SED
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Swiss Seismological Service

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Fault Representation Methods for Spontaneous Dynamic Rupture Simulation

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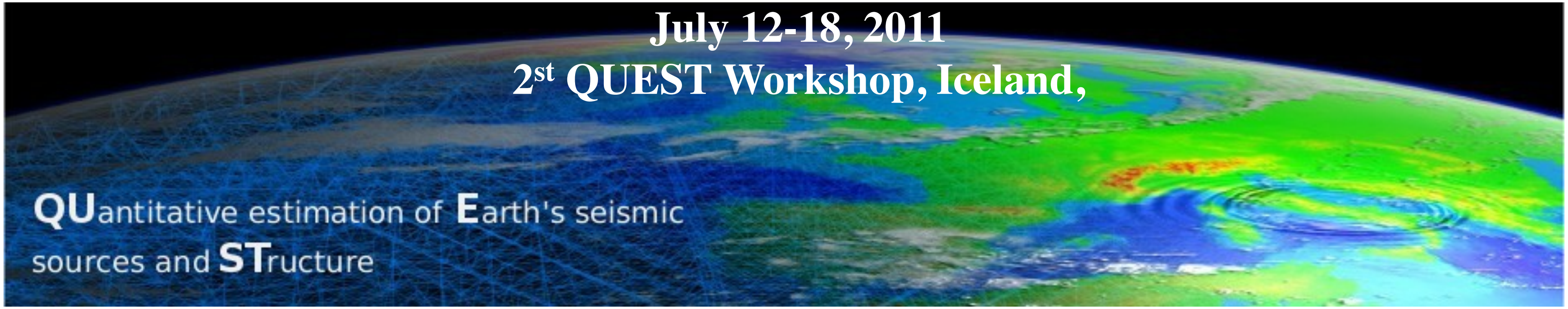
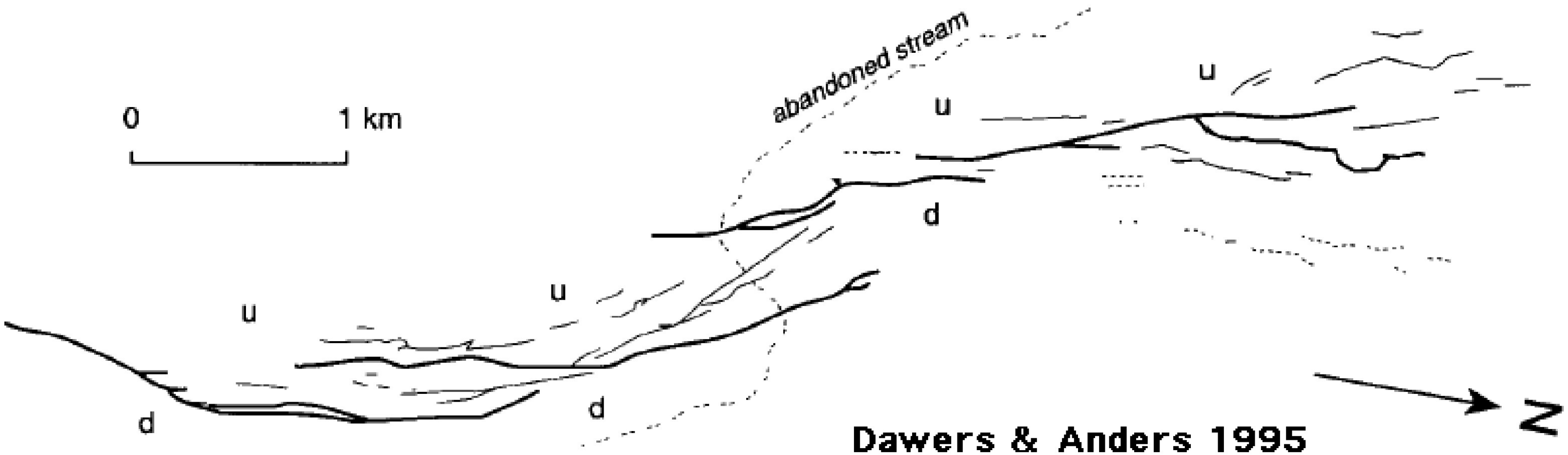


July 12-18, 2011
2st QUEST Workshop, Iceland,

QUantitative estimation of Earth's seismic sources and STructure

Earthquakes are complex at all scales

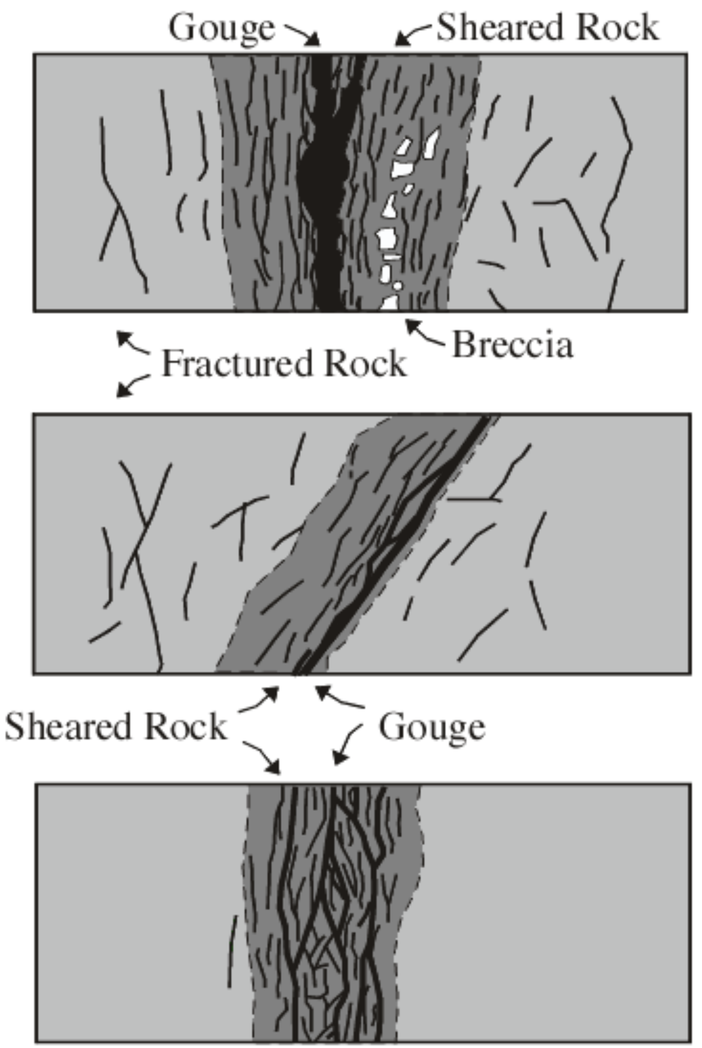
- Faults are not isolated (segmented and linked, irregular and rough at all scales)
- How does local characteristics of these complexities influence ground motion?



