Resolution analysis in full waveform inversion

by

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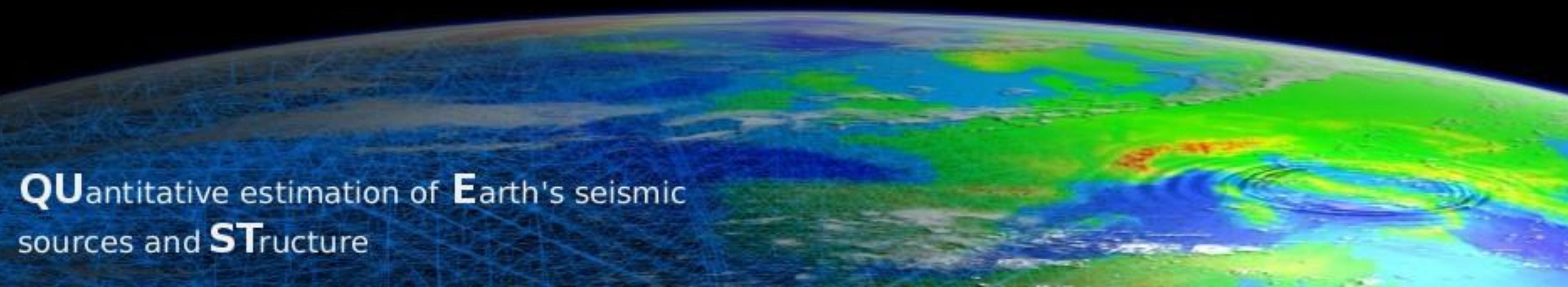
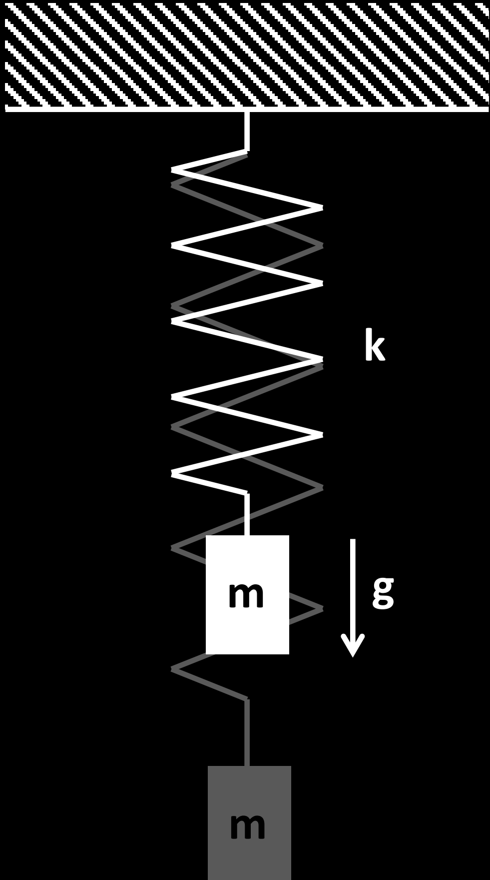


Welcome to the beginner‘s practicals !

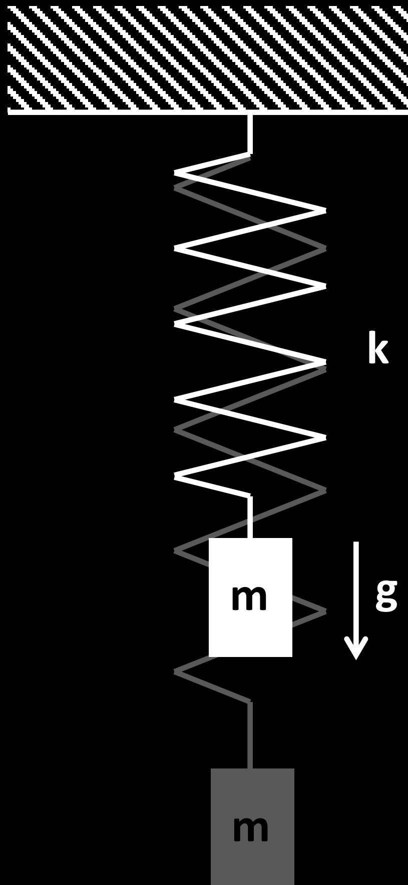


Initial-value problem

$$mg = kx + m\ddot{x}, \qquad x(0) = \dot{x}(0) = 0$$

Oscillatory solution

$$x(t) = \frac{1}{2} x_{max} \left[1 - \cos(\omega t)\right], \quad k = \omega^2 m$$



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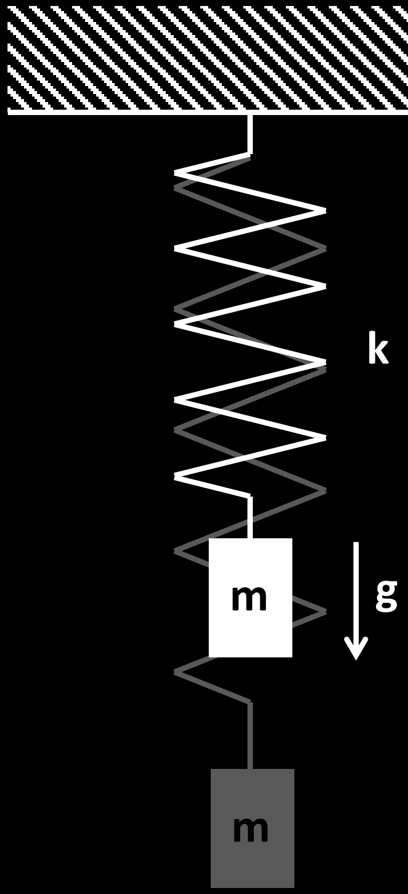
Simplistic error propagation

$$\Delta k = 2\omega m \, \Delta\omega + \omega^2 \, \Delta m$$



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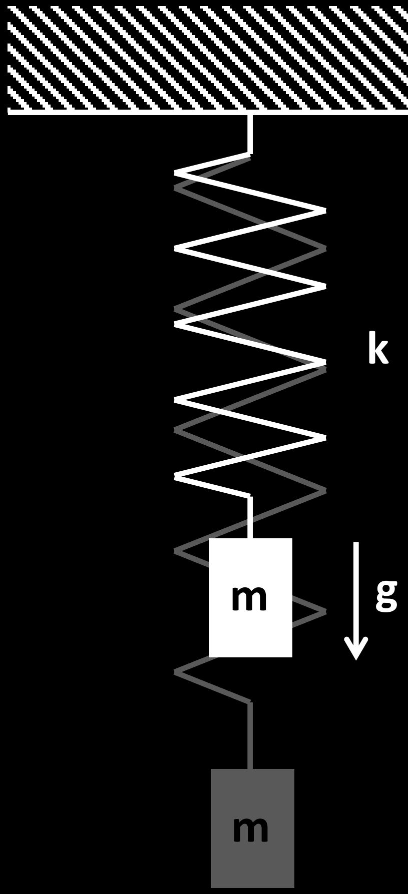
  → Relevance of results, experimental design.



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Classical linear inverse theory

- Backus & Gilbert, 1968. **The resolving power of gross Earth data.** Geophys. J.
- Kennett & Nolet, 1978. **Resolution analysis for discrete systems.** Geophys. J.
- Deal & Nolet, 1996. **Nullspace shuttles.** Geophys. J. Int.
- Nolet, Montelli & Virieux, 1999. **Explicit approximate expressions for the resolution and a posteriori covariance of massive tomographic systems.** Geophys. J. Int.
- Boschi, 2003. **Measures of resolution in global body wave tomography.** Geophys. Res. Lett.

Non-linear inverse problems

- Duane & Kennedy, 1987. **Hybrid Monte Carlo.** Phys. Lett. B
- Sambridge, 1999. **Geophysical inversion with a neighborhood algorithm.** Geophys. J. Int.
- Devilee, Curtis & Roy-Chowdhury, 1999. **An efficient, probabilistic neural network approach to solving inverse problems.** J. Geophys. Res.
- Sambridge & Mosegaard, 2002. **Monte Carlo methods in geophysical inverse problems.** Rev. Geophys.
- Tarantola, 2005. **Inverse problem theory and methods for model parameter estimation.**

„classical“ tomography:

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Nature of synthetic inversions

- Very easy to perform
- Optimistic (show what you can recover, hide what you cannot see)

Complexity of alternatives

- Require large efforts
- Difficult to interpret (posterior covariance, probabilities)

Cultural

- Lack of (self-) criticism
- Seductive power of colourful pictures

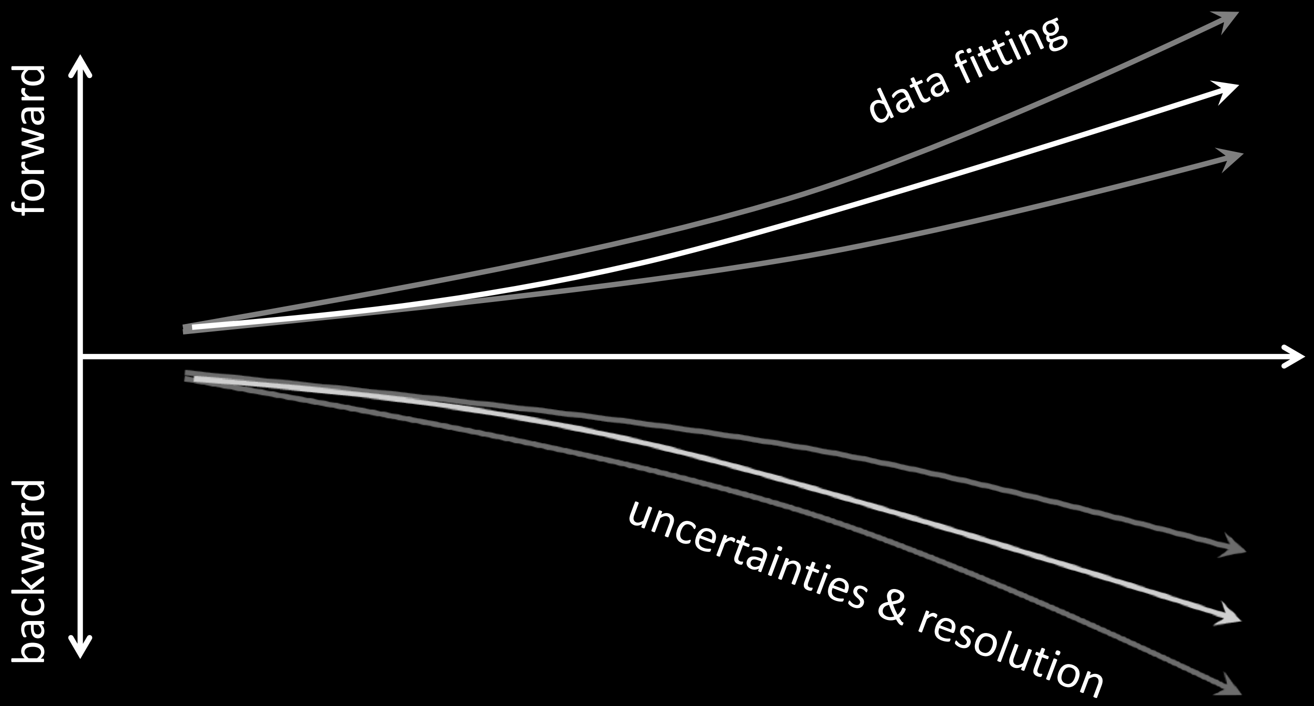
„classical“ tomography:

