

A 3D Earth globe showing continents and oceans. Overlaid on the globe are concentric red and blue circular lines representing seismic waves emanating from a source. A vertical yellow line runs through the center, and a horizontal yellow waveform representing a seismic signal is overlaid across the middle of the globe.

SOURCE INVERSION IN A 3D EARTH

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with: Jeroen Tromp Princeton,
Qinya Liu Univ Toronto



ChiChi earthquake, Taiwan | 1999

http://www.webpages.uidaho.edu/~simkat/geol345_files/2010lecture9.html



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How?
Why?

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SOURCE PARAMETER ESTIMATION

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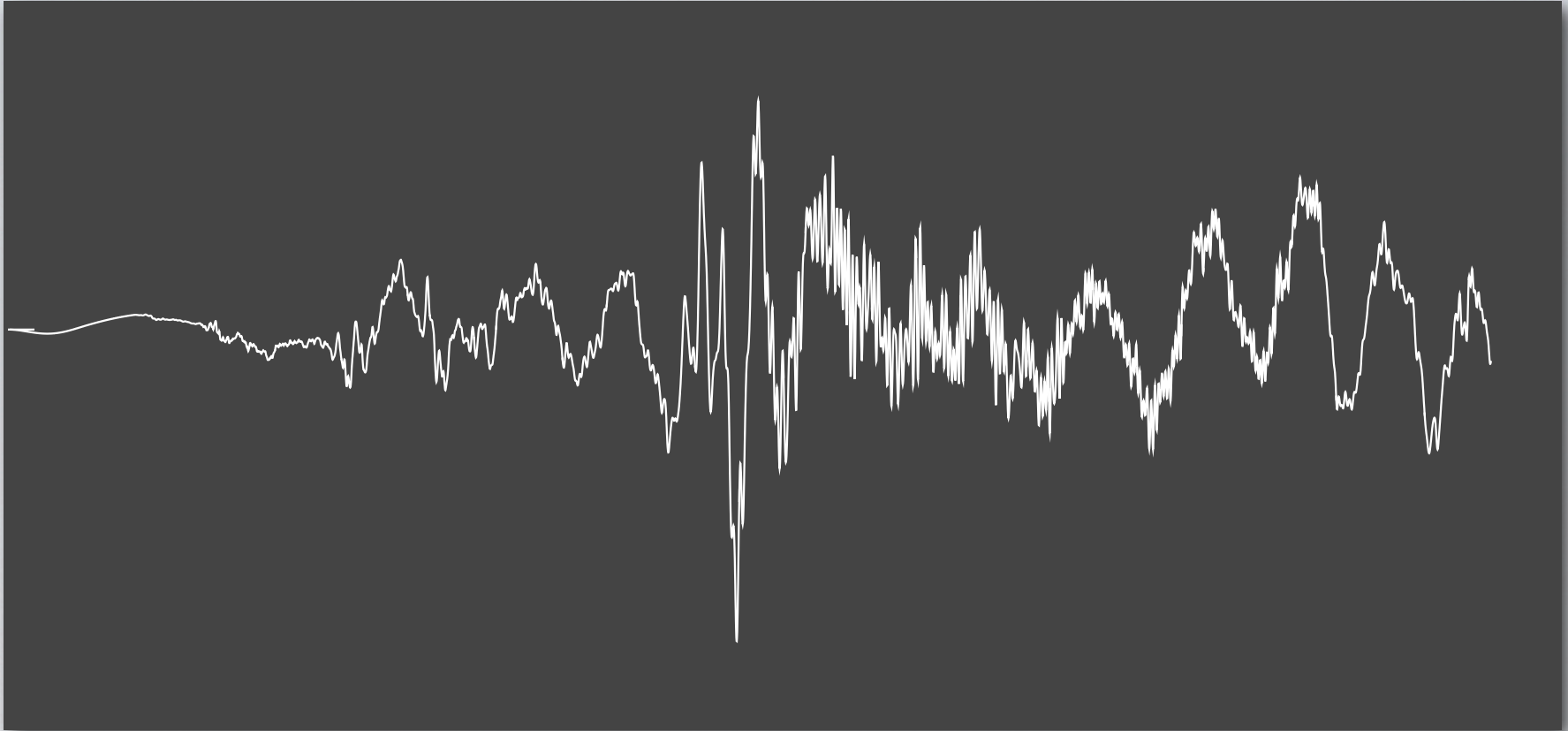
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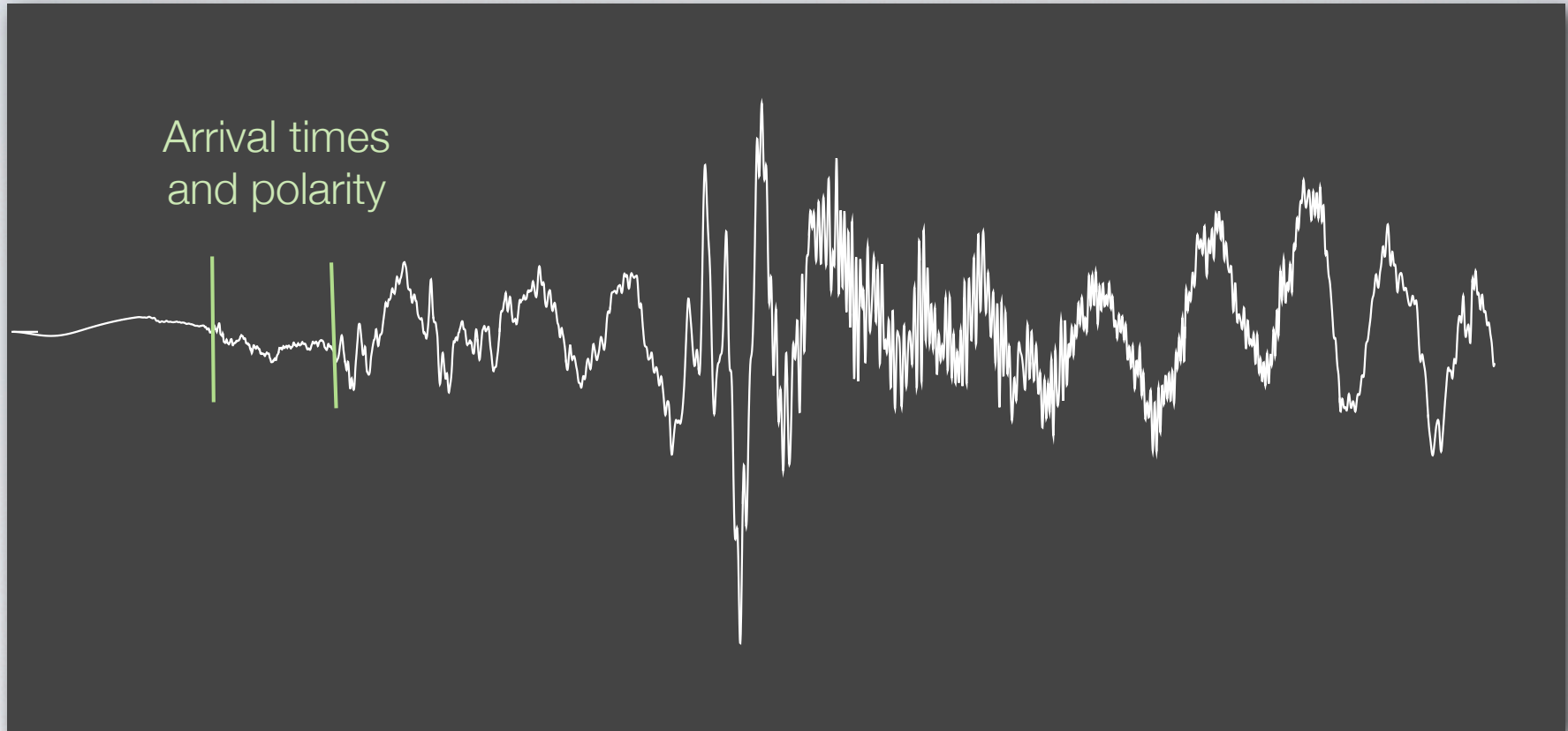
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SEISMOGRAM