Dawers & Anders 1995

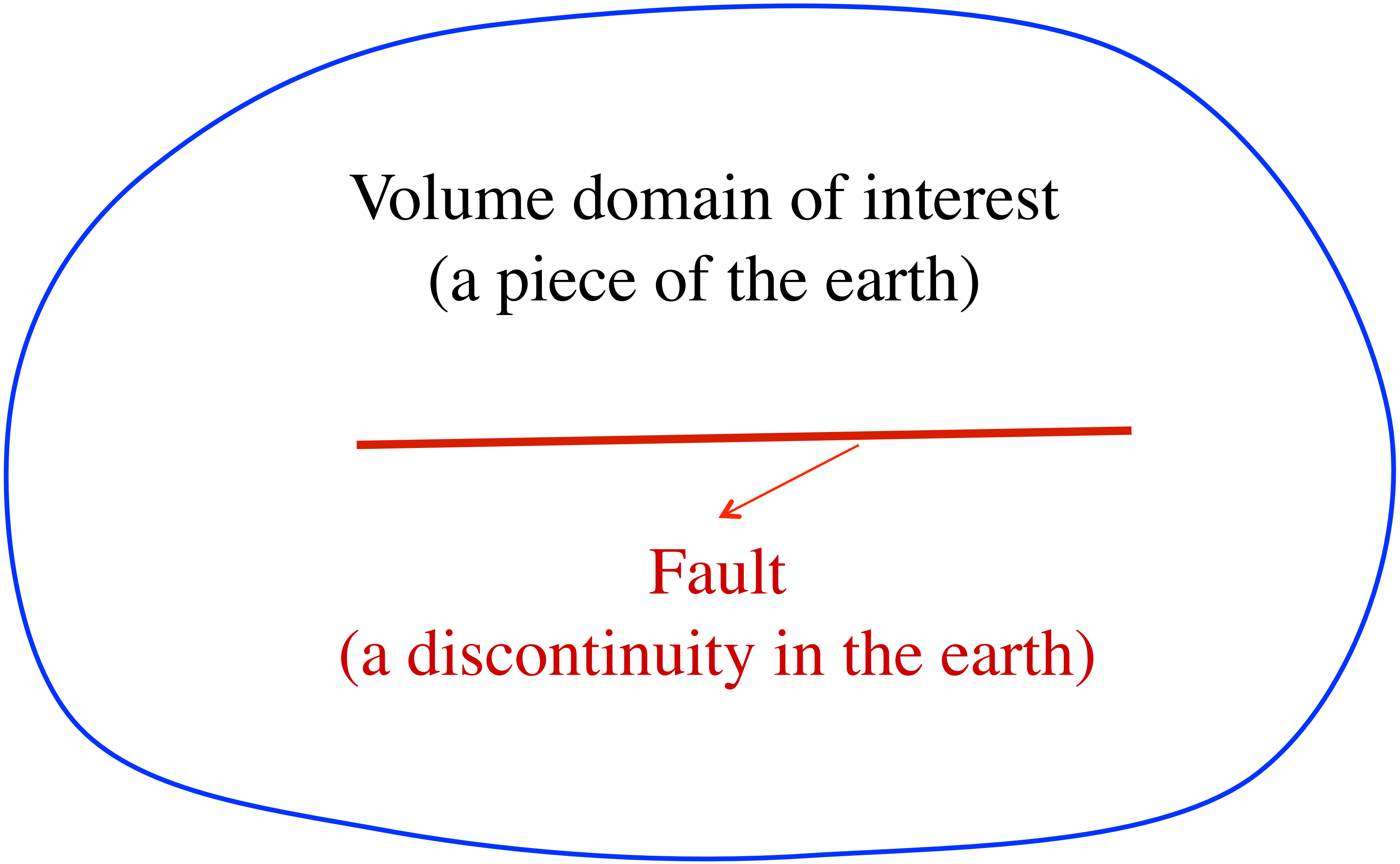
Internal Structure of Faults

- Detail observation of fault may provides important insight on the physics of rupture and the process of dynamic weakening
- Smaller-scale frictional processes during high-speed rupture?
- Distributed-shearing (Zones of distributed damage)
- How does these complexities influence ground motion?



Wallace and Morris, 1986

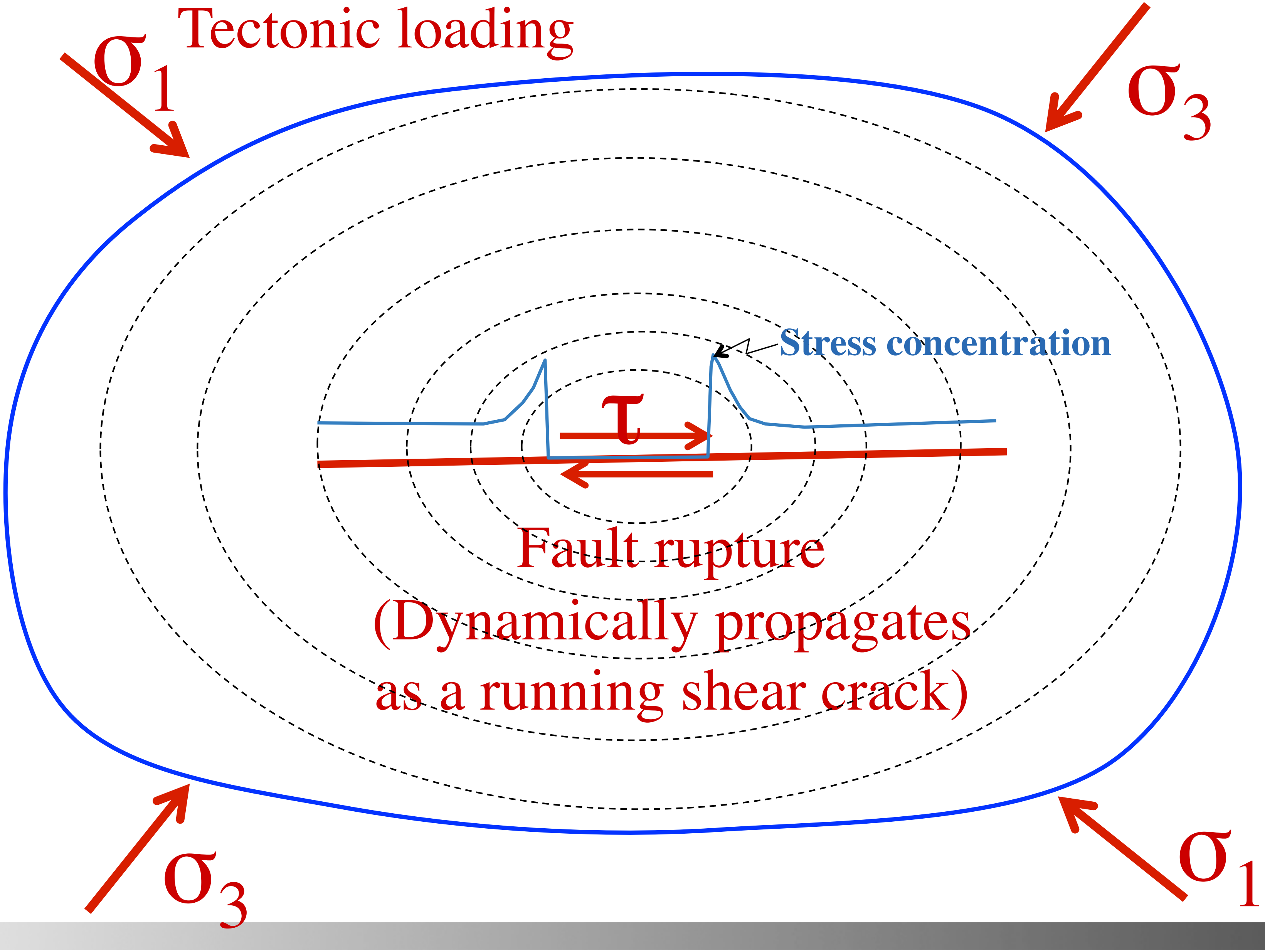
Problem statement



Volume domain of interest
(a piece of the earth)

Fault
(a discontinuity in the earth)

Problem statement



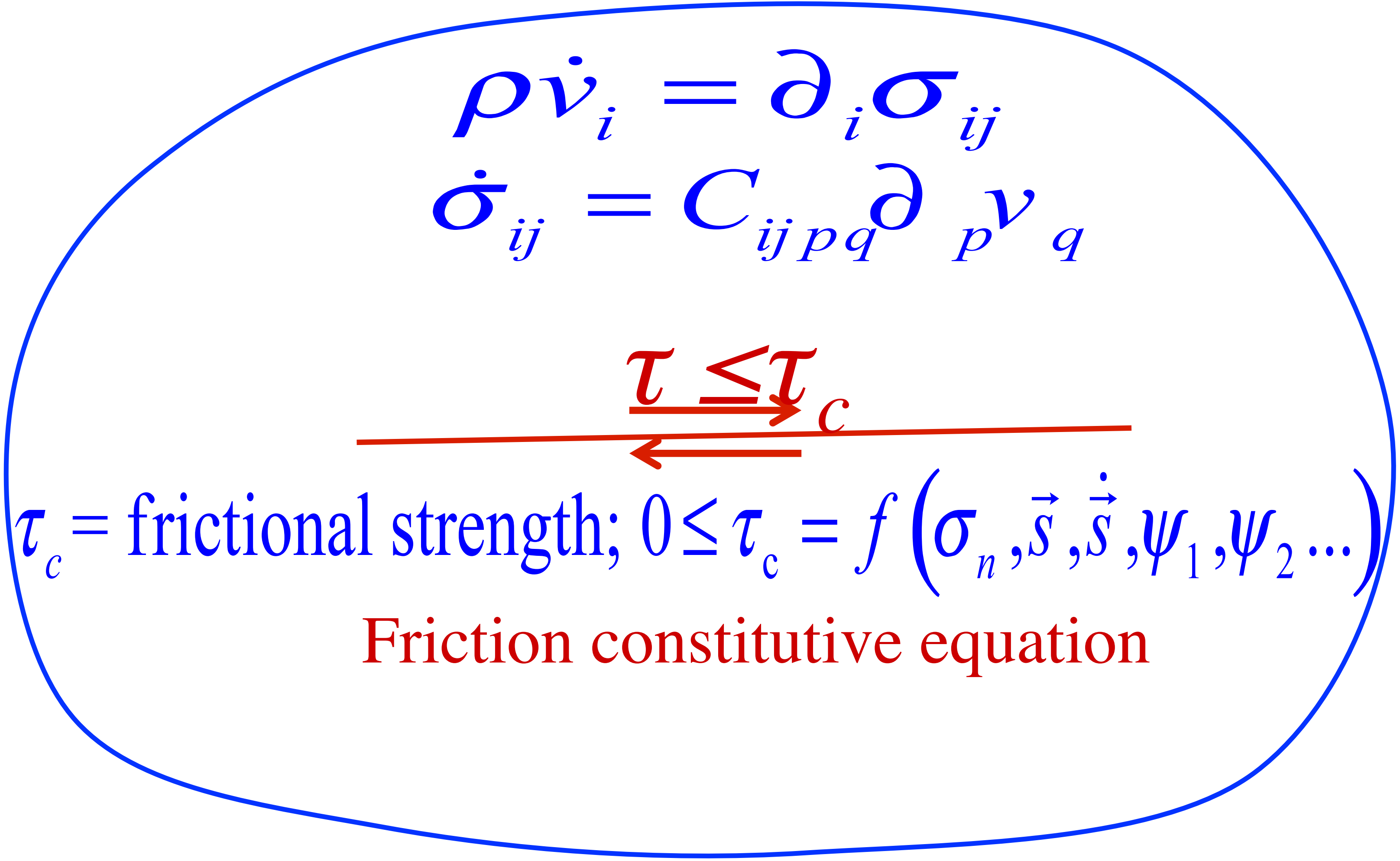
Tectonic loading

Fault rupture
(Dynamically propagates
as a running shear crack)

