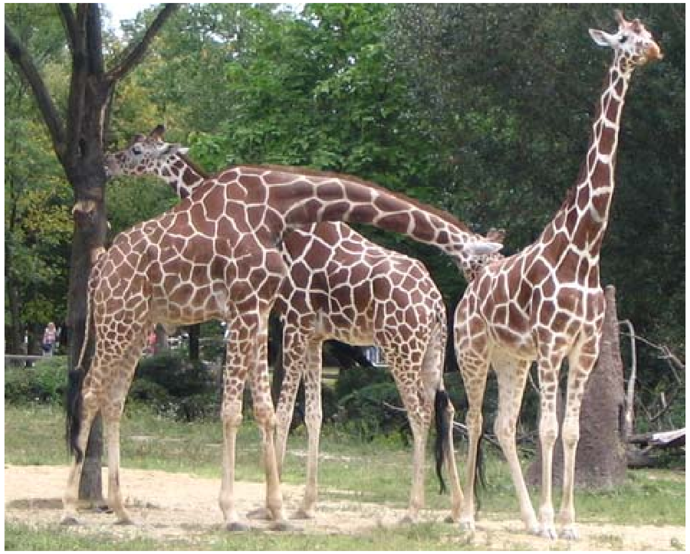


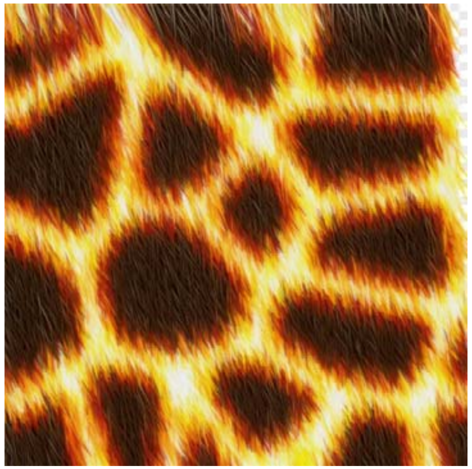
Voronoi cells are everywhere





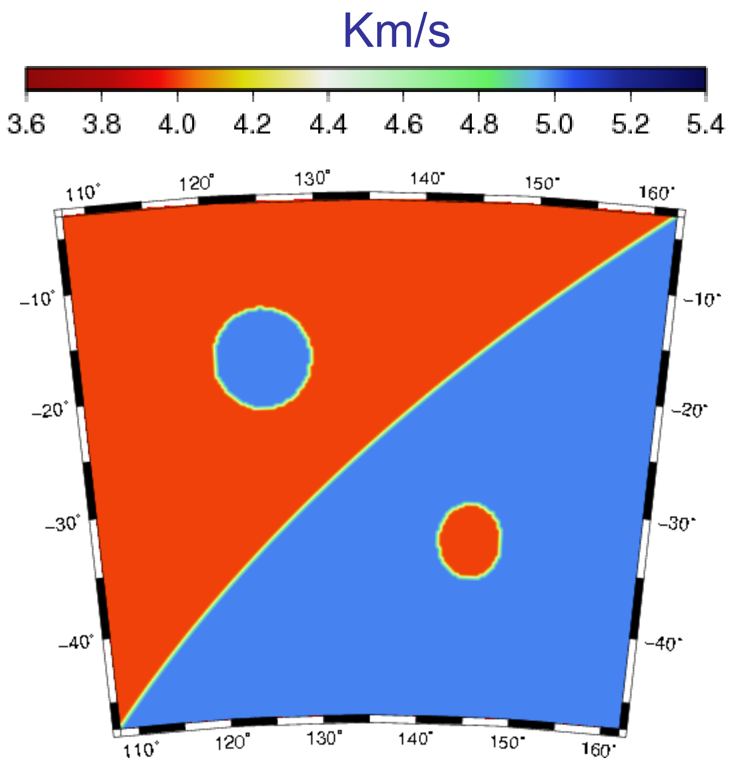
Voronoi cells are everywhere

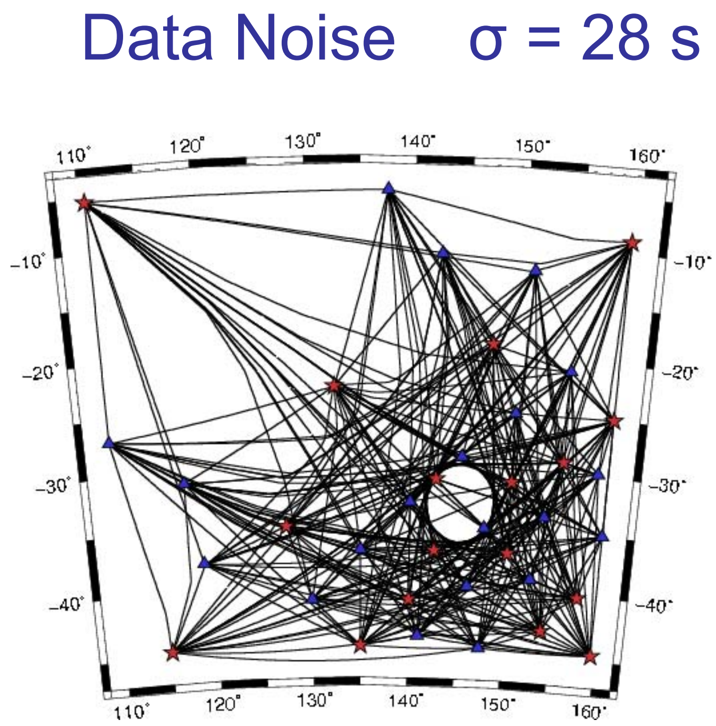




Voronoi cells are everywhere

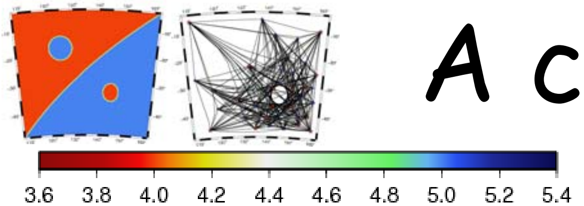
Synthetic experiment



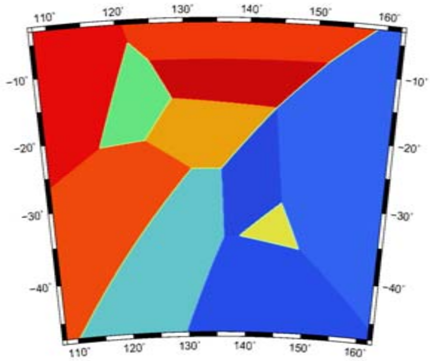


True velocity model