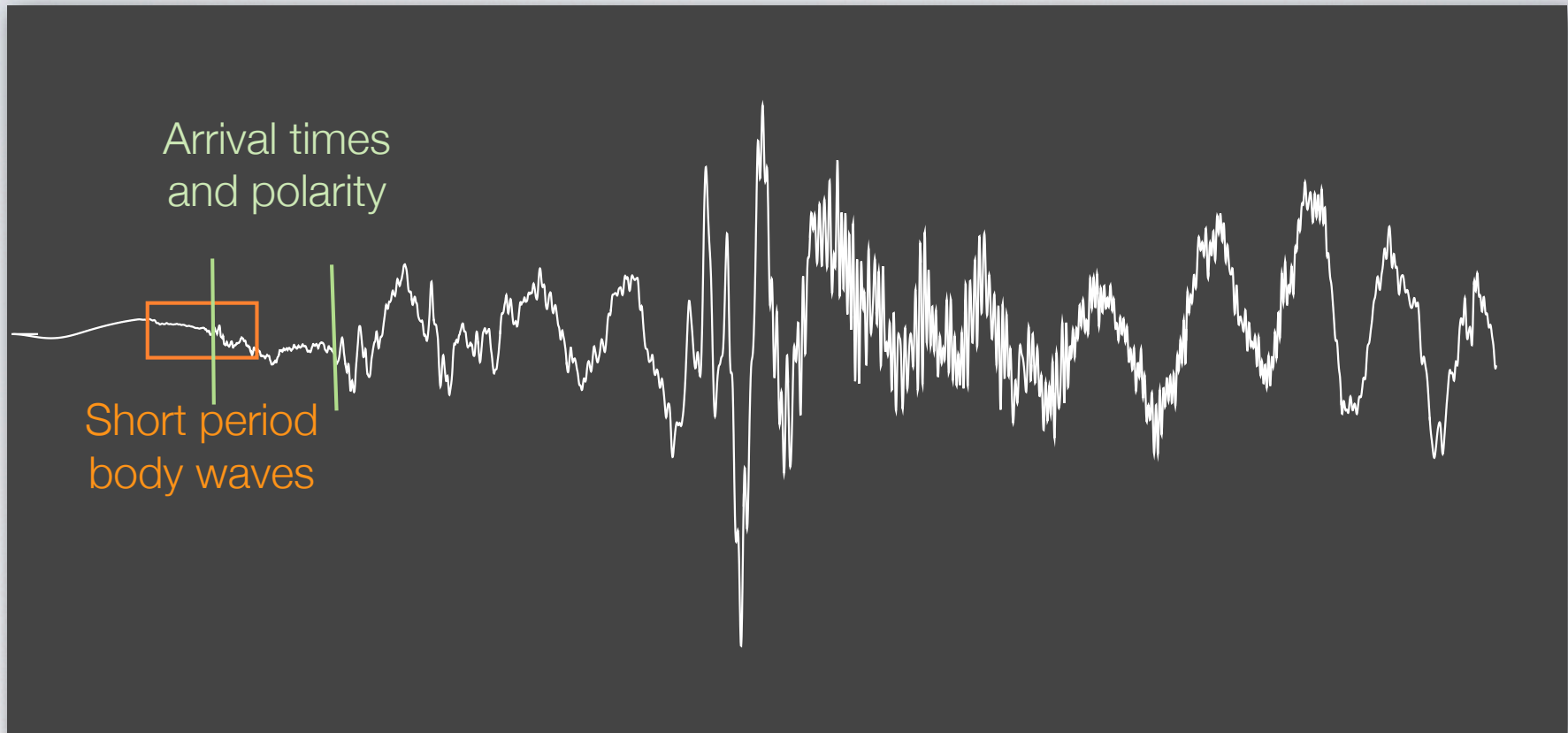
Sumatra 2004, vertical displ, Palmer Station Antarctica