Mathematical Model

Elastodynamic coupled to frictional sliding

(Highly non-linear problem)



$$\rho \dot{v}_i = \partial_i \sigma_{ij}$$

$$\dot{\sigma}_{ij} = C_{ijpq} \partial_p v_q$$

$$\tau \le \tau_c$$

$$\tau_c = \text{frictional strength; } 0 \le \tau_\mathrm{c} = f\left(\sigma_n, \vec{s}, \dot{\vec{s}}, \psi_1, \psi_2 \ldots\right)$$

Friction constitutive equation

Fault-surface boundary conditions

Definitions

$\vec{\tau}$ = shear stress vector ($\tau \equiv |\vec{\tau}|$)

$\dot{\vec{s}}$ = tangential slip velocity ($\dot{s} = |\dot{\vec{s}}|$)

σ_n = normal stress (positive in compression)

U_n = opening displacement discontinuity

τ_c = frictional strength; $0 \le \tau_c = f\left(\sigma_n, \vec{s}, \dot{\vec{s}}, \psi_1, \psi_2 \ldots\right)$

For shear (nonlinear)

$$\tau - \tau_c \le 0$$

$$\vec{\tau}\dot{s} - \tau_c \dot{\vec{s}} = 0$$

For opening (nonlinear)

$$\sigma_n \ge 0$$

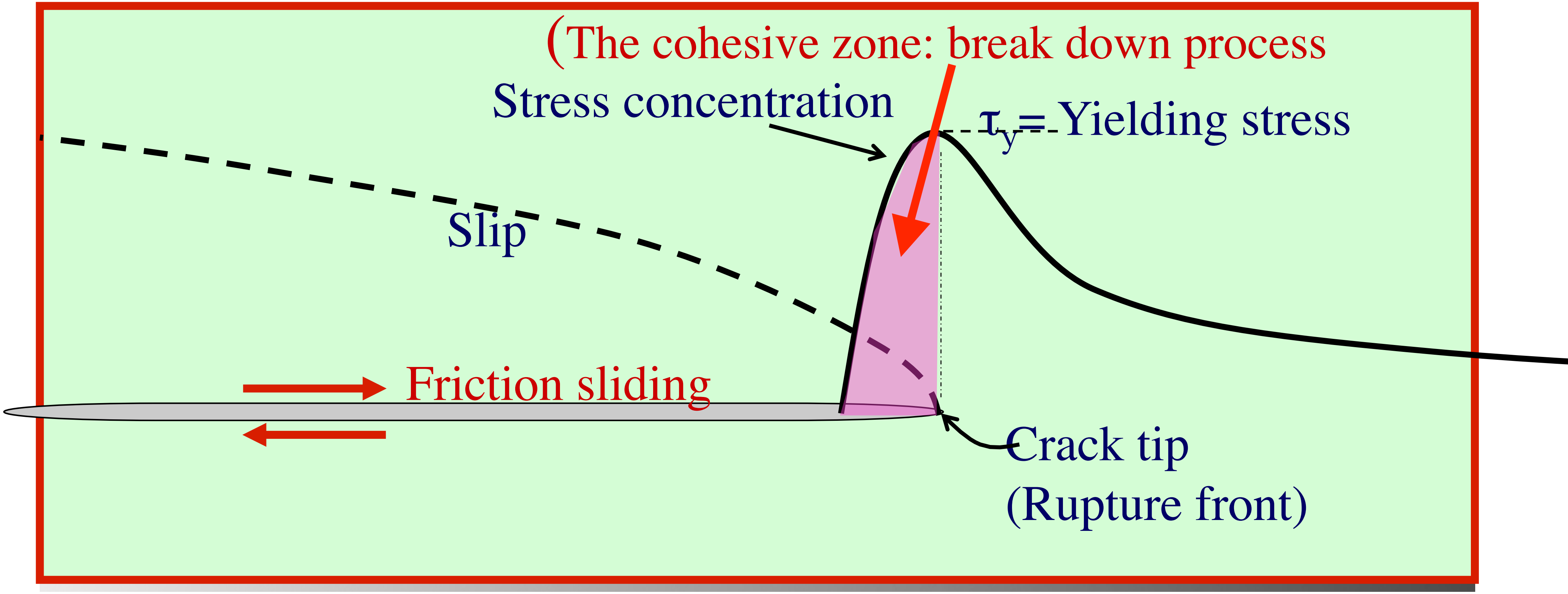
$$U_n \ge 0$$

$$\sigma_n U_n = 0$$

Stress and friction on the fault

(Interaction between the two sides of the fault)

The earthquake rupture can be described as a two-step process: (1) formation of crack and (2) propagation or growth of the crack. The crack tip serves as a stress concentrator due to driving force; if the stress at the crack tip exceeds some critical value, then the crack grows unstably accompanied by a sudden slip and stress drops.



(The cohesive zone: break down process

Stress concentration

τ_y = Yielding stress

Slip

Friction sliding

Crack tip
(Rupture front)

Cohesive zone (Fracture mechanics) and friction model

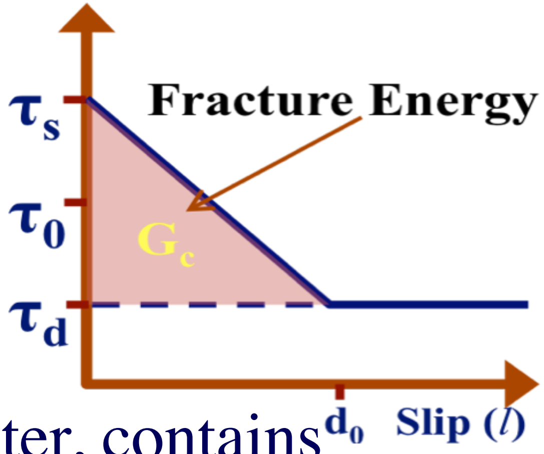
- Models

-Constant (Barenblatt, 1959)

-Linearly dependent on distance to crack tip (Palmer and Rice, 1973; Ida, 1973)

-Linearly dependent on slip (Ida, 1973 Andrews; 1976)

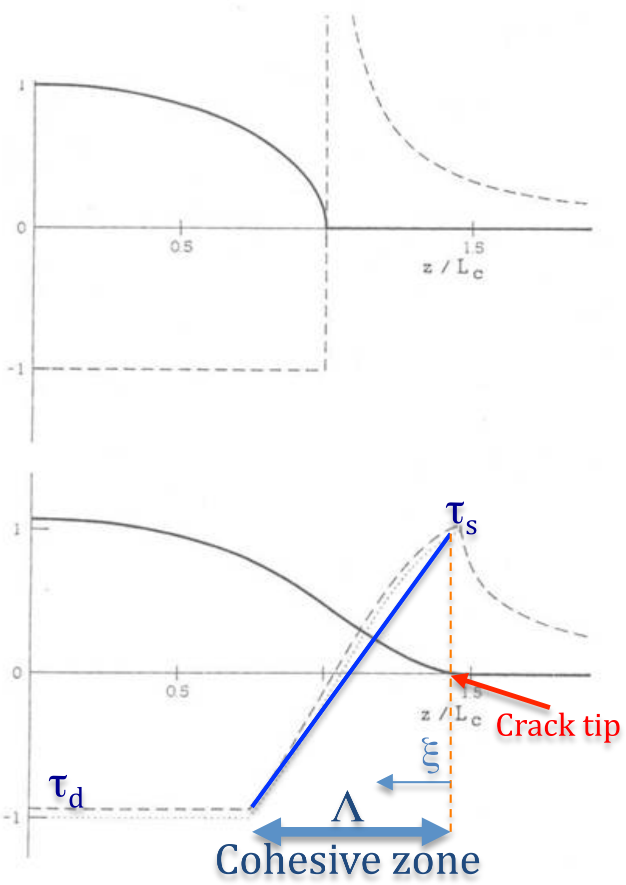
$$G_c = \frac{d_0(\tau_s - \tau_d)}{2}$$



G_c is a mesoscopic parameter, contains all the dissipative processes in the volume around the crack tip: off-fault yielding, damage, micro-cracking etc.

-They are mapped on the fault plane.