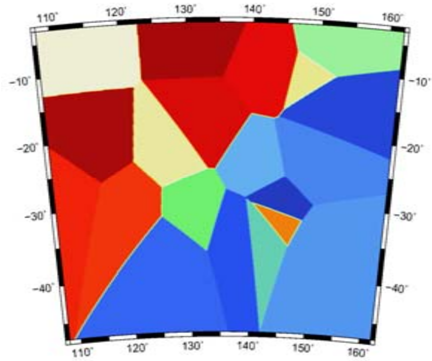
Ray geometry



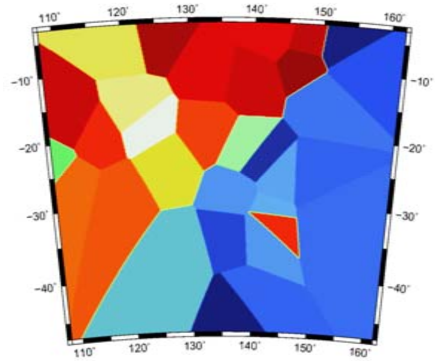
A convergent Markov chain



11 cells

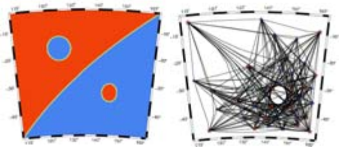


20 cells

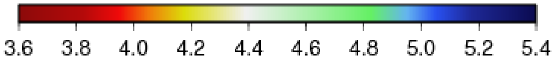


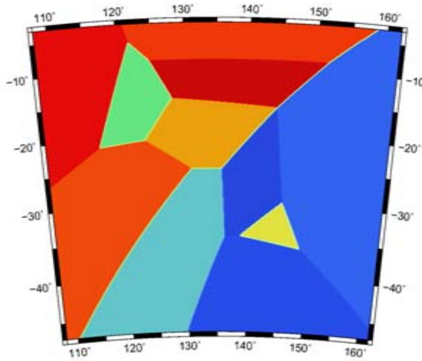
32 cells

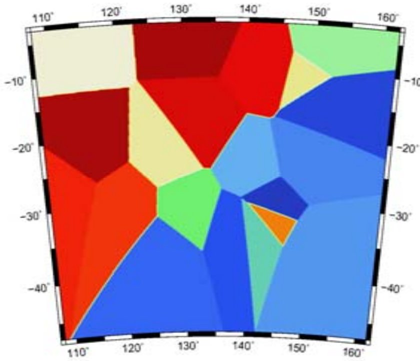
Current model at different points along the chain