SEISMOGRAM



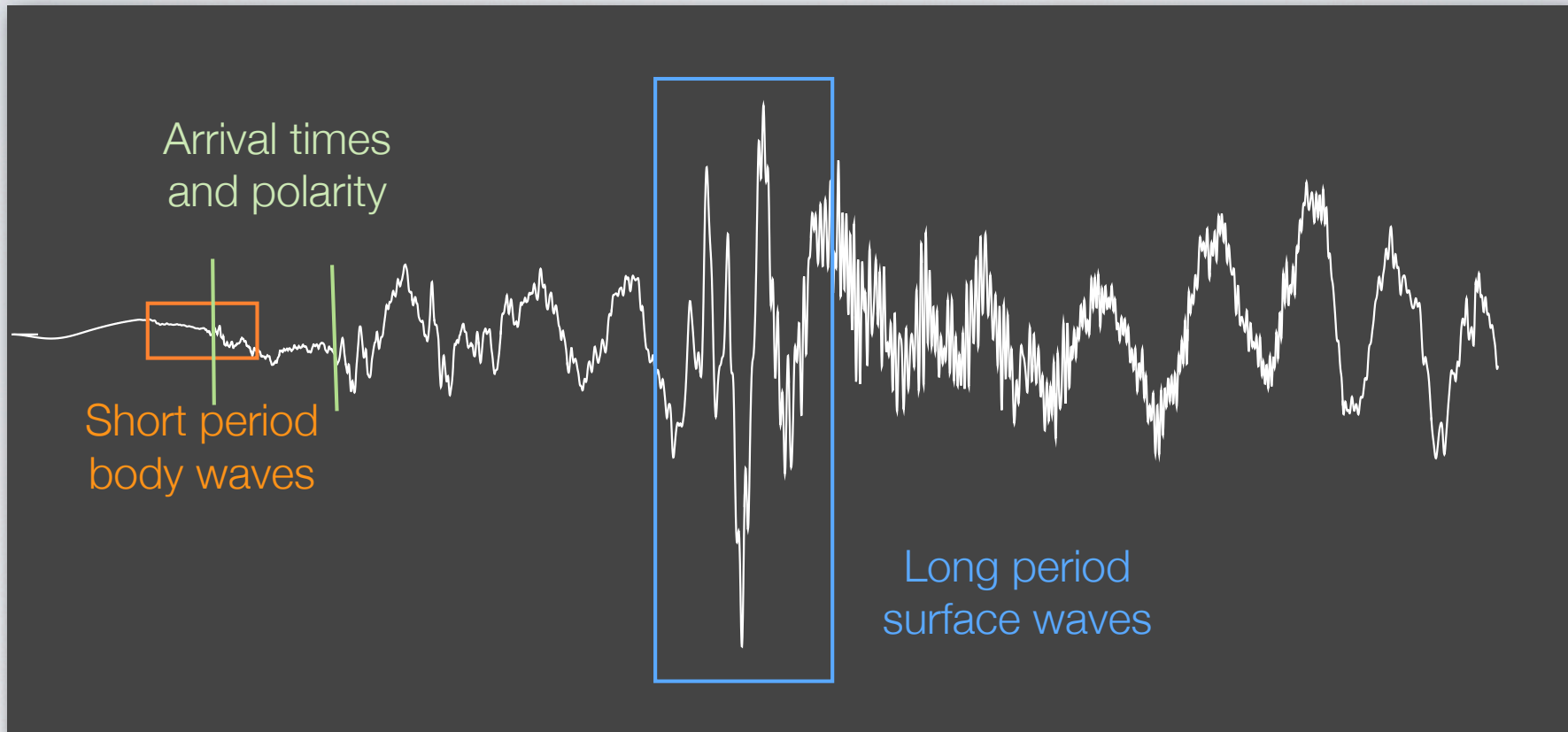
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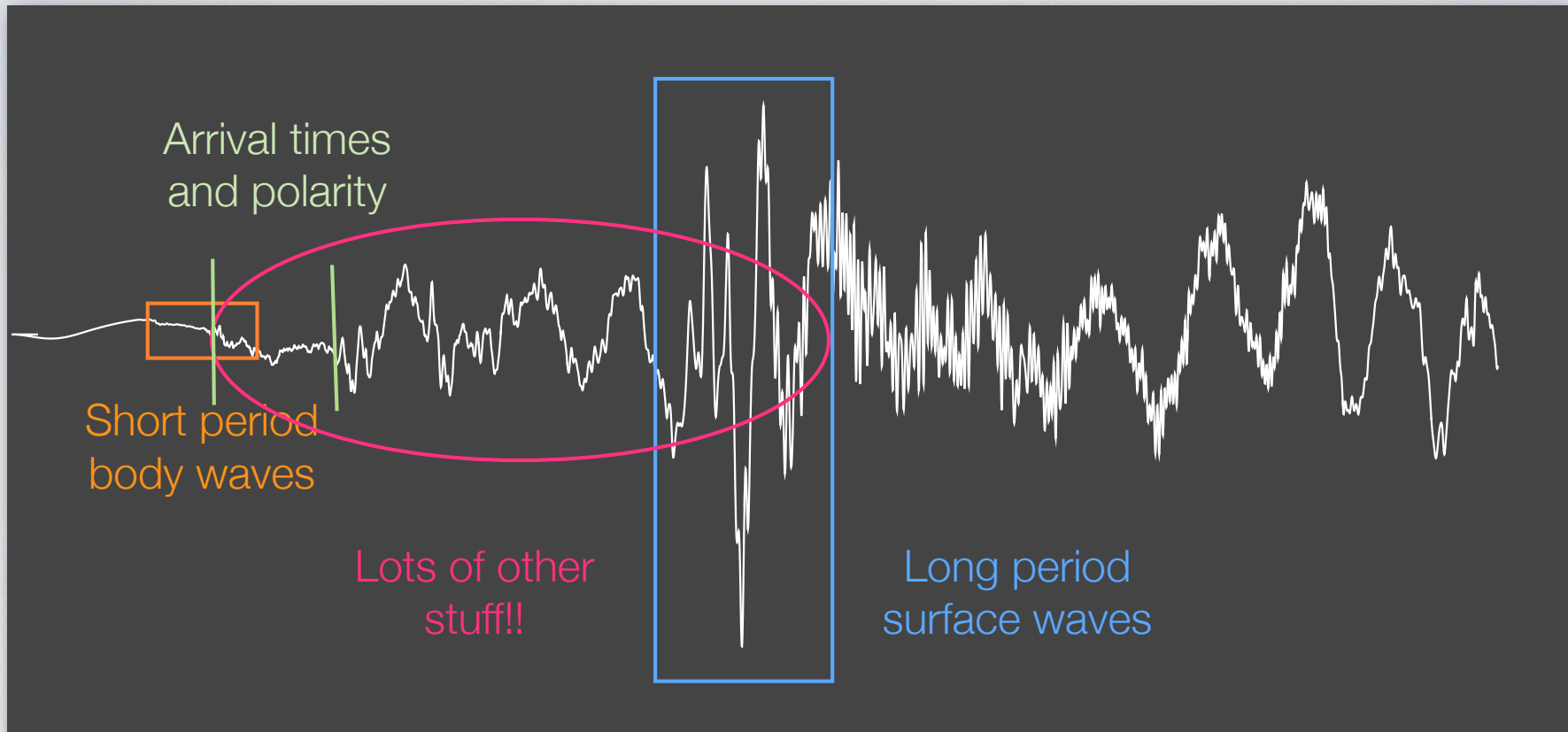
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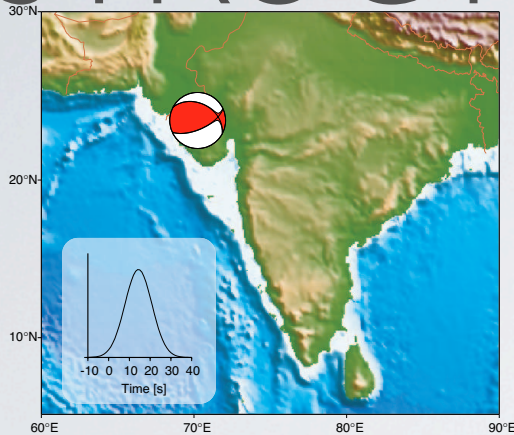
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QUESTIONS