- tendency to ignore these developments
- restriction to synthetic inversions (chequer boards, hardly informative and misleading)

full waveform inversion (numerical modelling + adjoint or scattering integral methods):

- no resolution analysis
- often without synthetic inversions
- visual analysis and data fit

INVERSION = DATA FITTING + UNCERTAINTY ANALYSIS + …



„classical“ linear & smaller-scale tomographic problems

full waveform and adjoints, large-scale

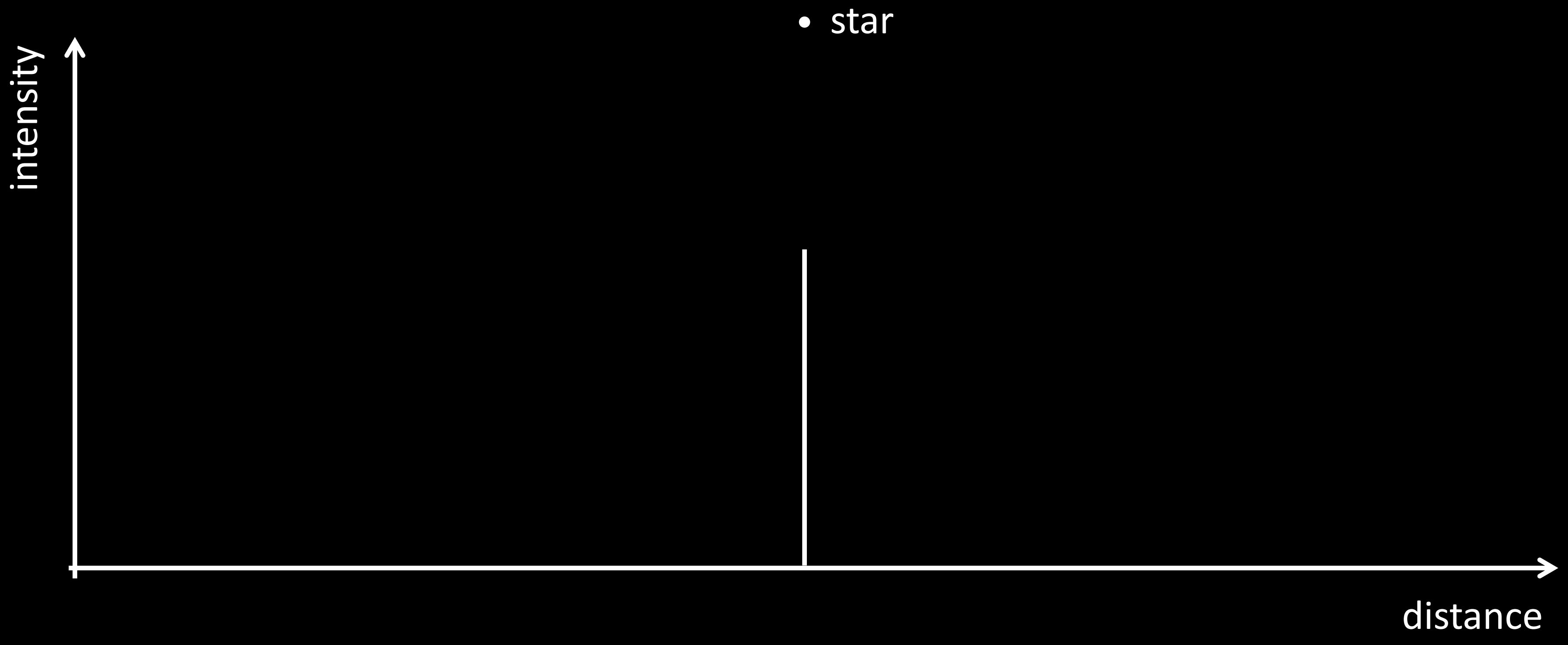
Have we really made progress

in solving inverse rather than just data fitting

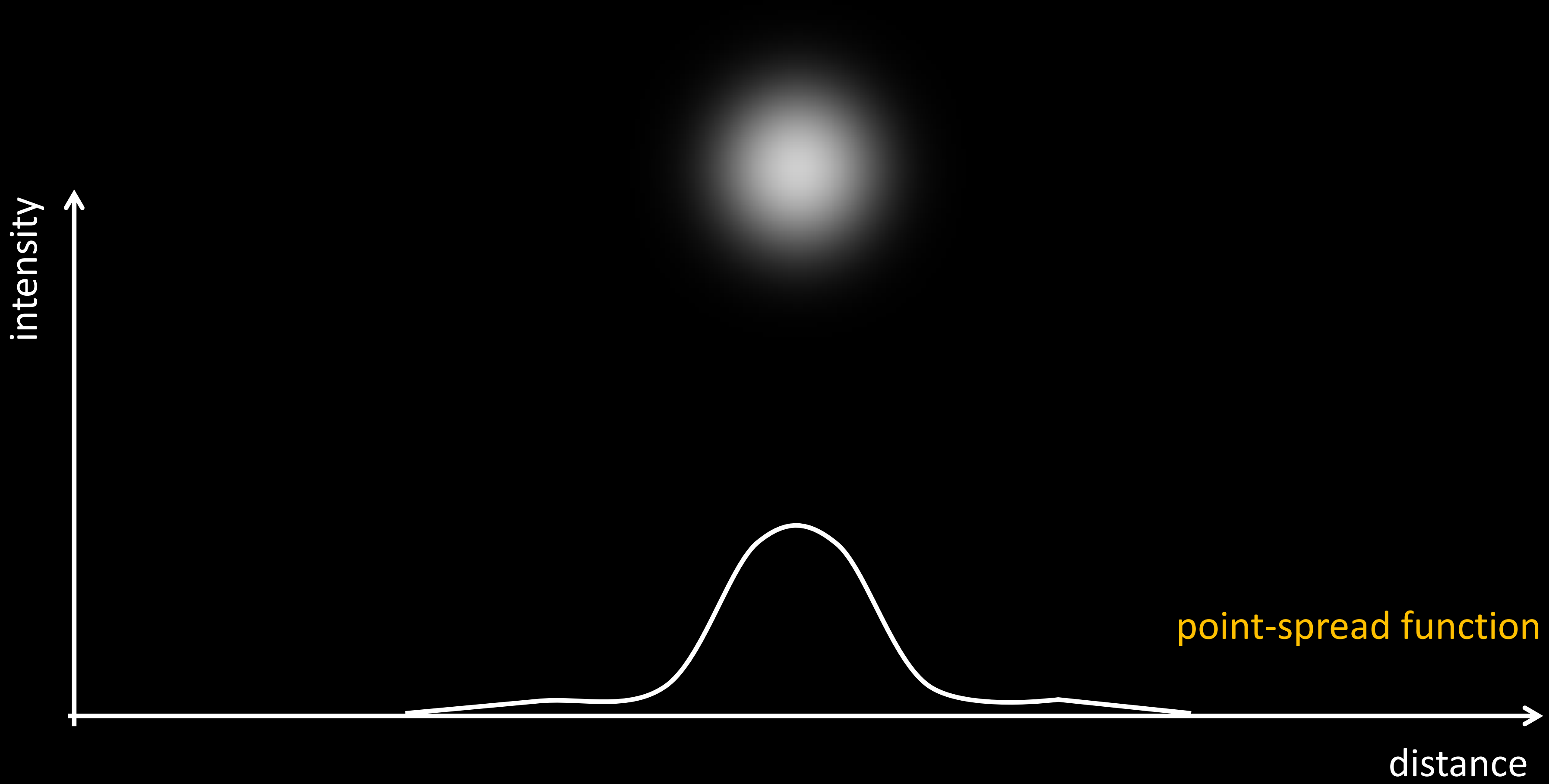
problems ?