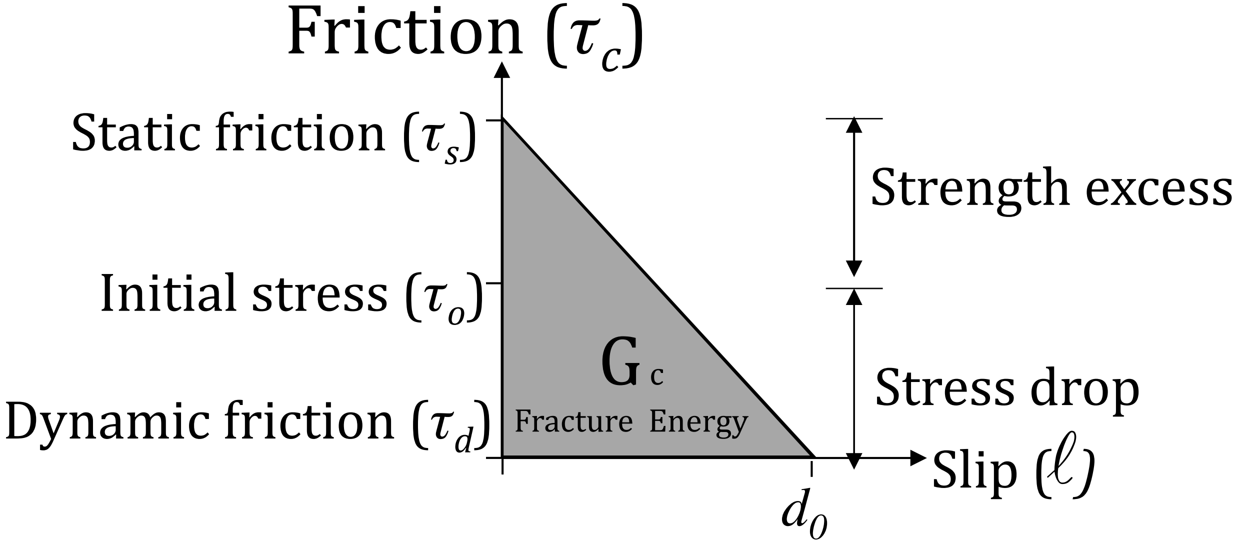
-G_c is not the surface energy defined by Griffith



Slip weakening friction model
(In the form given by Andrews, 1976)

$$\tau_c = \sigma\mu_f(\ell)$$

$$\mu_f(\ell) = \begin{cases} \mu_s - (\mu_s - \mu_d)\ell / d_0 & \ell < d_0 \\ \mu_d & \ell \geq d_0 \end{cases}$$



Friction (τ_c)

Static friction (τ_s)

Initial stress (τ_o)

Dynamic friction (τ_d)

G_c Fracture Energy

Strength excess

Stress drop

Slip (ℓ)

d_0

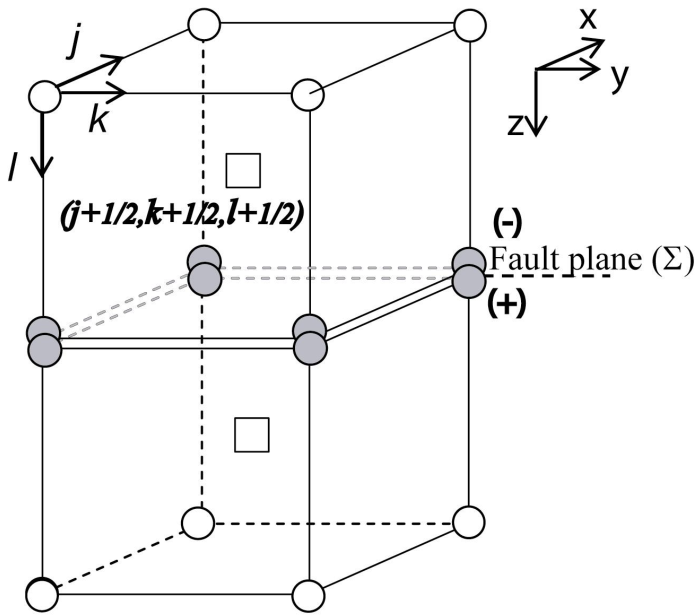
Fault representation

- Traction at Split-node method
  Fault Discontinuity explicitly incorporated
  (Andrews, 1973; DFM model: Day, 1977, 1982; SGSN model, Dalguer and Day, 2007)

- “Inelastic-zone” methods:
  Fault Discontinuity <u>not</u> explicitly incorporated
  - Thick-fault method (TF) (Madariaga et al., 1998)
  - Stress-glut (SG) method (Andrews 1976, 1999)

Fault representation

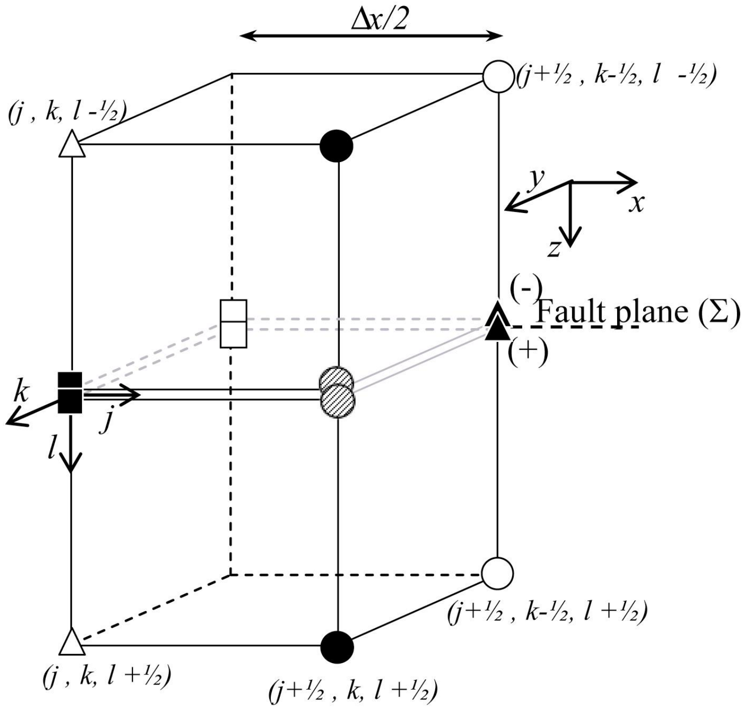
Traction at Split-Node method



○ $u_x, u_y, u_z, \dot{u}_x, \dot{u}_y, \dot{u}_z,$ R_x, R_y, R_z, M

□ $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}$

● $u_x^\pm, u_y^\pm, u_z^\pm, \dot{u}_x^\pm, \dot{u}_y^\pm, \dot{u}_z^\pm,$ $R_x^\pm, R_y^\pm, R_z^\pm, M^\pm, T_x, T_y, T_z$



■ $\dot{u}_x^\pm, R_x^\pm, T_x$

▲ $\dot{u}_y^\pm, R_y^\pm, T_y$

● $\dot{u}_z$

△ σ_{xz}

○ σ_{yz}

□ $\sigma_{xy}^\pm$

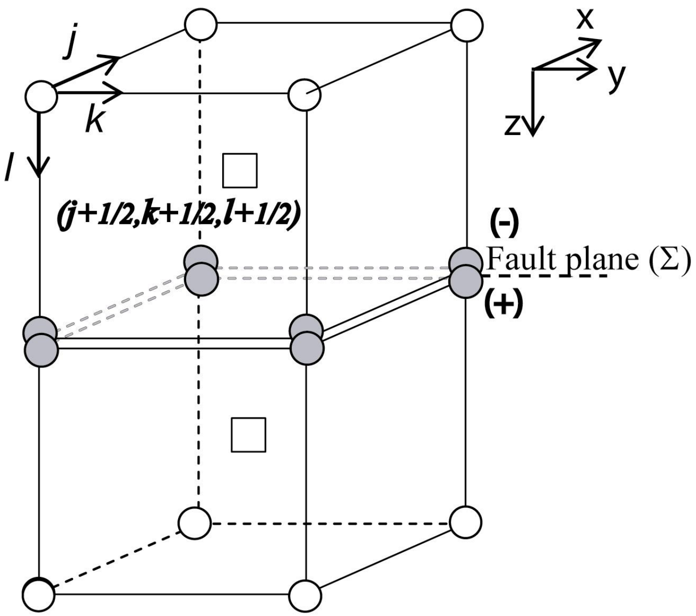
▨ $\sigma_{zz}, \sigma_{yy}^\pm, \sigma_{xx}^\pm$

For partially Staggered Grid
(e.g, model DFM
Day, 1982; Day et al, 2005)

For Staggered Grid
Staggered-Grid Split-Node Method (SGSN)
(Dalguer and Day 2007, JGR)