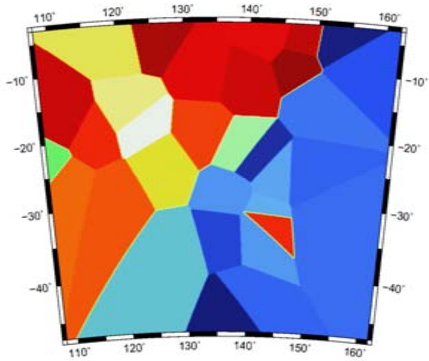


A convergent Markov chain



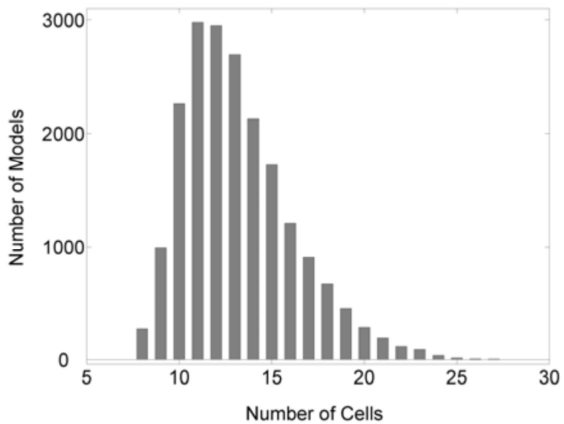






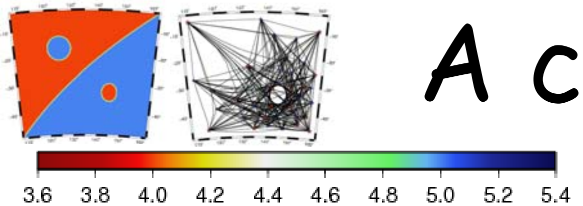
11 cells 20 cells) 32 cells

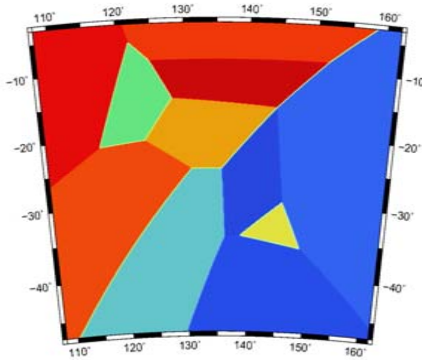
Current model at different points along the chain



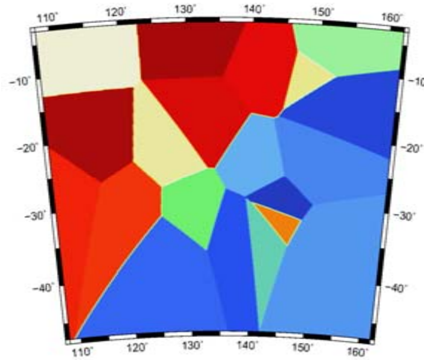
Estimated distribution for the number of cells