Astronomical detour

perfect (non-existent) **telescope**



imperfect (real) **telescope**



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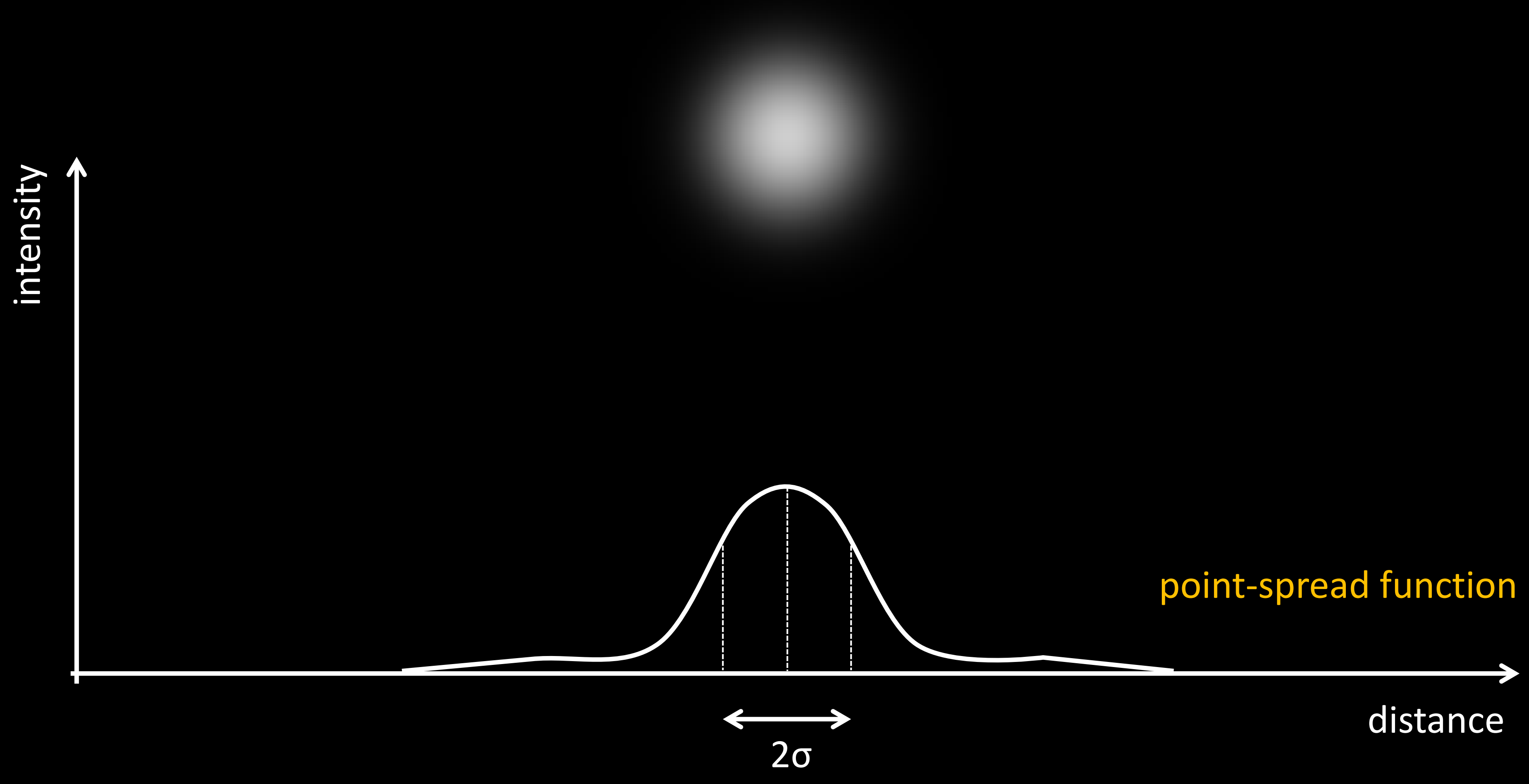
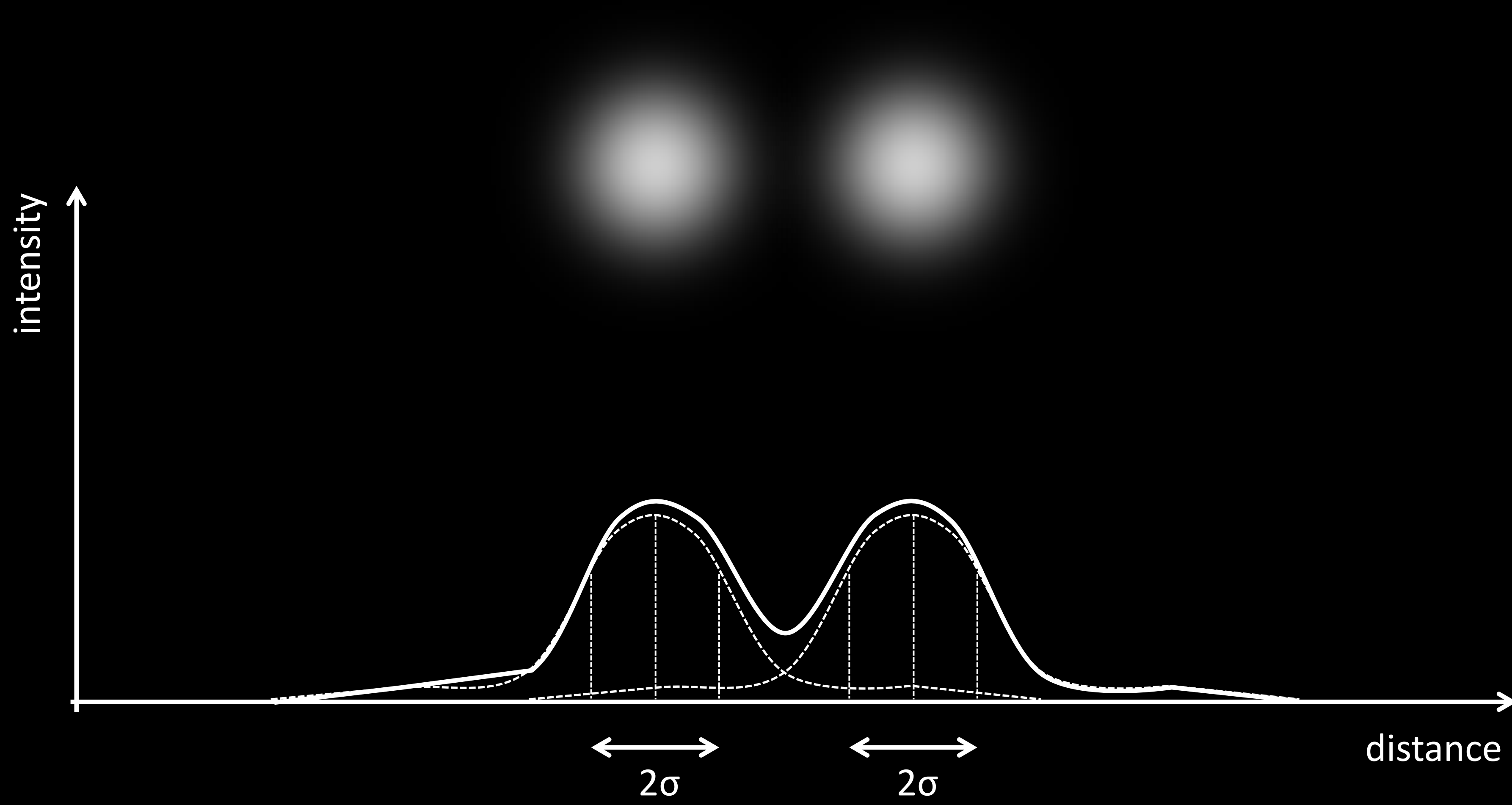
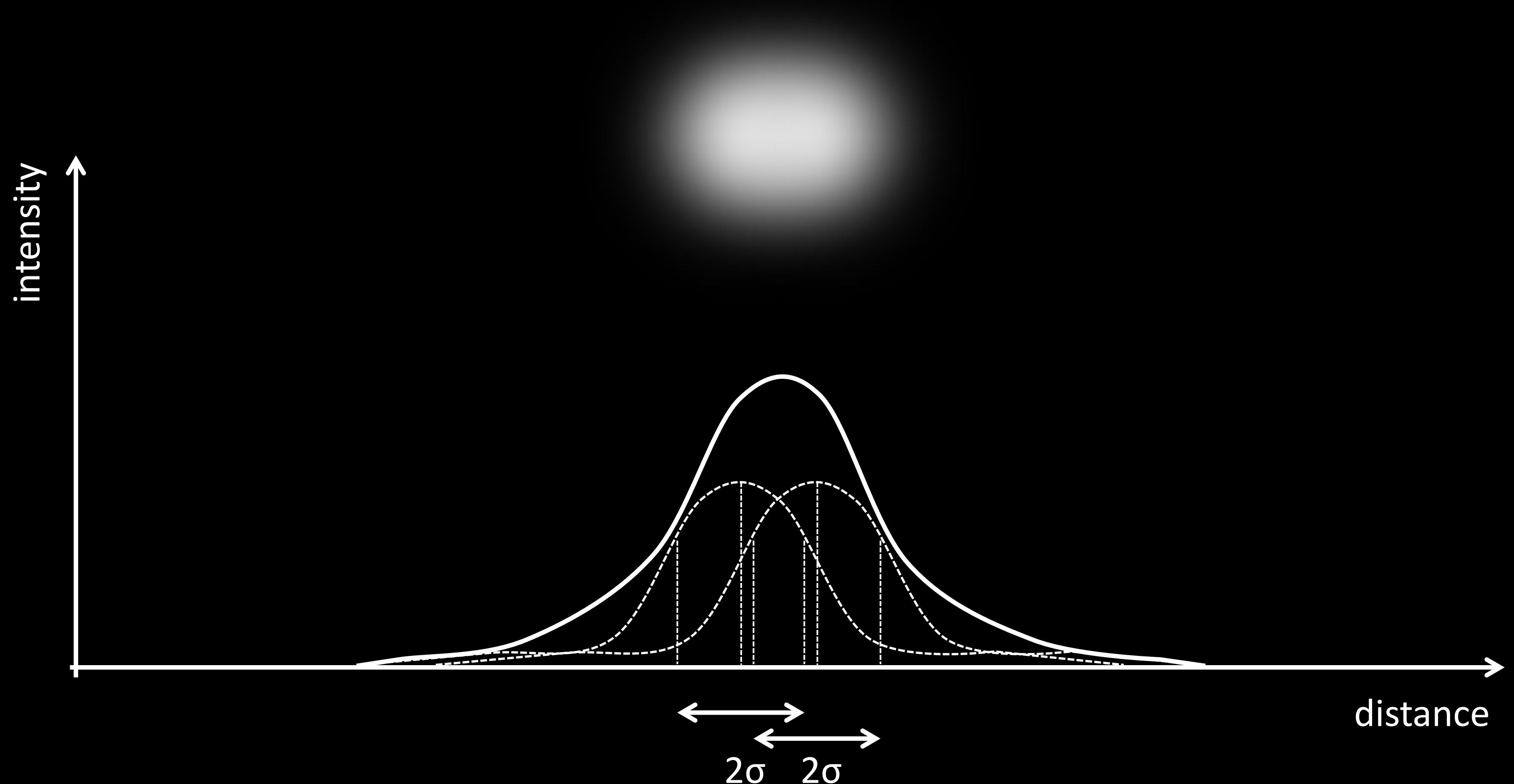




intensity
distance
2σ
2σ

Definition: resolution length = 2σ



Back to Earth

Waveform tomography

- **Misfit for model m:** $\chi(\mathrm{m})$
  - frequency-dependent traveltimes, time-frequency misfits, amplitudes, …
- **Iterative minimisation of χ to reach optimal model:** m_{opt}
  - steepest descent, conjugate gradients, Newton-like methods, …

Waveform tomography

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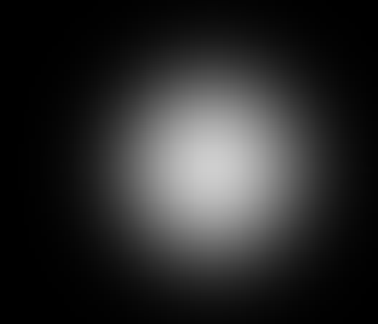
- **Approximate tomographic equivalent of the point-spread function:**

structural heterogeneity

• x_0

seen through the tomographic telescope



Hessian of χ at $\mathrm{m}_{\mathrm{opt}}$: $H(\vec{x}, \vec{x}_0)$

Waveform tomography

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- **Iterative minimisation of χ to reach optimal model:** m_{opt}
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- **Approximate tomographic equivalent of the point-spread function:**
- **Interpretations of the Hessian:**
  - Point-spread function
  - Inverse posterior covariance
  - Extremal bounds analysis