Fault representation

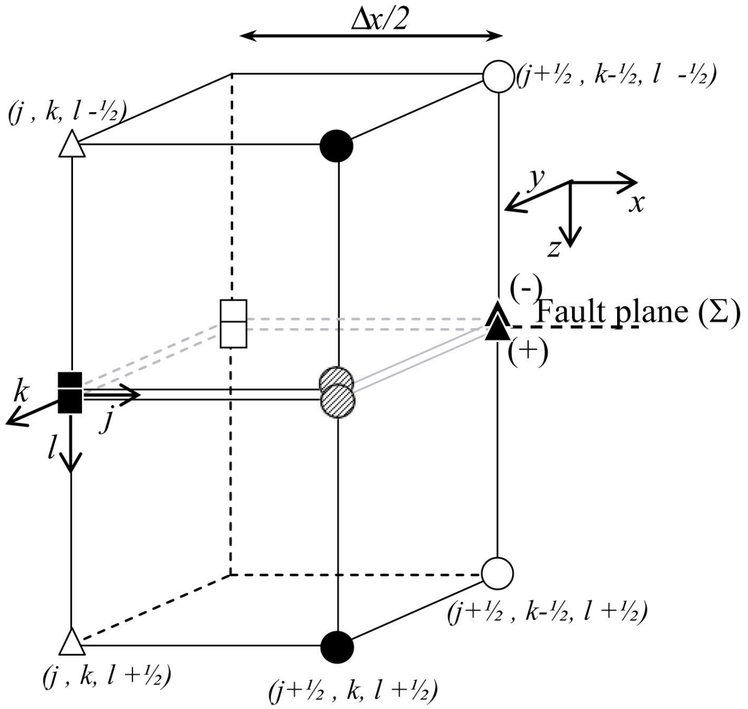
Traction at Split-Node method



○ $u_x, u_y, u_z, \dot{u}_x, \dot{u}_y, \dot{u}_z,$
R_x, R_y, R_z, M

☐ $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}$

● $u_x^{\pm}, u_y^{\pm}, u_z^{\pm}, \dot{u}_x^{\pm}, \dot{u}_y^{\pm}, \dot{u}_z^{\pm},$
$R_x^{\pm}, R_y^{\pm}, R_z^{\pm}, M^{\pm}, T_x, T_y, T_z$



■ $\dot{u}_x^{\pm}, R_x^{\pm}, T_x$

▲ $\dot{u}_y^{\pm}, R_y^{\pm}, T_y$

● $\dot{u}_z$

△ σ_{xz}

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▨ $\sigma_{zz}, \sigma_{yy}^{\pm}, \sigma_{xx}^{\pm}$

For partially Staggered Grid
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For Staggered Grid
Staggered-Grid Split-Node Method (SGSN)
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Fault representation

Traction at Split-Node method

Discrete representation of equation of motion on the fault
(Central Differencing in time)

$$\dot{u}_v^{\pm}(t+\Delta t/2) = \dot{u}_v^{\pm}\left(t-\Delta t/2\right) + \Delta t(M^{\pm})^{-1}\left\{R_v^{\pm}(t) \mp a\left[T_v(t) - T_v^0\right]\right\}$$

$$\dot{s}_v = \dot{u}_v^{+}\left(t+\Delta t/2\right) - \dot{u}_v^{-}\left(t+\Delta t/2\right)$$ (Slip velocity)

Compute “trial” traction $\tilde{T}_v$ that enforces continuity of tangential velocity and continuity of normal displacement ($\dot{s}_v = 0$)

Then the actual nodal traction T_v (tangential components *v*=x,y) that satisfies b.c.’s is

$$T_v = \begin{cases} \tilde{T}_v & \text{for} \quad \left[(\tilde{T}_x)^2 + (\tilde{T}_y)^2\right]^{1/2} \le \tau_c \\ \tau_c \dfrac{\tilde{T}_v}{\left[(\tilde{T}_x)^2 + (\tilde{T}_y)^2\right]^{1/2}} & \text{for} \quad \left[(\tilde{T}_x)^2 + (\tilde{T}_y)^2\right]^{1/2} > \tau_c \end{cases}$$

$\vec{\dot{u}}^{\pm}$ = split-node velocities (+,- side of fault, respectively)

$\vec{R}^{\pm}$ = stress divergence terms from FD eqns (+,- side)

$M^{\pm}$ = nodal mass factors from FD eqns (+,- side)

$\vec{T}$ = split-node traction vector (no jump)

a = interface area of split node

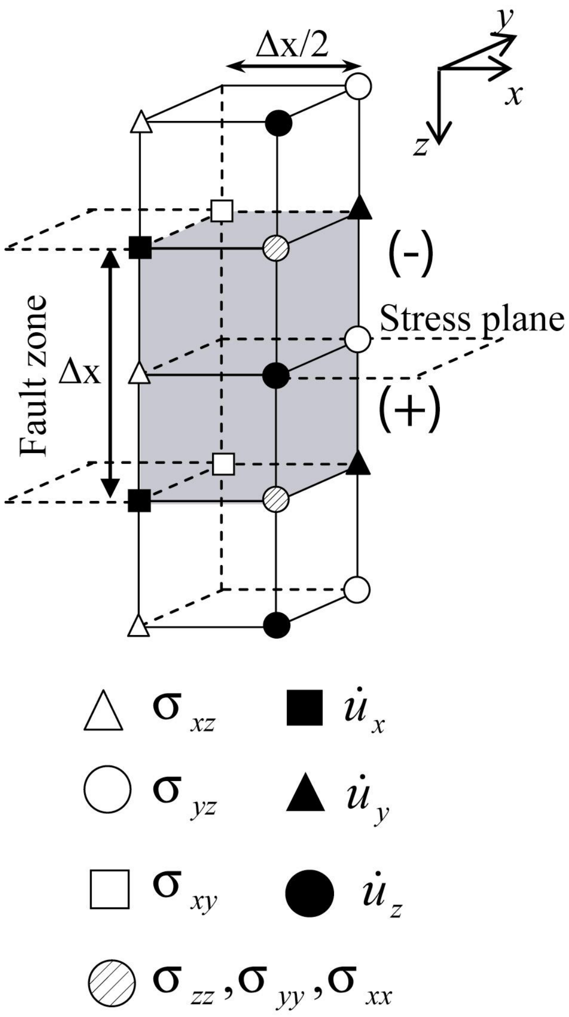
Δt=time step

T_v^0 =Initial traction

Fault representation

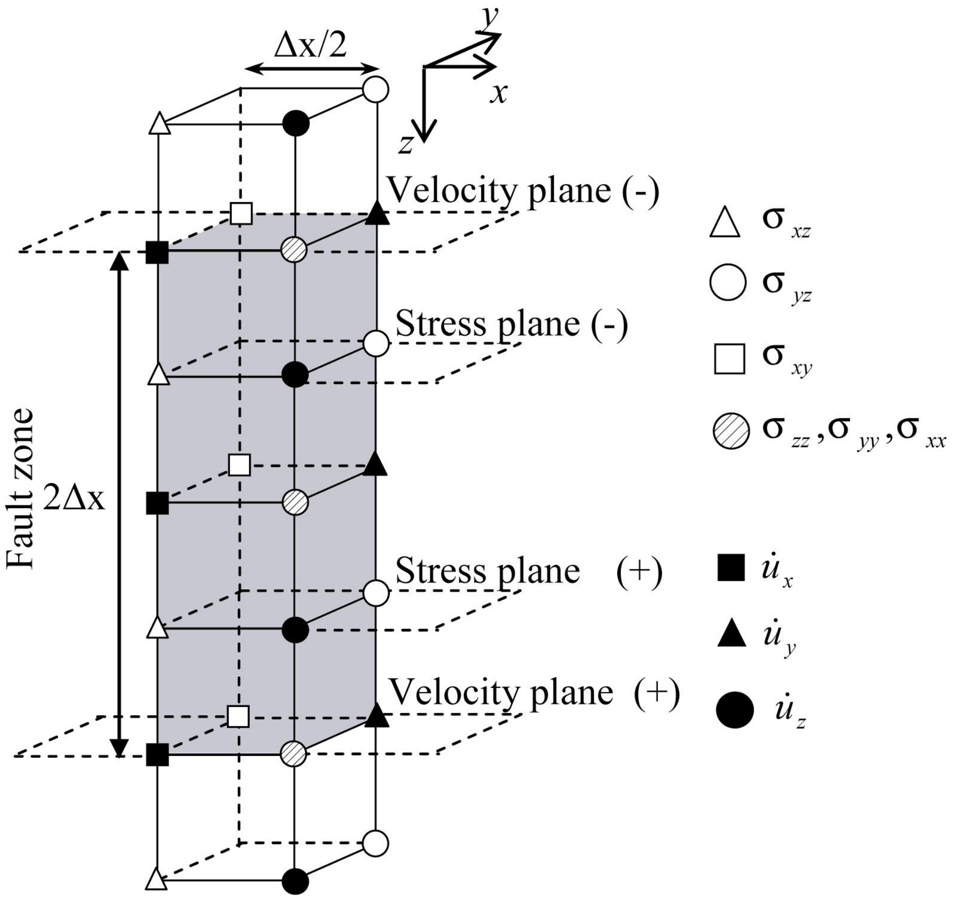
"Inelastic-zone" Fault models

(Dalguer and Day, 2006, BSSA)



Stress-glut method (SG)

(Andrews 1976, 1999)



Thick-fault method (TF)

(Madariaga et al., 1998)

Fault representation

“Inelastic-zone” Fault models

Nodal Stress by Central Differencing in time gives (example σ_{xz})

$$\sigma_{xz}(t) = \sigma_{xz}\left(t - \Delta t\right) + \Delta t 2\mu \dot{\varepsilon}_{xz}\left(t - \Delta t/2\right)$$

addition of an inelastic component to the total strain rate $\left(T_x = \sigma_{xz}\right)$

$$\sigma_{xz} = T_x(t) = T_x\left(t - \Delta t\right) + \Delta t 2\mu\left[\dot{\varepsilon}_{xz}\left(t - \Delta t/2\right) - \dot{\varepsilon}^{p}_{xz}\left(t - \Delta t/2\right)\right]$$