A convergent Markov chain

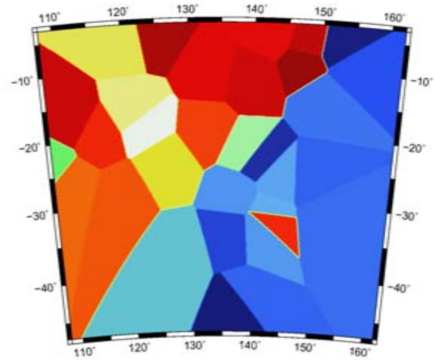




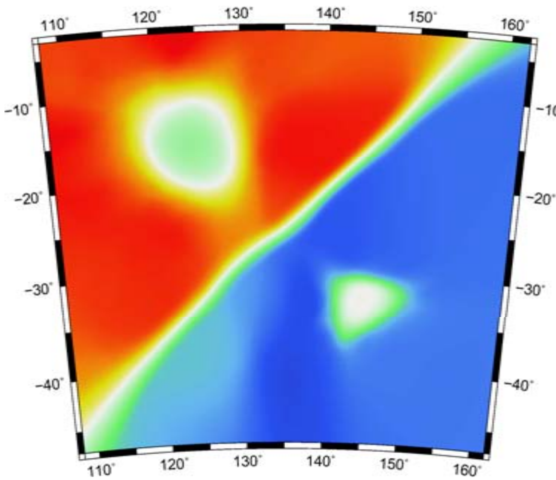
11 cells



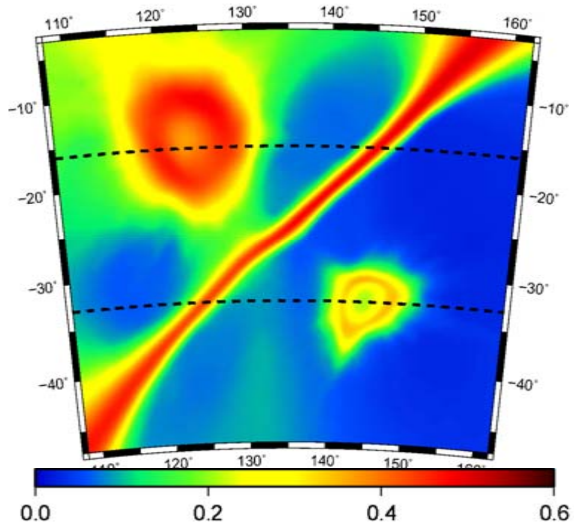
20 cells



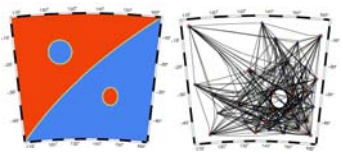
32 cells



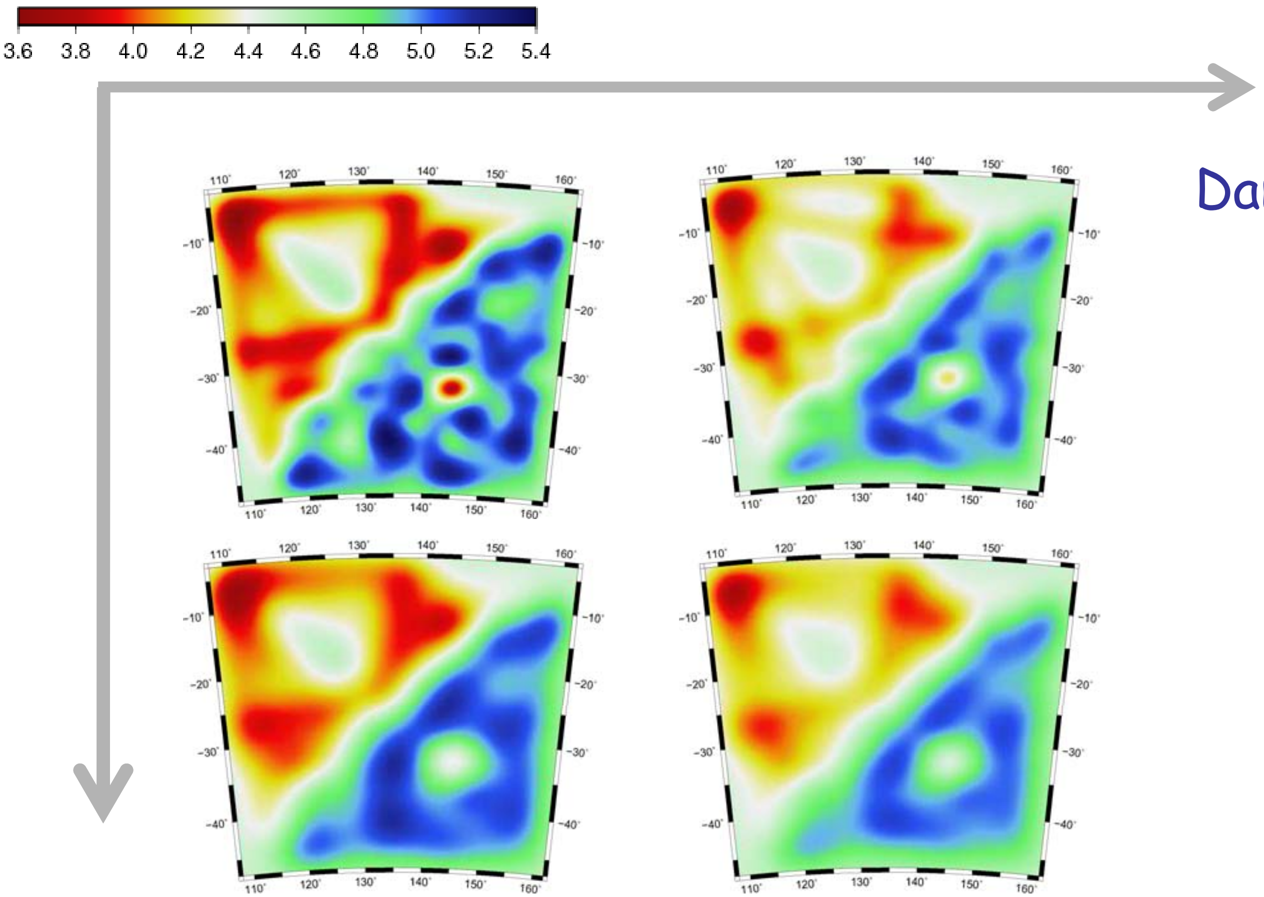
Average



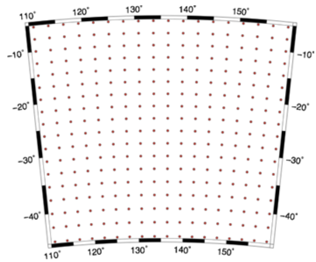
Standard deviation



The Standard approach with a fixed Regular grid (20*20 nodes)