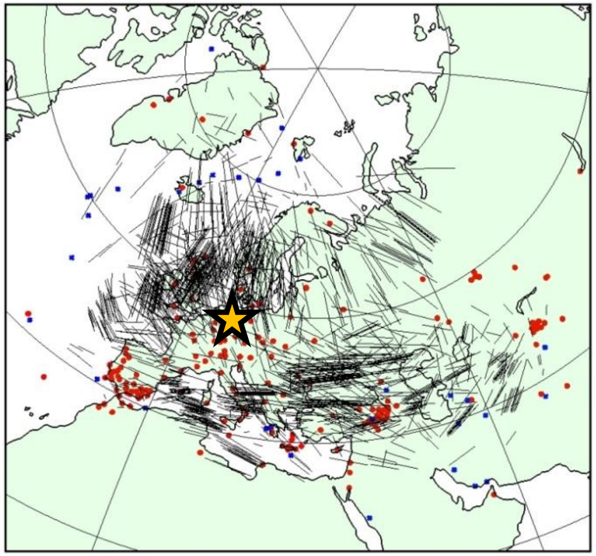
➡ **To make a first step towards a quantitative resolution analysis, we have to get efficient access to the Hessian!**

- **Computation of $H \cdot \delta m$ via an extension of the adjoint method**
  - **(1) 2 forward simulations**
  - **(2) 2 adjoint simulations (time-reversed)**

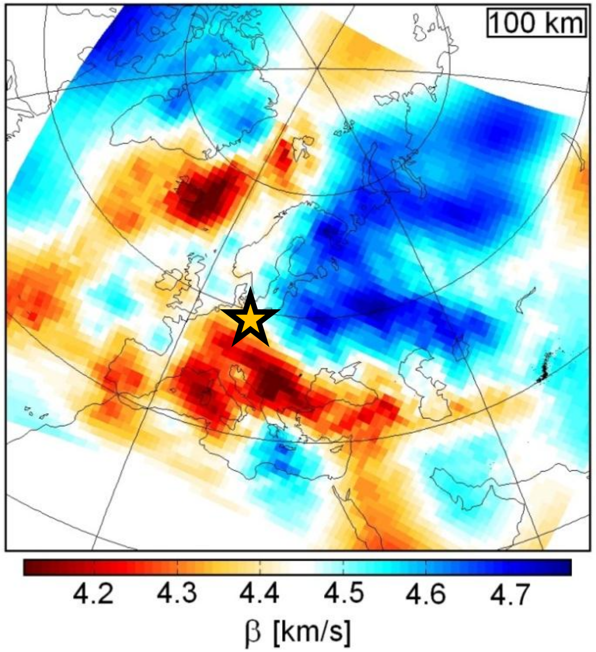
Fichtner & Trampert, 2011. **Hessian kernels of seismic data functionals based upon adjoint techniques**. Geophys. J. Int.

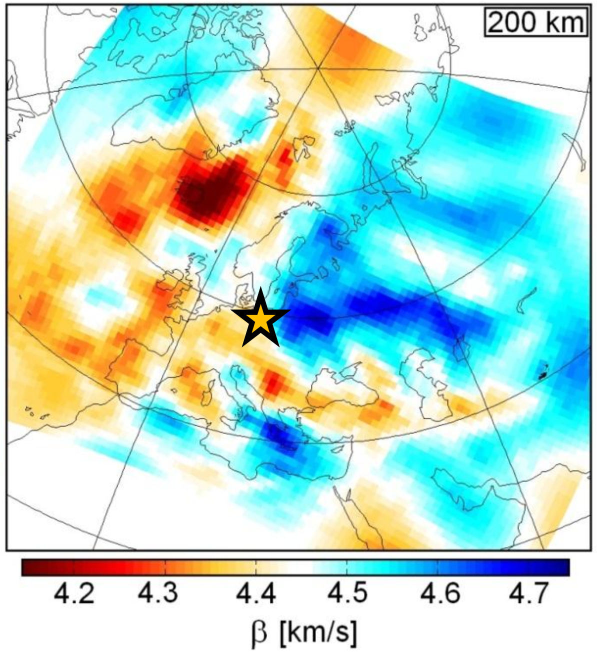
- **Efficient computation of H·δm via an extension of the adjoint method**
- **Example:** δm = point-localised S velocity perturbation at ★

central ray segments



S velocity





dominant period: T = 100 s

- **Efficient computation of H·δm via an extension of the adjoint method**
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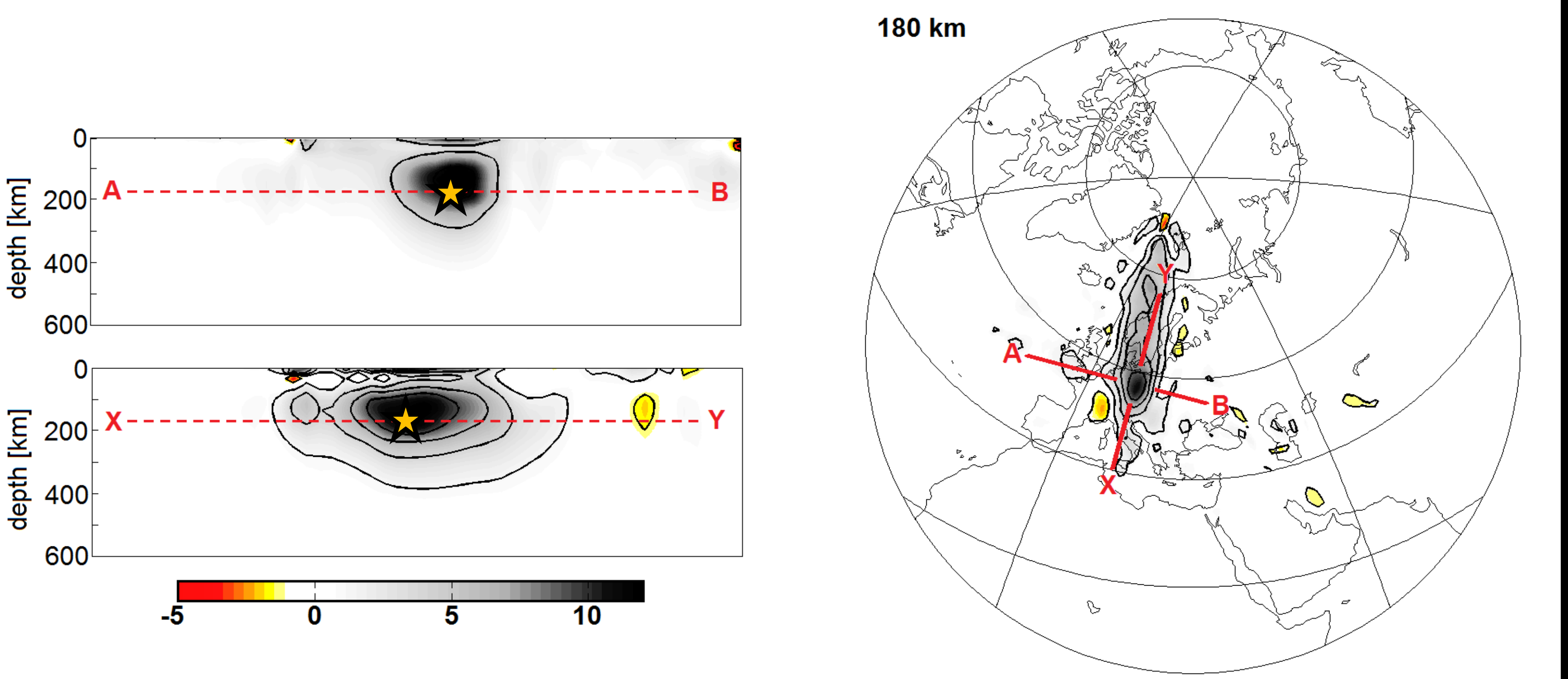

1st order point-spread function

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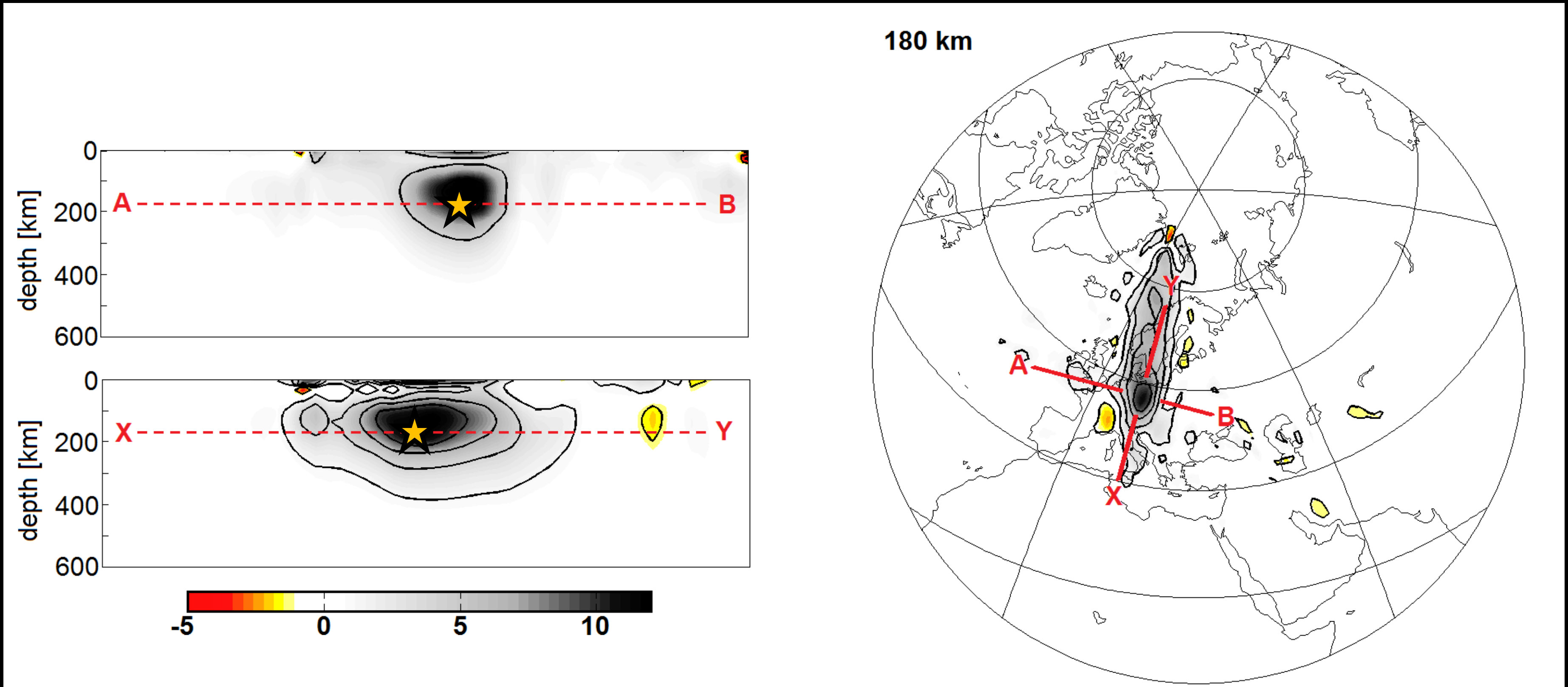