Compute “trial” traction $\tilde{T}$ setting $\dot{\varepsilon}^{p}_{xz}\left(t - \Delta t/2\right) = 0$

$$\tilde{T}_x(t) = T_x\left(t - \Delta t\right) + \Delta t 2\mu \dot{\varepsilon}_{xz}\left(t - \Delta t/2\right)$$

Then set the fault plane traction to

$$T_x\left(t\right) = \begin{cases} \tilde{T}_x\left(t\right) & \text{if } \tilde{T}_x\left(t\right) \le \tau_c \\ \tau_c & \text{if } \tilde{T}_x\left(t\right) > \tau_c \end{cases}$$

Fault representation

"Inelastic-zone" Fault models

Stress-glut method (SG)

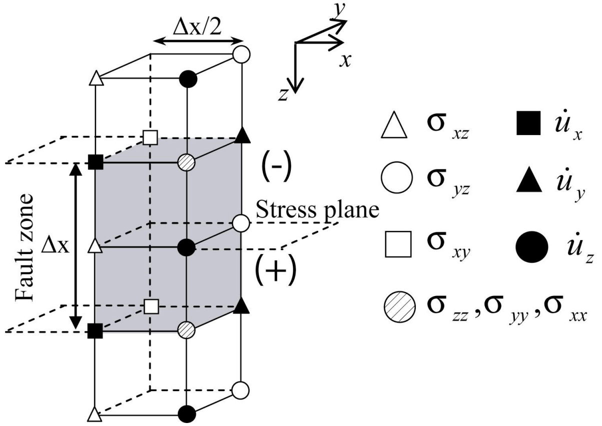
Frictional bound enforced on one plane of traction nodes

Calculate inelastic component $\dot{\varepsilon}^{p}_{xz}$

$$\dot{\varepsilon}^{p}_{xz}(t-\Delta t/2)=\frac{\tilde{T}_x(t)-T_x(t)}{2\mu\Delta t}$$

Calculate the total slip rate by integrating $\dot{\varepsilon}^{p}_{xz}$ over the spatial step

$$\dot{s}_x\left(t-\Delta t/2\right)=2\Delta x\dot{\varepsilon}^{p}_{xz}\left(t-\Delta t/2\right)$$

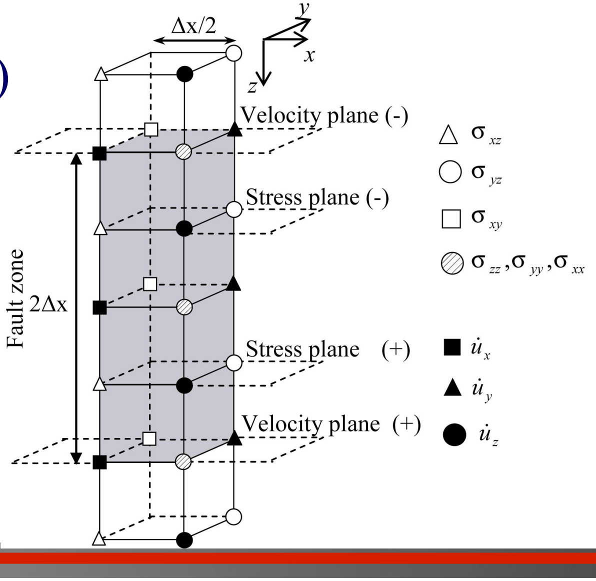


Thick-fault method (TF) (Madariaga et al, 1998)

Frictional bound enforced on 2 planes of traction nodes

Slip-velocity given by velocity difference across 2 unit-cell wide zone

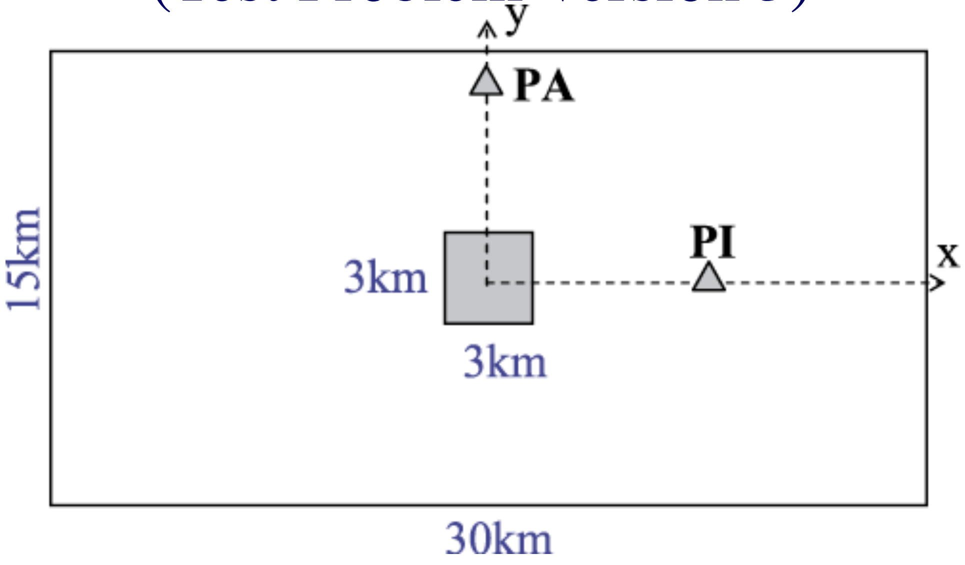
$$\dot{s}_x\left(t-\Delta t/2\right)=\dot{u}^{(+)}_x\left(t-\Delta t/2\right)-\dot{u}^{(-)}_x\left(t-\Delta t/2\right)$$



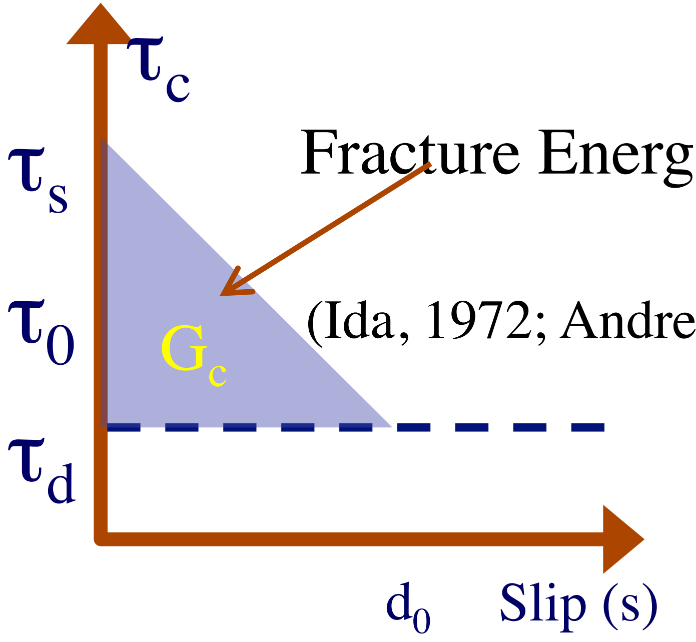
Assessment of Methods

SCEC 3D Rupture Dynamics Code Validation Project
(coordinators Ruth Harris, Ralph Archuleta)

Fault model
(Test Problem Version 3)



Slip Weakening Friction model



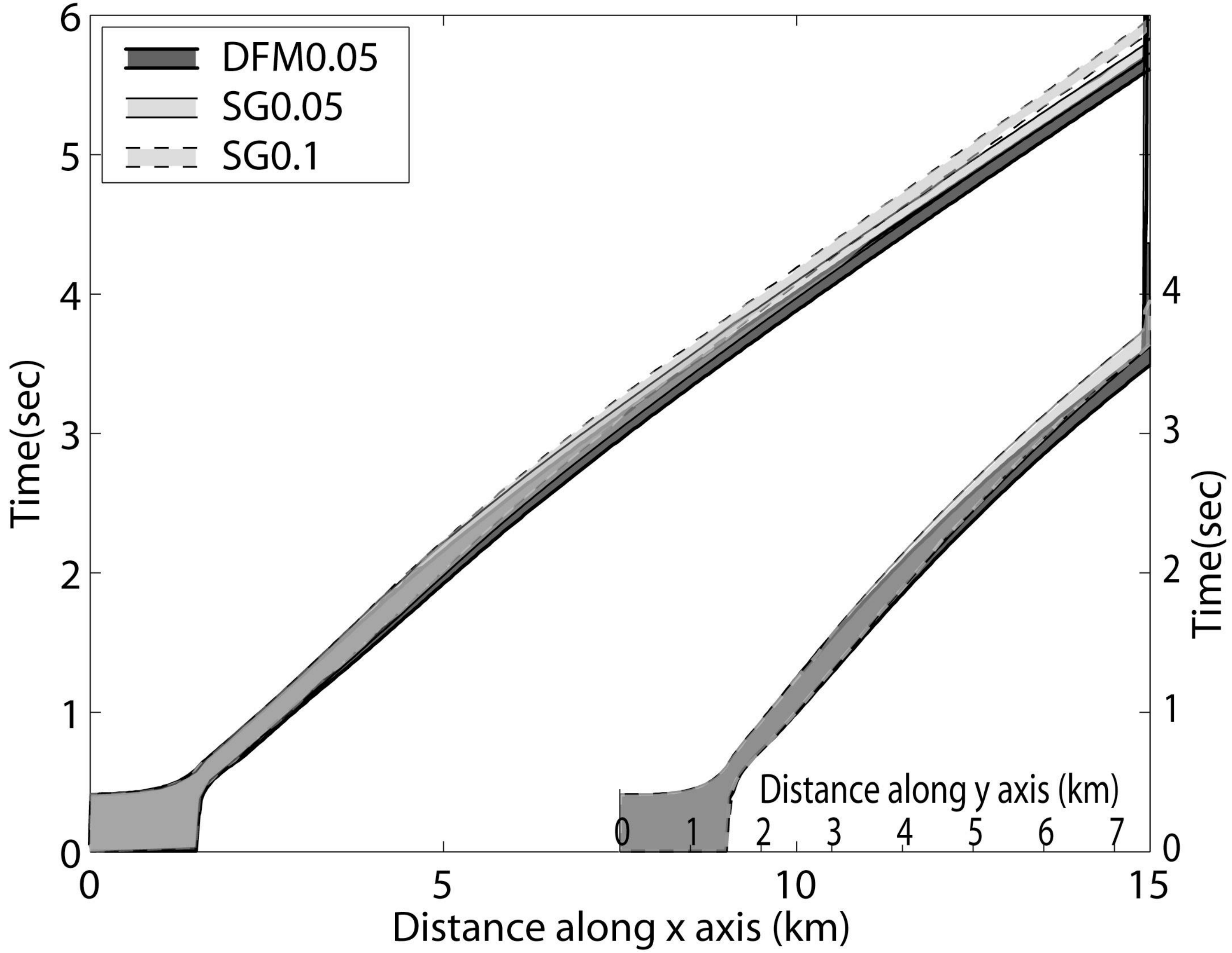
(Ida, 1972; Andrews, 1976)