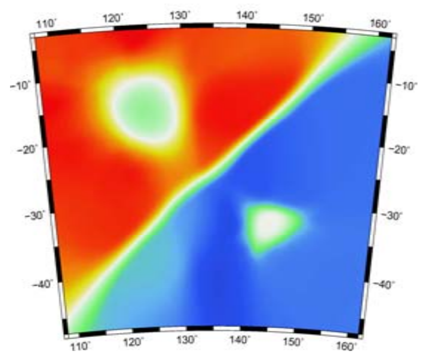


Damping



20 x 20
B-splines nodes

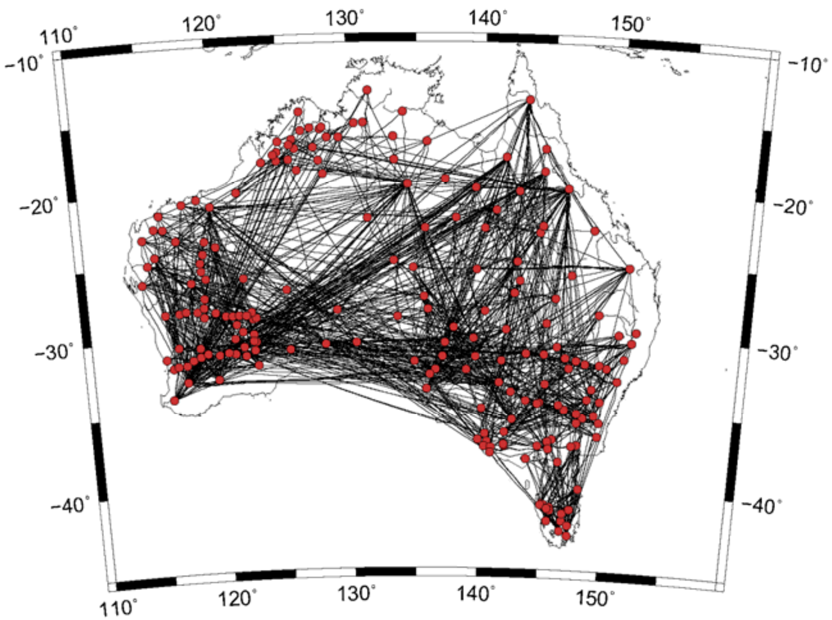
Smoothing



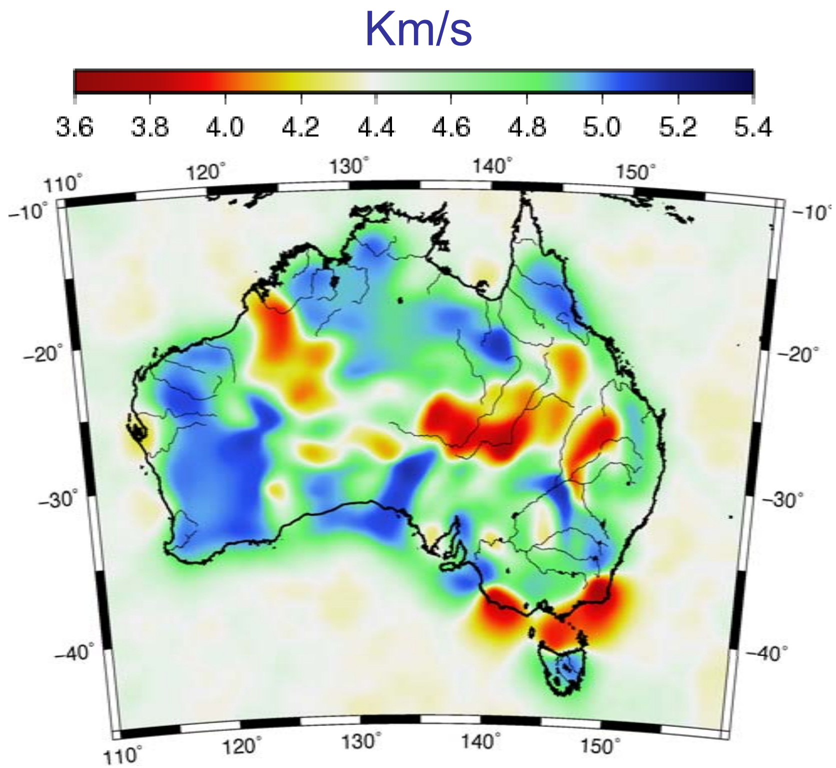
Transdimensional solution

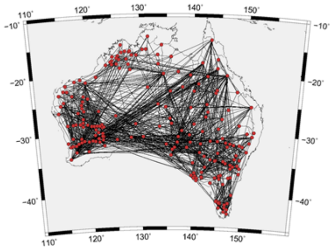
Real Data Application

Cross correlation of seismic ambient noise for Rayleigh wave group velocity at 5s



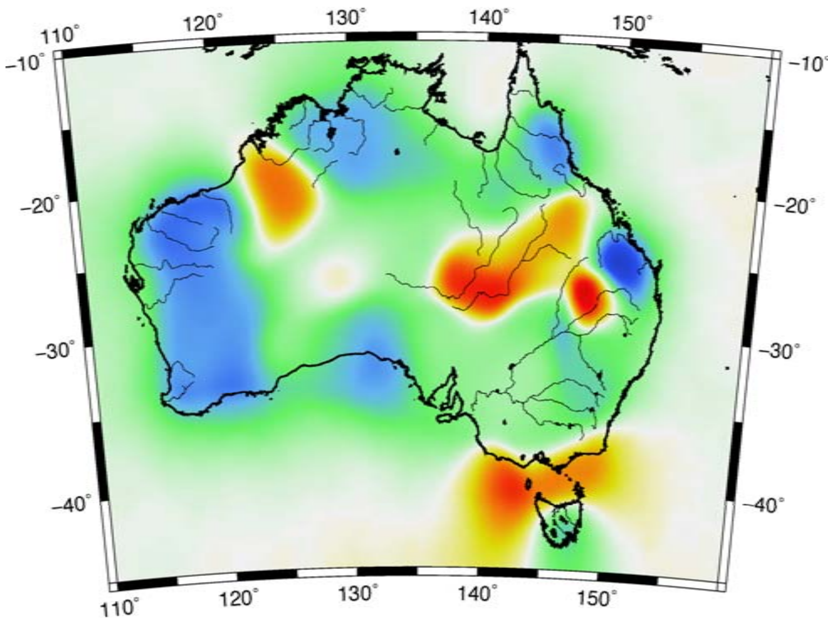
(Saygin & Kennett ,2009)