1st order point-spread function

- Tomographic point-spread functions are not generally
  - symmetric
  - positive definite

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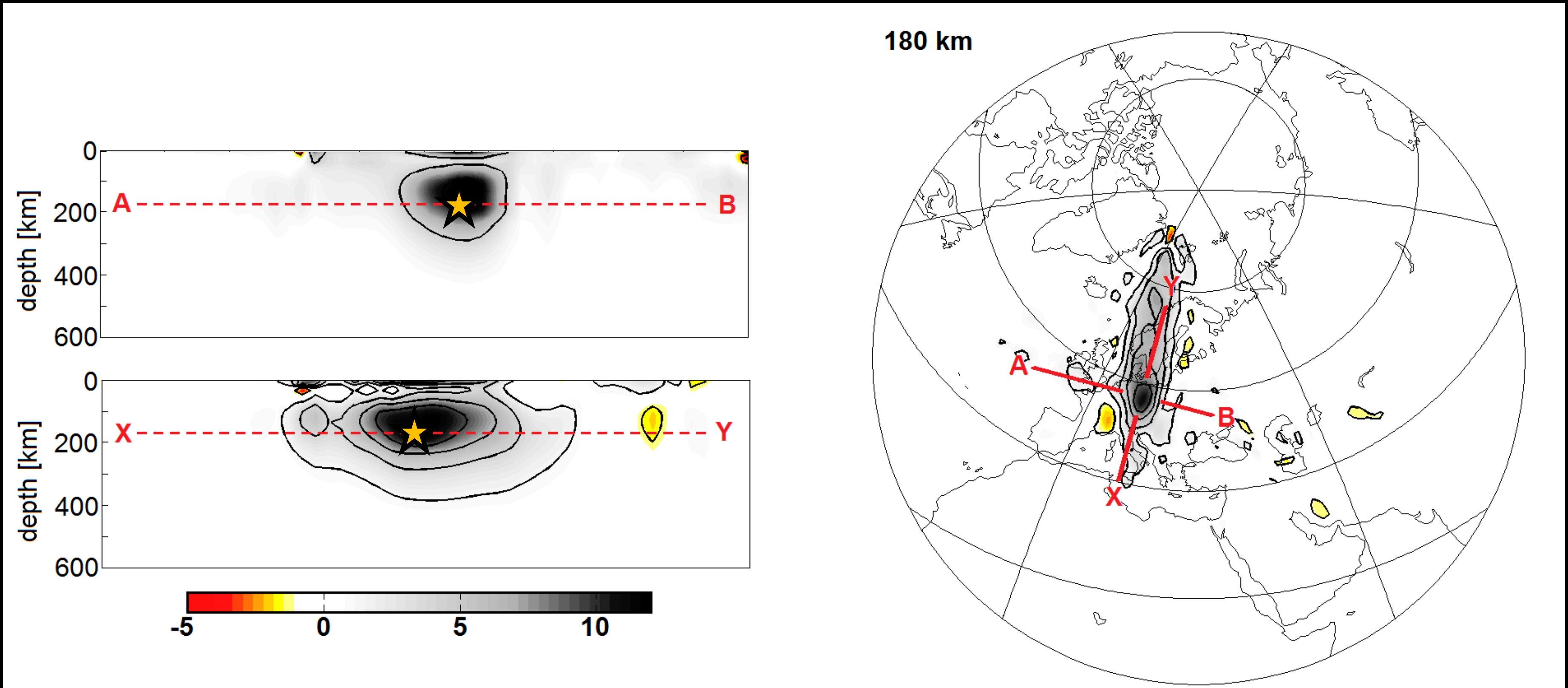
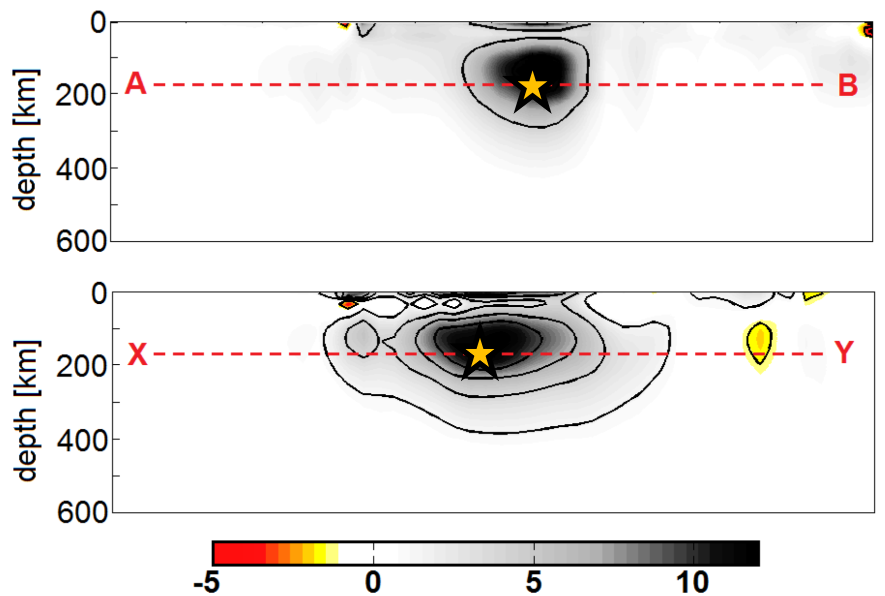
- **Example:** δm = point-localised S velocity perturbation at ★



1st order point-spread function

- Ideally for every point in the model volume
- Prohibitively expensive

- **Parameterise the Hessian by a position-dependent Gaussian:**



1st order point-spread function

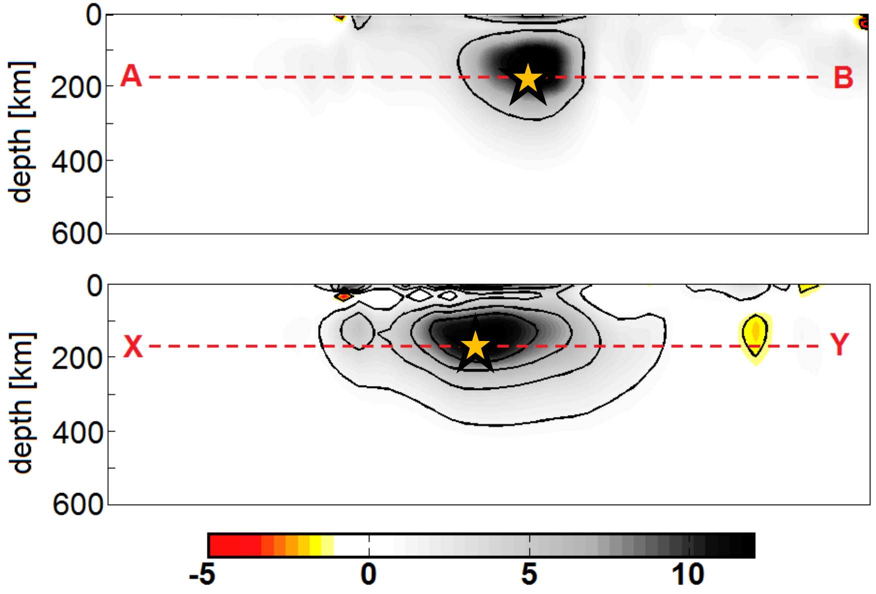
$$H(x,y) \approx A(x)\, e^{-(x-y)^T C(x)\,(x-y)}$$

A(x): position-dependent **amplitude**
C(x): position-dependent **width**

- **Generalisation:** - Gram-Charlier expansion of H
  - sum over a parent function and its successive derivatives

- **How can we compute the position-dependent parameters of the approximation?**

- **Parameterise the Hessian by a position-dependent Gaussian:**
- **Determine the parameters using Fourier transforms of the Hessian**



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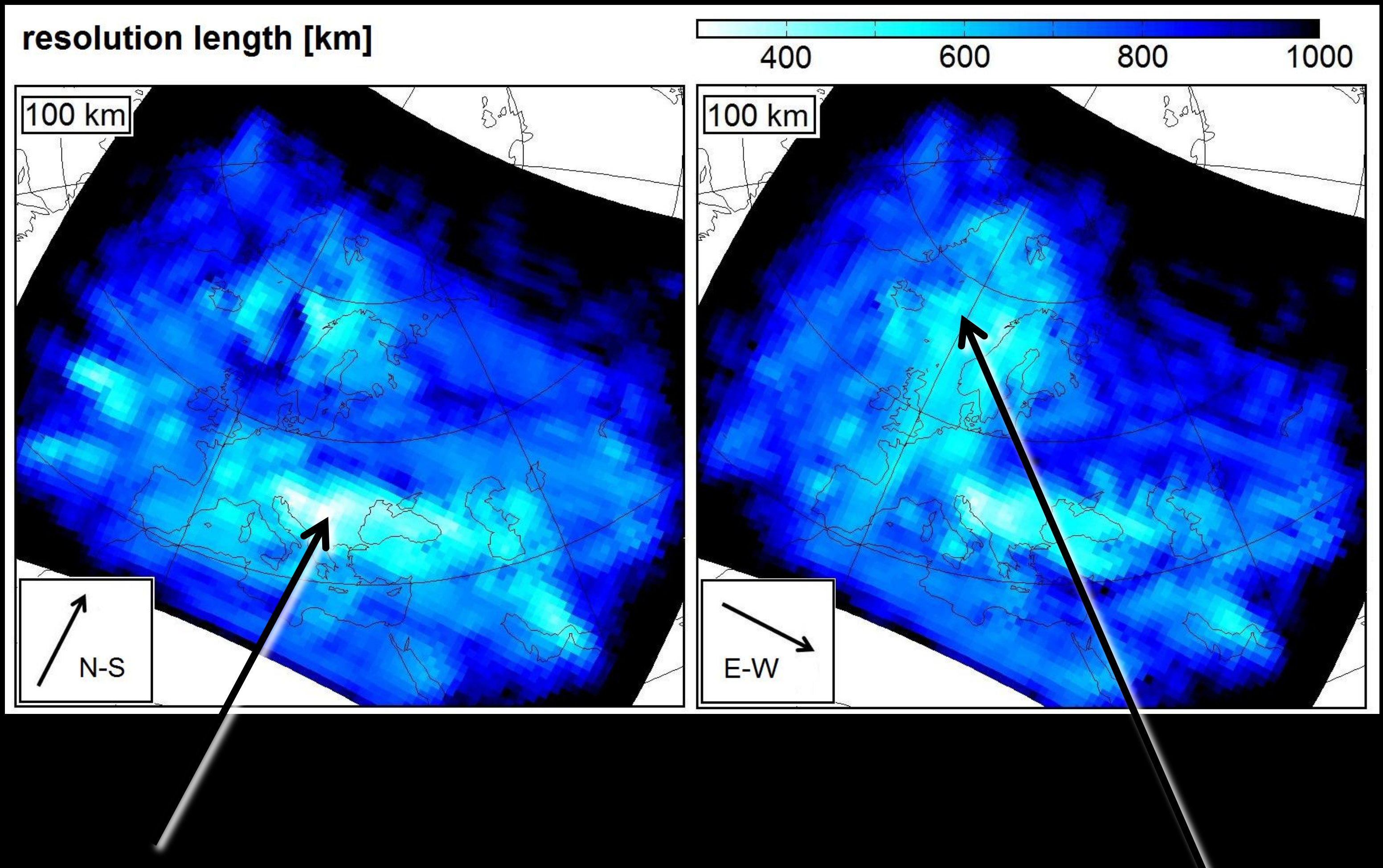
$$\widetilde{H}(x,k) = \underbrace{\int H(x,y)\, e^{-ik^T y}\, dy}\ \propto\ A(x)\, e^{i\,k^T C^{-1}(x)\, k}$$