Numerical resolution measured by

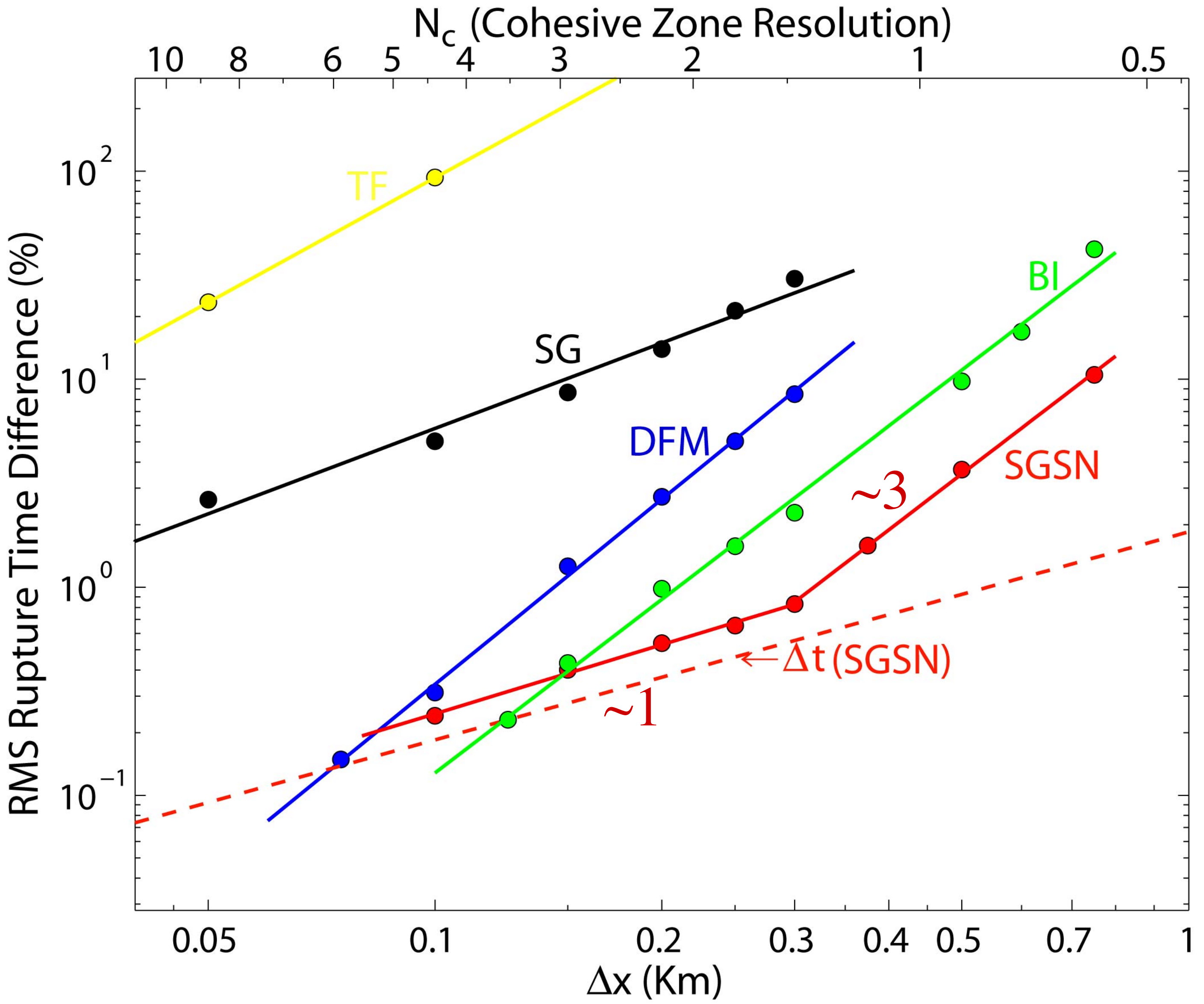
Λ = cohesive-zone width (normal to rupture front)

Δx = spatial step size (in numerical solution)

SG inelastic zone - vs - Split-node models
Cohesive zone development



Assessment of Methods

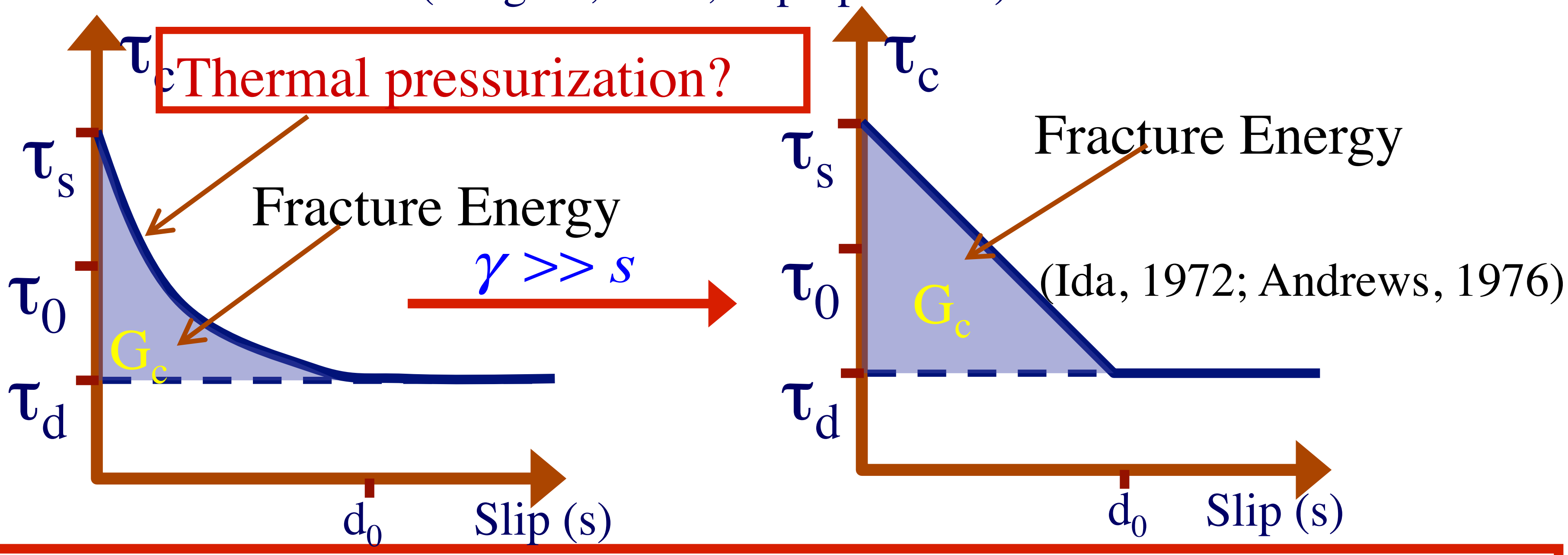


Summary of series of papers:
(Day, Dalguer, et al, 2005, JGR; Dalguer and Day, 2006, BSSA; 2007, JGR)

Other Friction laws

No-linear Slip weakening

(Dalguer, 2011, in preparation)



$$\tau_c = \begin{cases} \dfrac{\gamma\tau_s(d_0 - s)}{d_0(s+\gamma)} + \dfrac{\tau_d s(d_0+\gamma)}{d_0(s+\gamma)} & s < d_0 \qquad \text{for } \gamma >> s \qquad \approx \dfrac{\tau_s(d_0 - s)}{d_0} + \dfrac{\tau_d s}{d_0} \\ \tau_d & s \geq d_0 \end{cases}$$