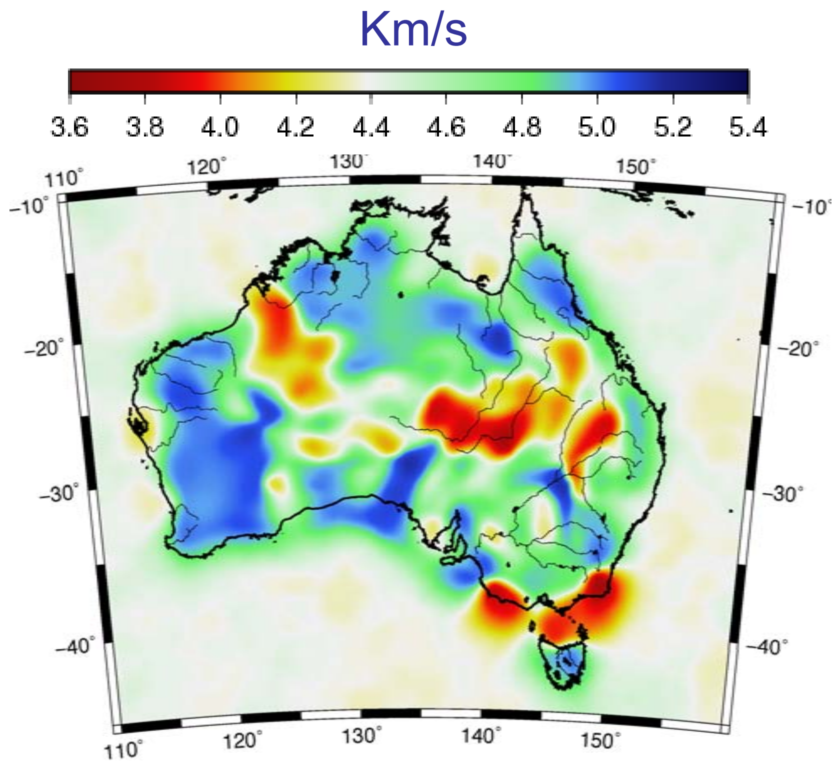


Real Data Application

The choice of model complexity is automatic and depends on estimated data noise



Data Noise = 22 s



Data Noise = 8 s

Hierarchical Bayesian Formulation

Account for the uncertainty in
the level of the data noise

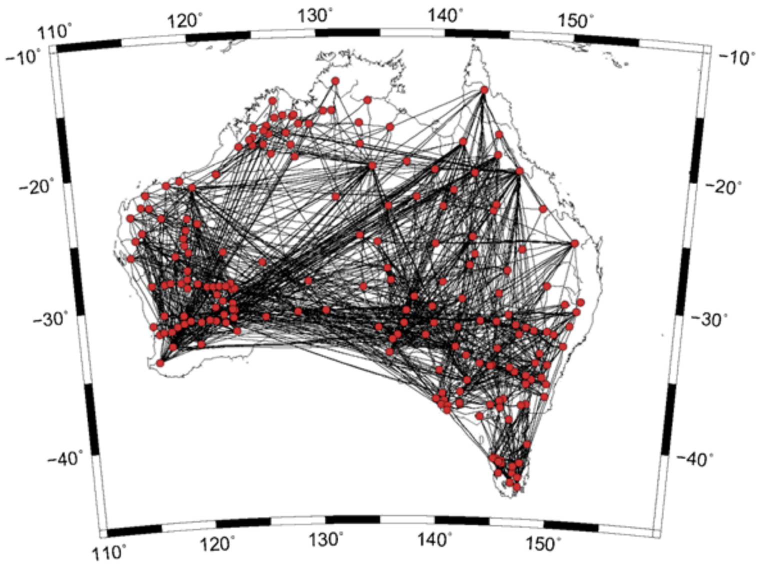
$$p(m \mid d) \propto \frac{1}{\sqrt{2\pi}\,\sigma^{N}} \exp\left[-\frac{1}{2}\sum_{i=1}^{N}\left(\frac{d_i - g(m)_i}{\sigma}\right)^2\right]$$

Malinverno & Parker (2004)

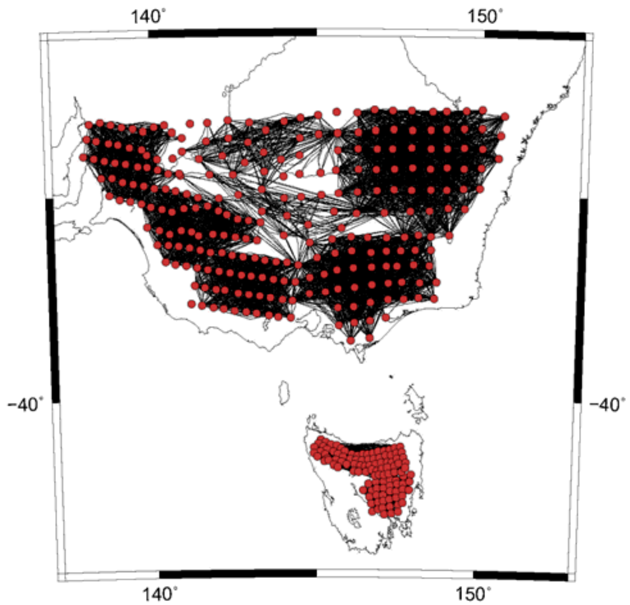
Level of Data noise is treated as an unknown in the problem

Multi-scale Tomography with Field Data

Cross correlation of seismic ambient noise for Rayleigh wave group velocity at 5s

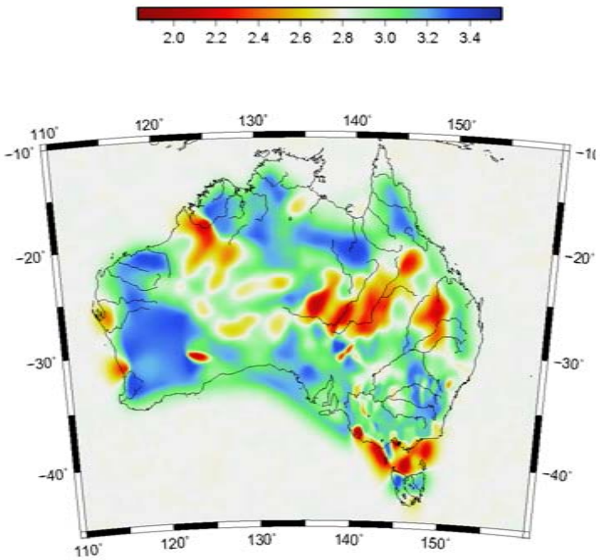


(Saygin & Kennett ,2009)