Hessian applied to a sinusoidal model perturbation, $H \cdot \delta m$ - easy to compute

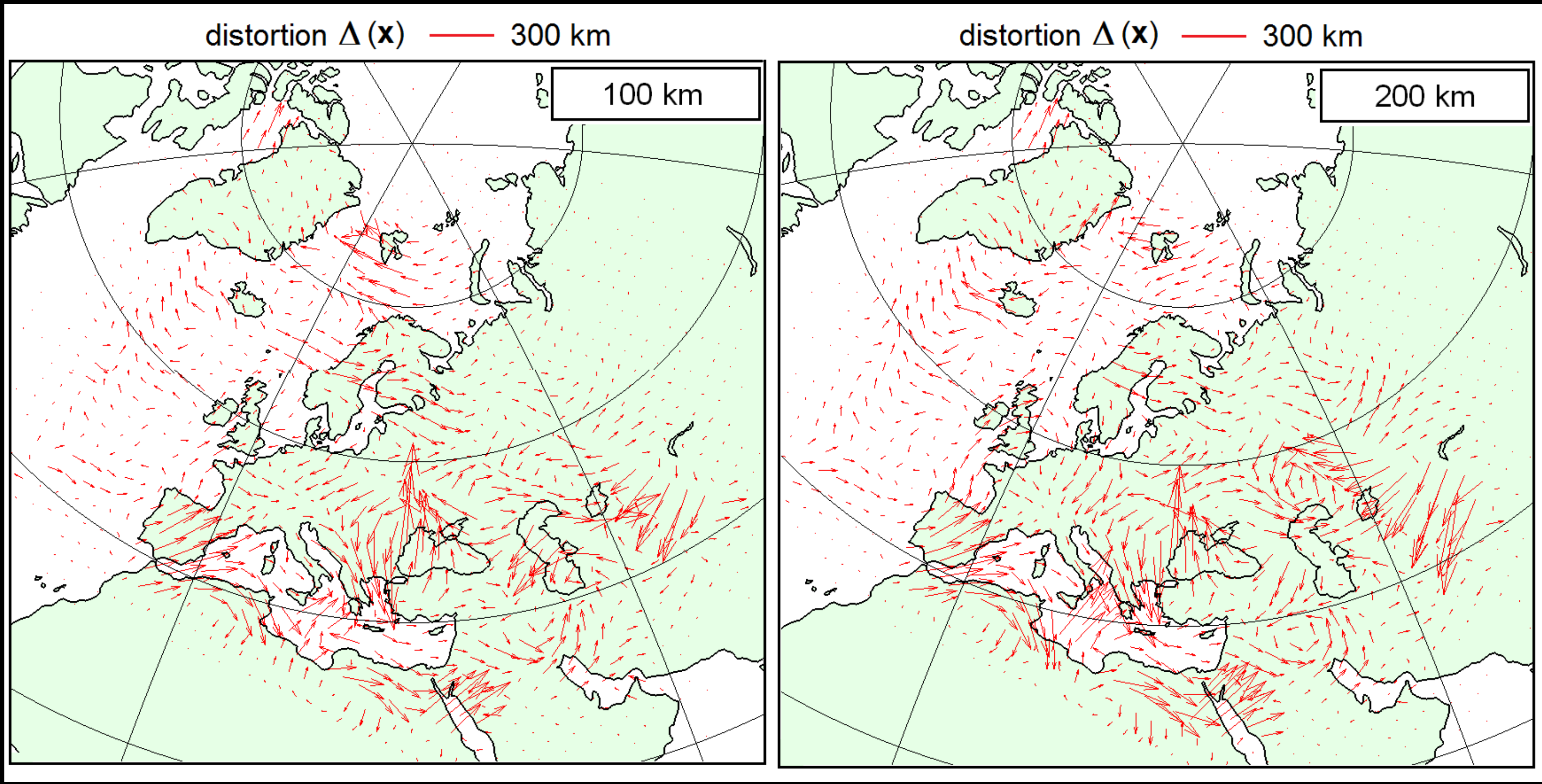
- **Resolution length:** The direction-dependent width of the point-spread function



high resolution in N-S direction
information propagating E-W

high resolution in E-W direction
information propagating N-S

- Point-perturbations are displaced through the tomographic imaging.
- Distortion = [position of point perturbation] – [centre of mass of its blurred image]



➡ **What you see may actually be somewhere else!**

Conclusions

- Resolution analysis based on the Hessian of the misfit functional.
- Quantitative measures of resolution independent from misleading synthetic inversions.
- Method built on
  - Parametrisation of the Hessian (borrowed from astronomy)
  - Computation of the parameters via Fourier transforms
- 3D distributions of **resolution length** and **distortion**

Fichtner & Trampert, 2011. **Resolution analysis in full waveform inversion**. Geophys. J. Int., accepted

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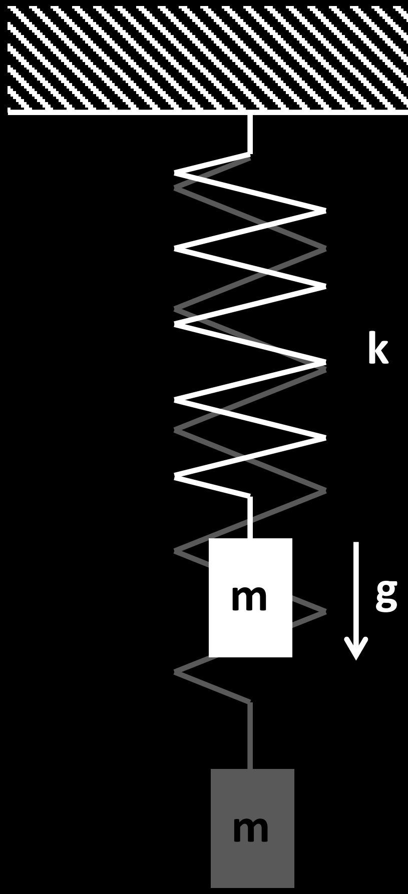
Corollaries:

- New family of Newton-like methods with an explicitly computed approximate Hessian
- Efficient pre-conditioner for conjugate-gradient algorithms
- Approach to adaptive parametrisation independent from ray theory

Fichtner & Trampert, 2011. **Resolution analysis in full waveform inversion**. Geophys. J. Int., accepted



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  - → **Relevance of results, experimental design.**
- **Inconsistencies (non-overlapping error bars) indicate incomplete physics.**
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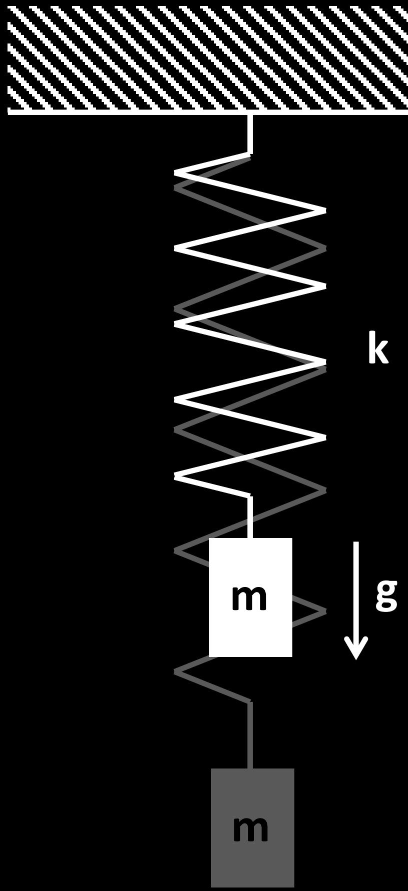

A. Tarantola

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Thank you very much for your attention !