Input requirement:

$\gamma =$ Define No-linearity $\gamma >> s$ then $\approx$ Linear weakening

$\tau_0 =$ Initial shear stress, $\tau_s =$ Static friction, $\tau_d =$ Dynamic friction

$d_0 =$ Critical slip-weakening

Other Friction laws

Rate and State

(its basis on the aging law: Dieterich, 1986; Ruina, 1983)

$$\tau_c = \tau_c(\sigma_n, \dot{s}, \psi) = \sigma_n\left[\mu_0 + a\ln\left(\dot{s}/V_0\right) + \psi\right]$$

$$\dot{\psi} = -G(\sigma_n, \dot{s}, \psi) \qquad \text{(Evolution equation)}$$

Input requirement:

$\psi =$ Initial state variable

$V_0 =$ Steady state reference velocity ,

$\mu_0 =$ Friction coefficient at steady state V_0

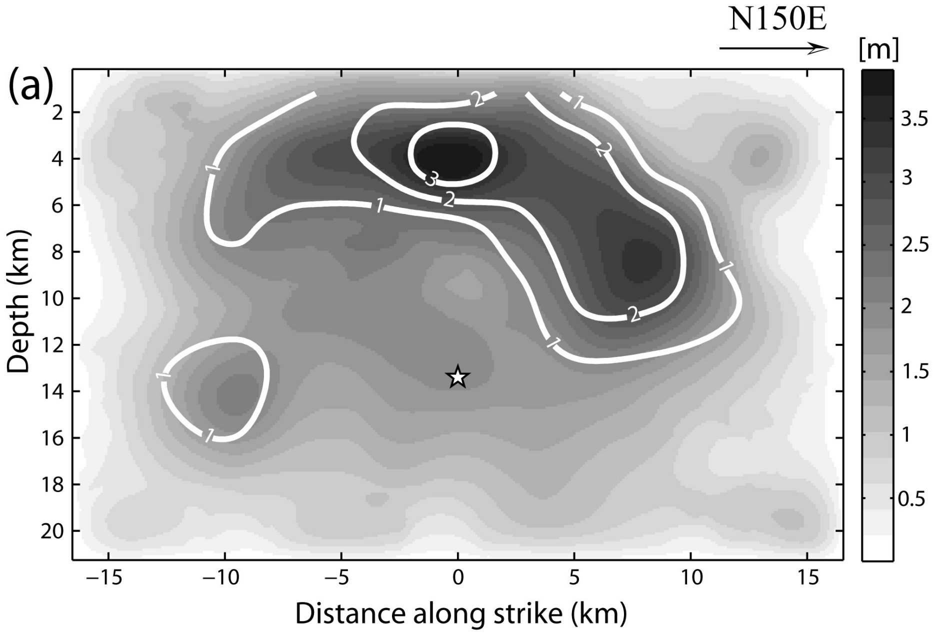
$a =$ friction parameter

Other considerations:

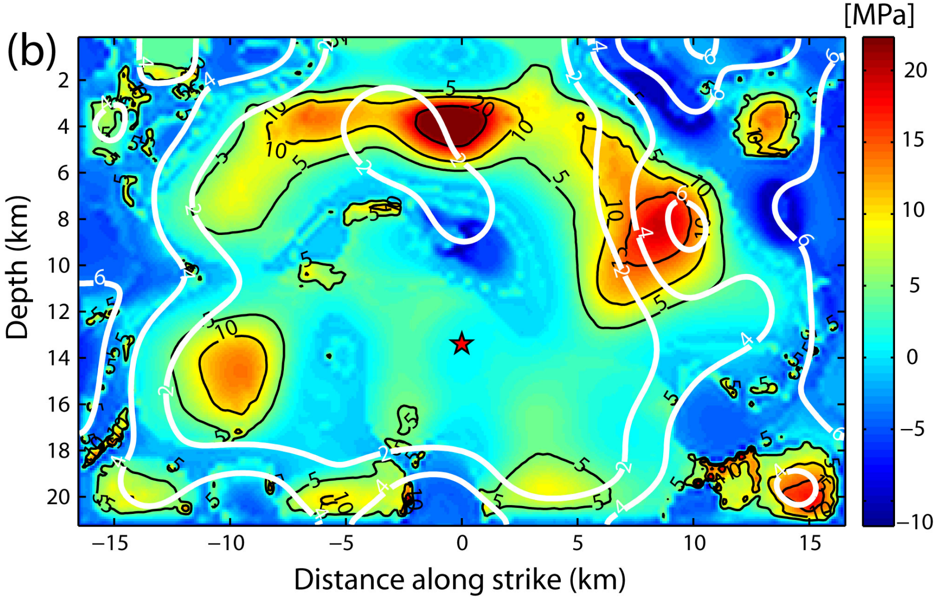
Flash heating

Thermal pressurization of pore fluid

A model for 2000 Tottori earthquake



Final slip of dynamic model
Overlapped by contour line of
Slip kinematic model

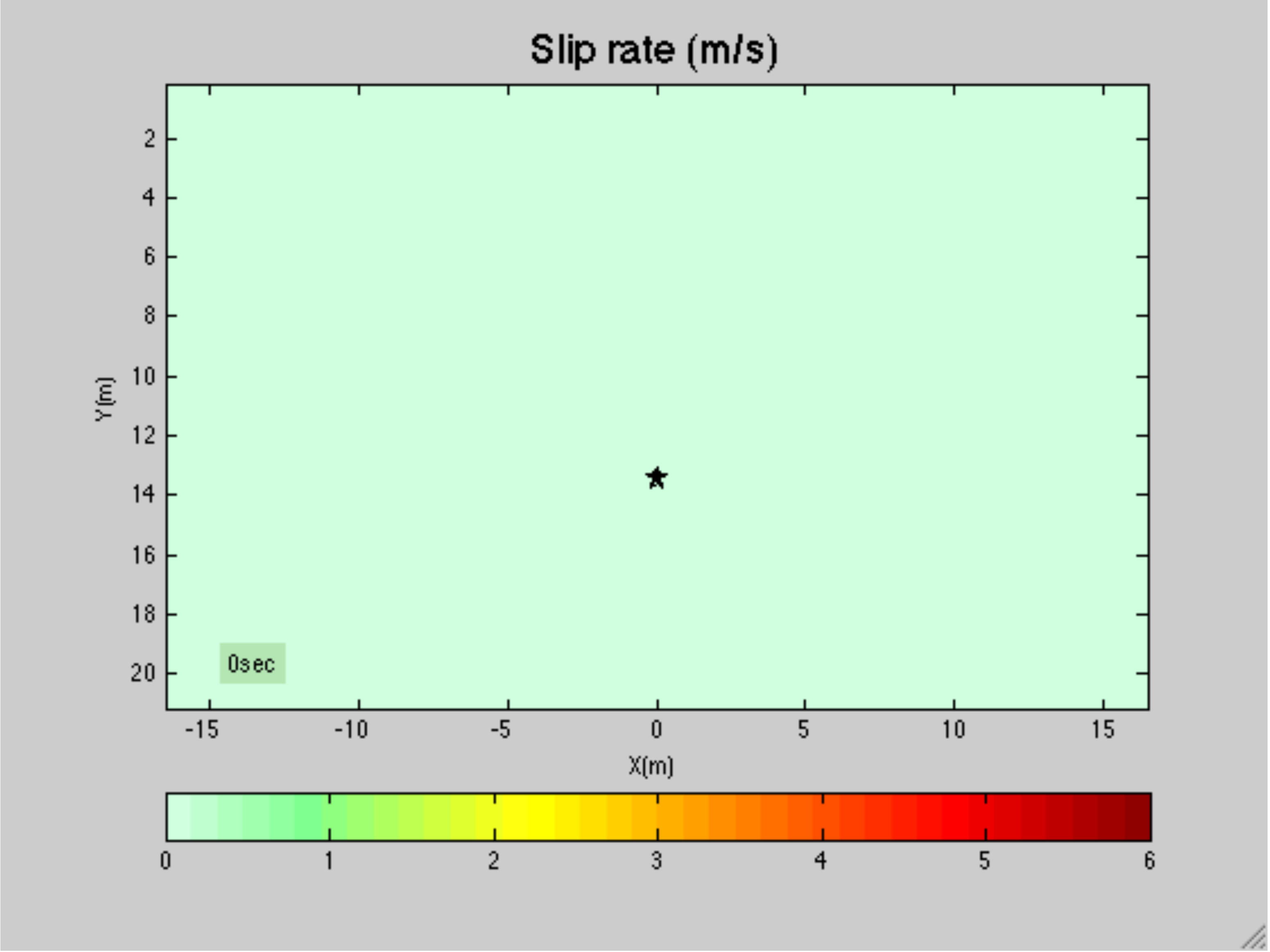


Dynamic stress drop
Overlapped by contour line of
Strength excess

(Pulido and Dalguer, 2009)

A model for 2000 Tottori earthquake

(Pulido and Dalguer, 2009)



A model for 2000 Tottori earthquake (velocity ground motion)

Pulido and Dalguer (2009)