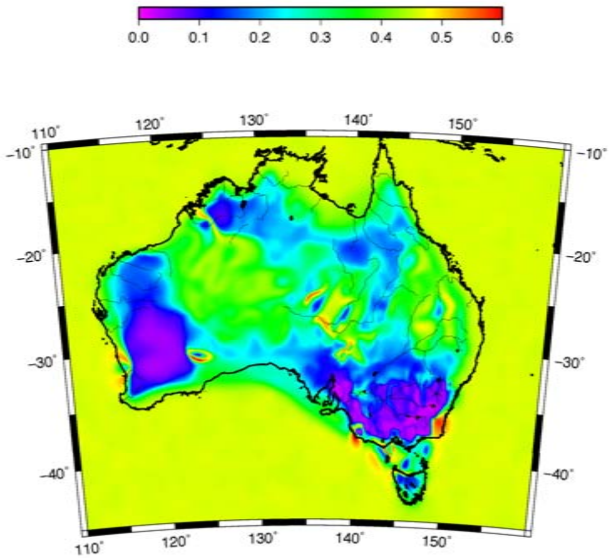


(Arroucau et al ,2010)

Multi-scale Tomography

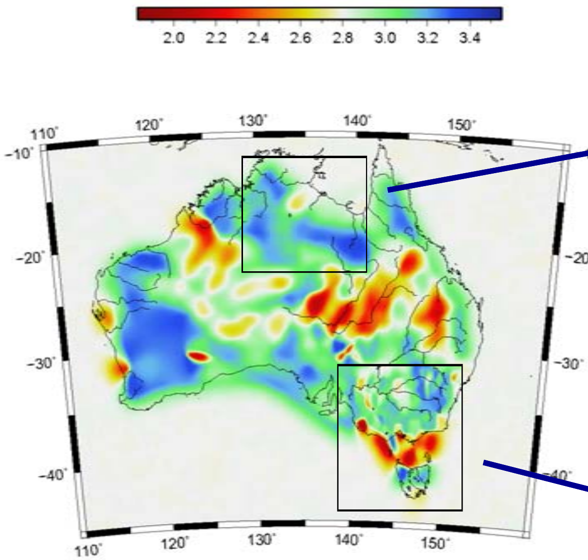


Average model (Km/s)

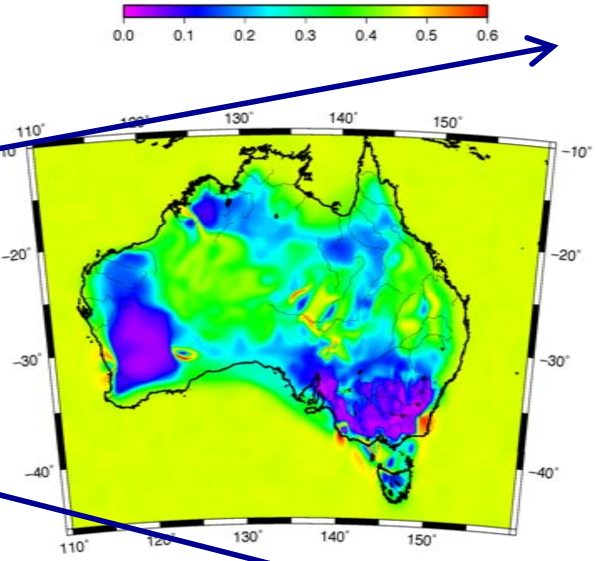


Error estimation (Km/s)

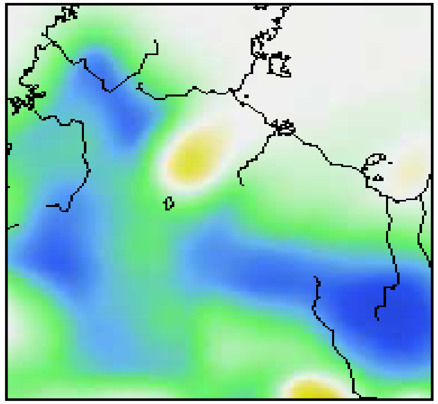
Multi-scale Tomography

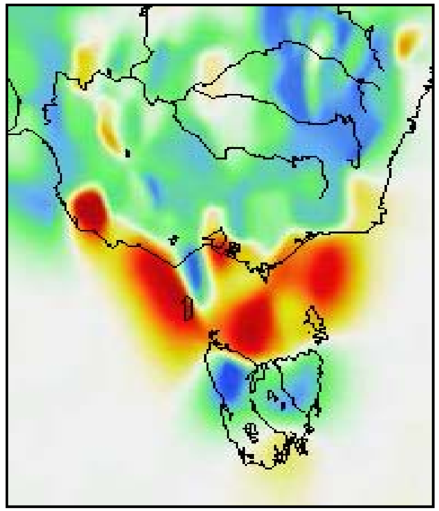


Average model (Km/s)