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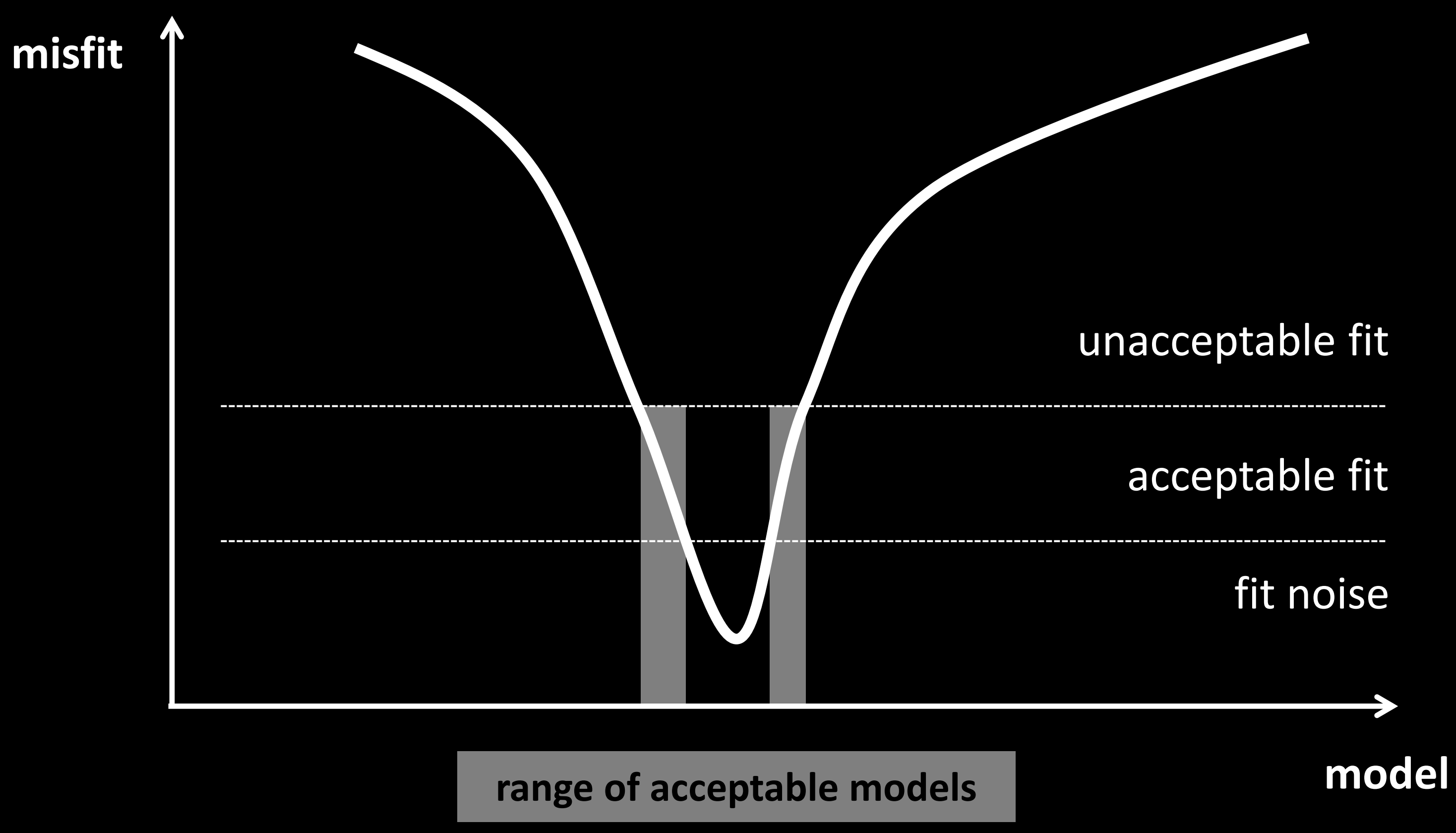

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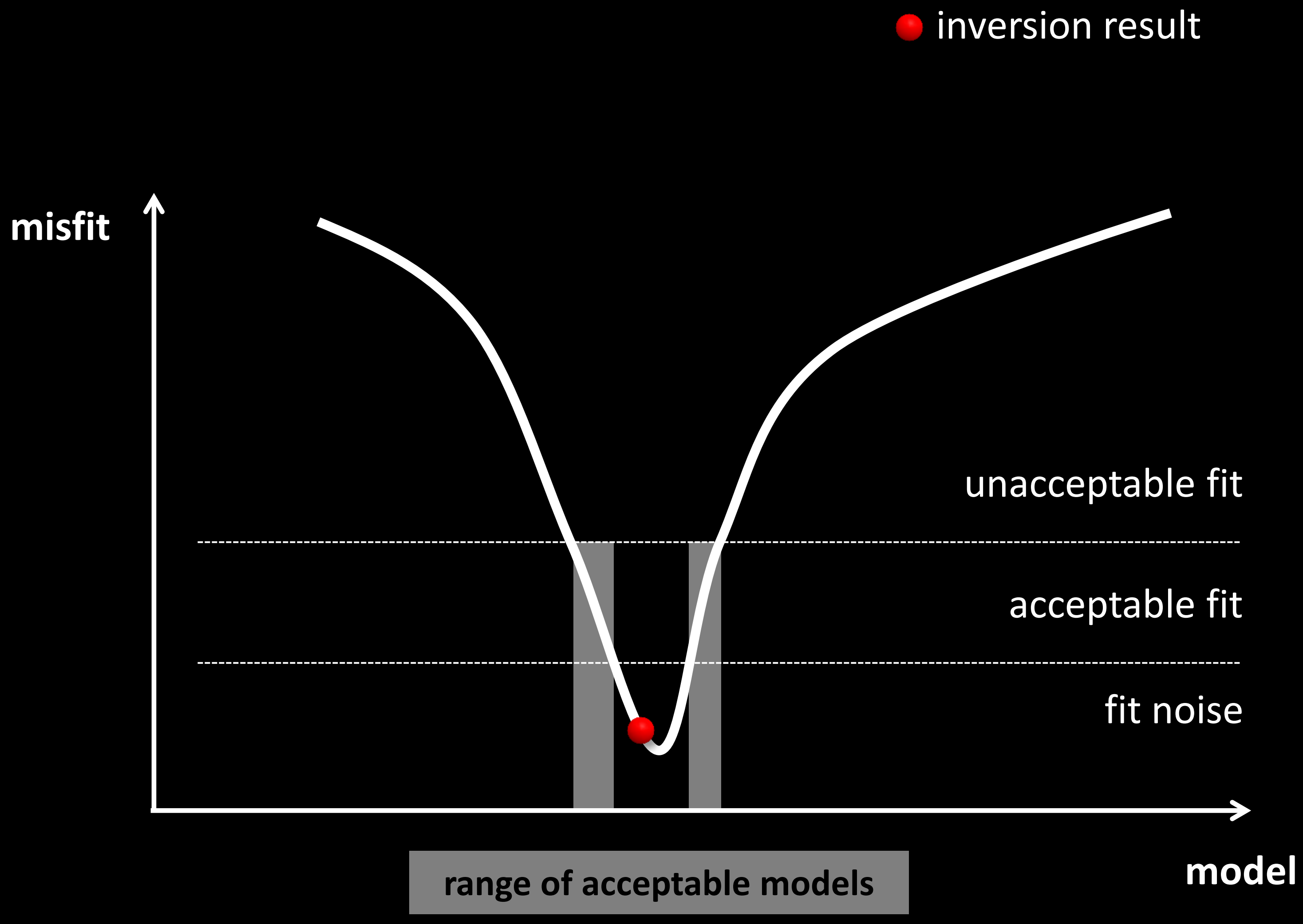

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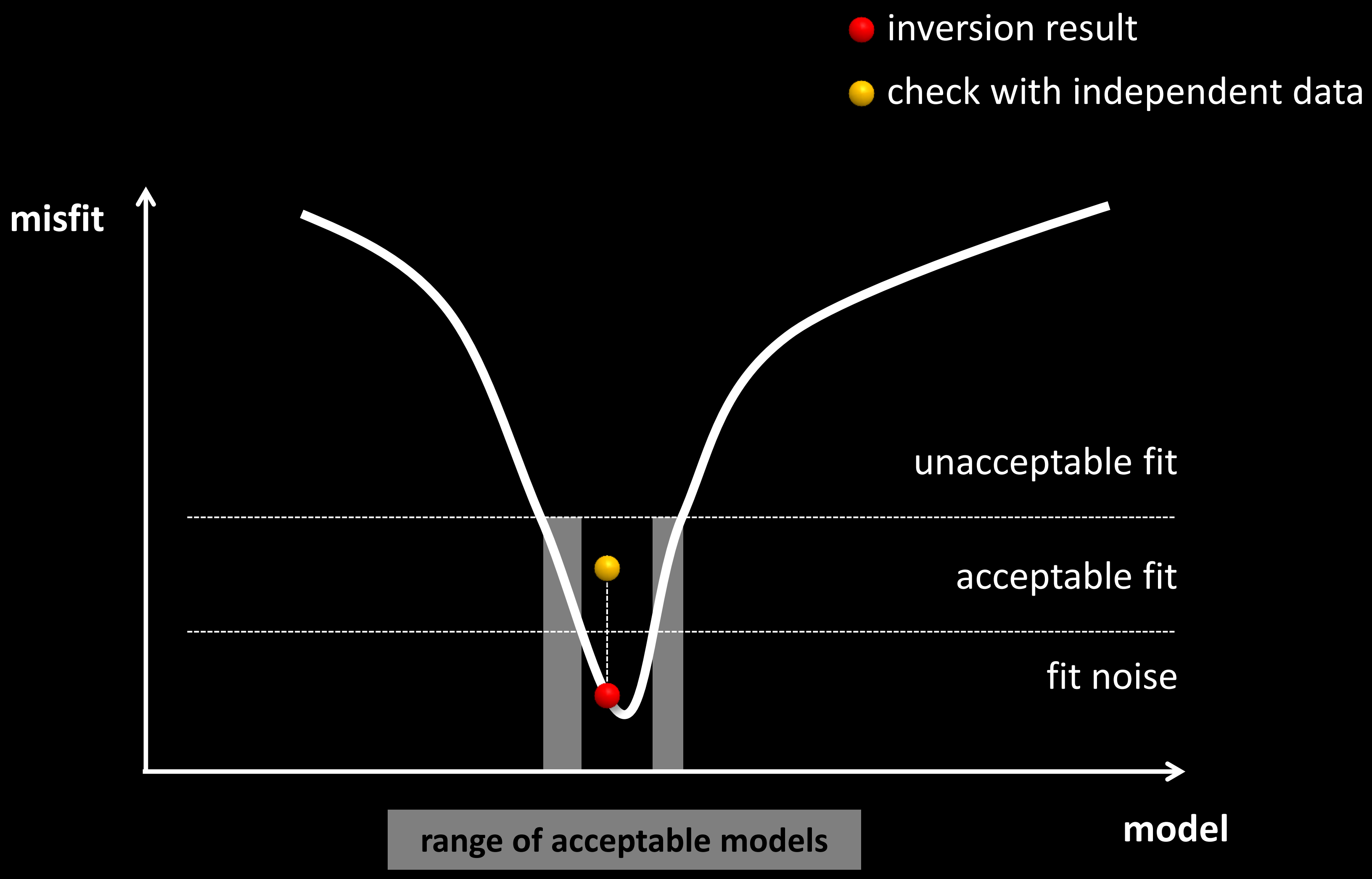

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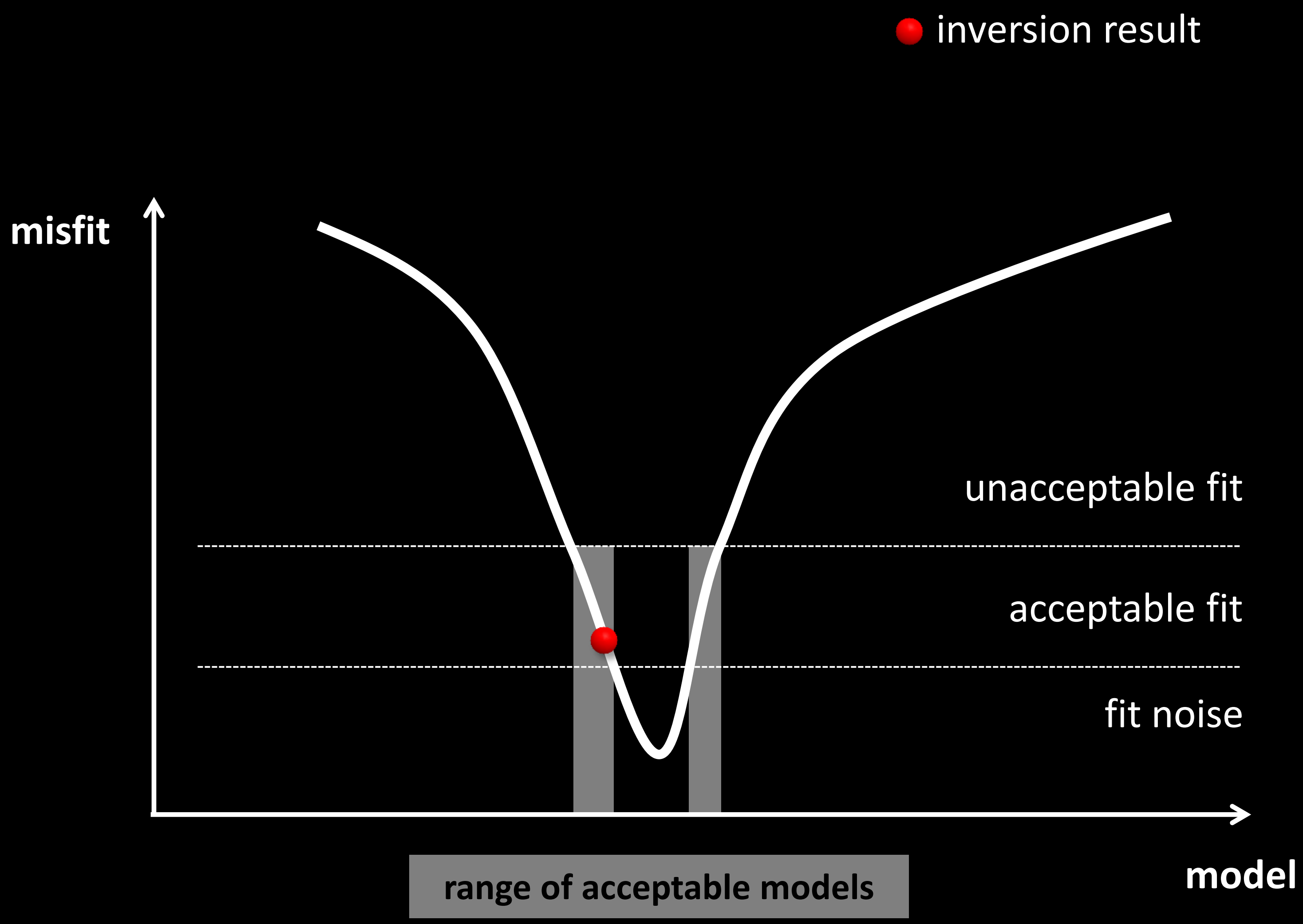

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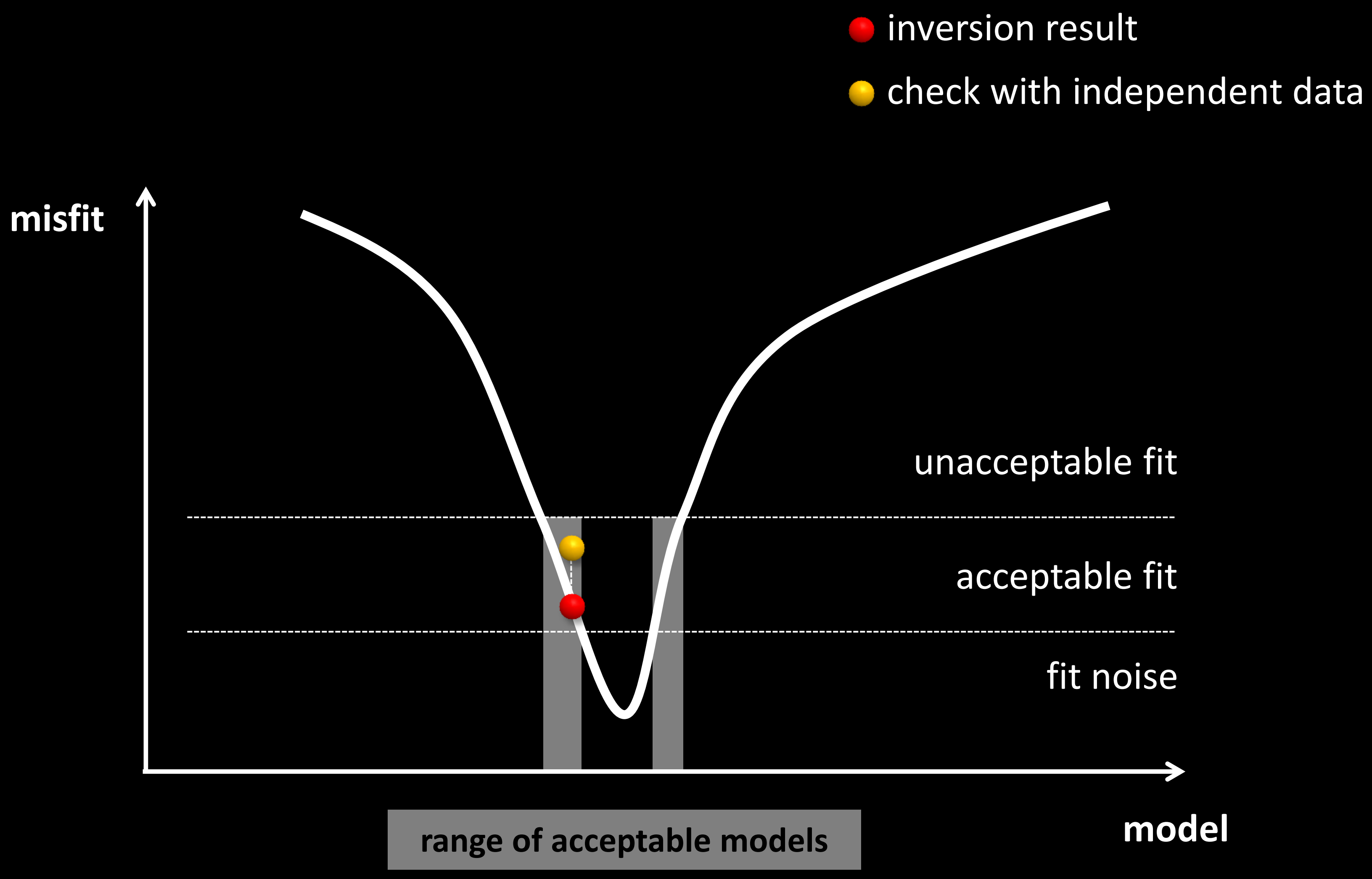

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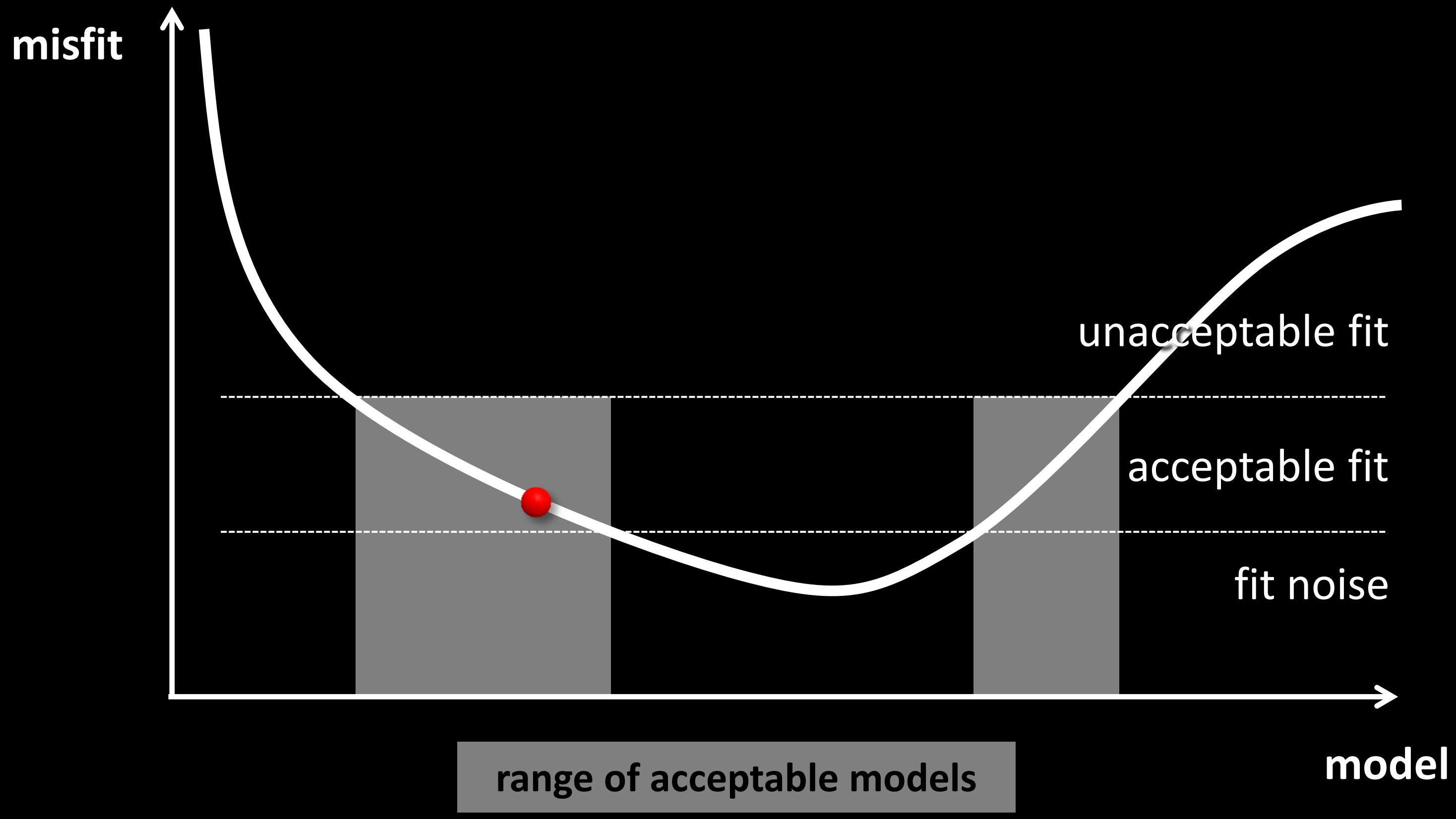

Reproduction of the errors statisitcs with independent data

- necessary to avoid over-fitting of data and over-structuring of models
- **not sufficient** to ensure the model is well-constrained