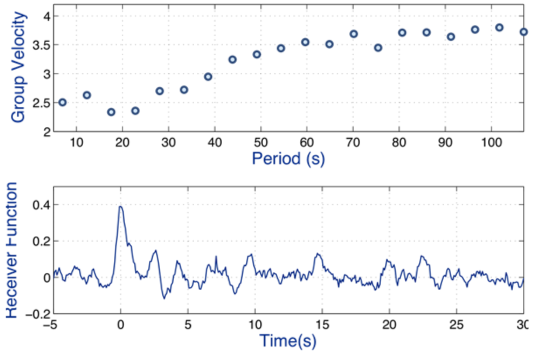
Error estimation (Km/s)



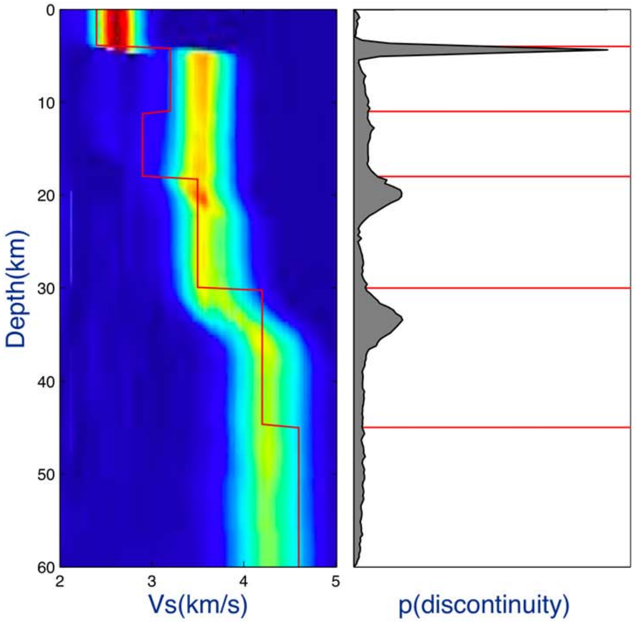


Algorithm finds automatically the correct model complexity and the correct level of data noise

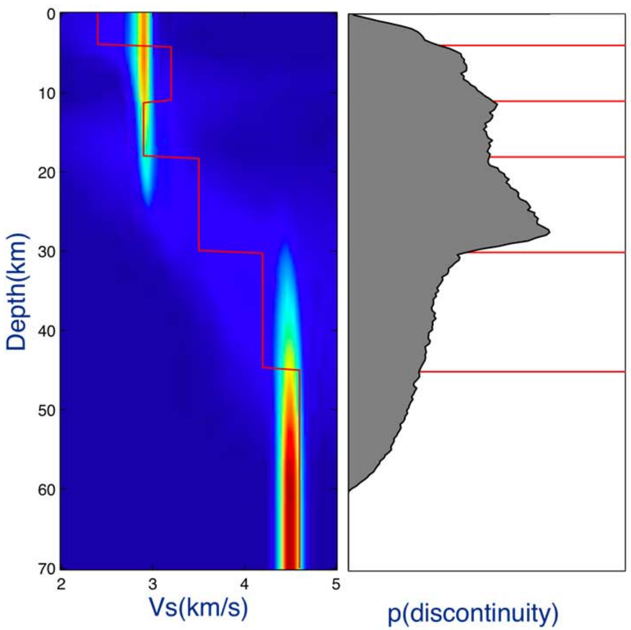
Application to Joint Inversion



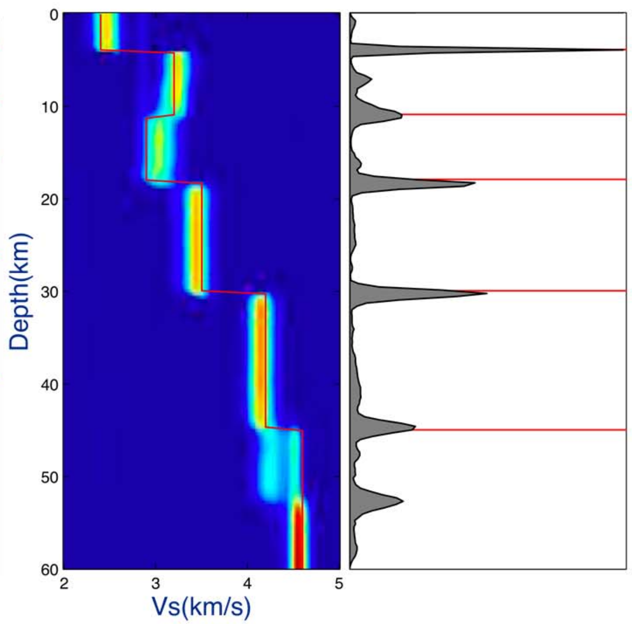
Receiver Function



Dispersion



Joint Inversion



Conclusion

Different features of trans-dimensional methods:

1. Adaptive parameterization
2. No need of regularization
3. Hierarchical Bayesian formulation enables to quantify the information brought by each data set.

This is a general inversion strategy. We have applied to other types of inverse problems in Earth Science

- Seismic tomography
- Receiver Functions
- Dispersions curves
- Electromagnetic data
- Regression of palaeoclimate data