For many purposes 1D Earth models are good approximations to the real Earth. In what period range do we need to worry about 3D effects?

How can we incorporate 3D seismograms into source inversions?

INDIA EQ - EFFECT OF 3D STRUCTURE ON WAVEFORMS

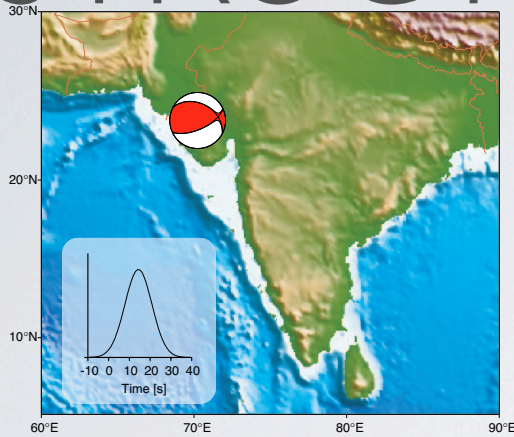


Multitaper measurements of **time shift** and **amplitude ratios**, between observed and synthetic seismograms, as a function of frequency

Source model: Bhuj, India, Mw 7.6, point source model (globalcmt.org)

Earth models: Combinations of Crust 2.0 (Bassin et al), S20RTS (Ritsema et al 1999) and PREM

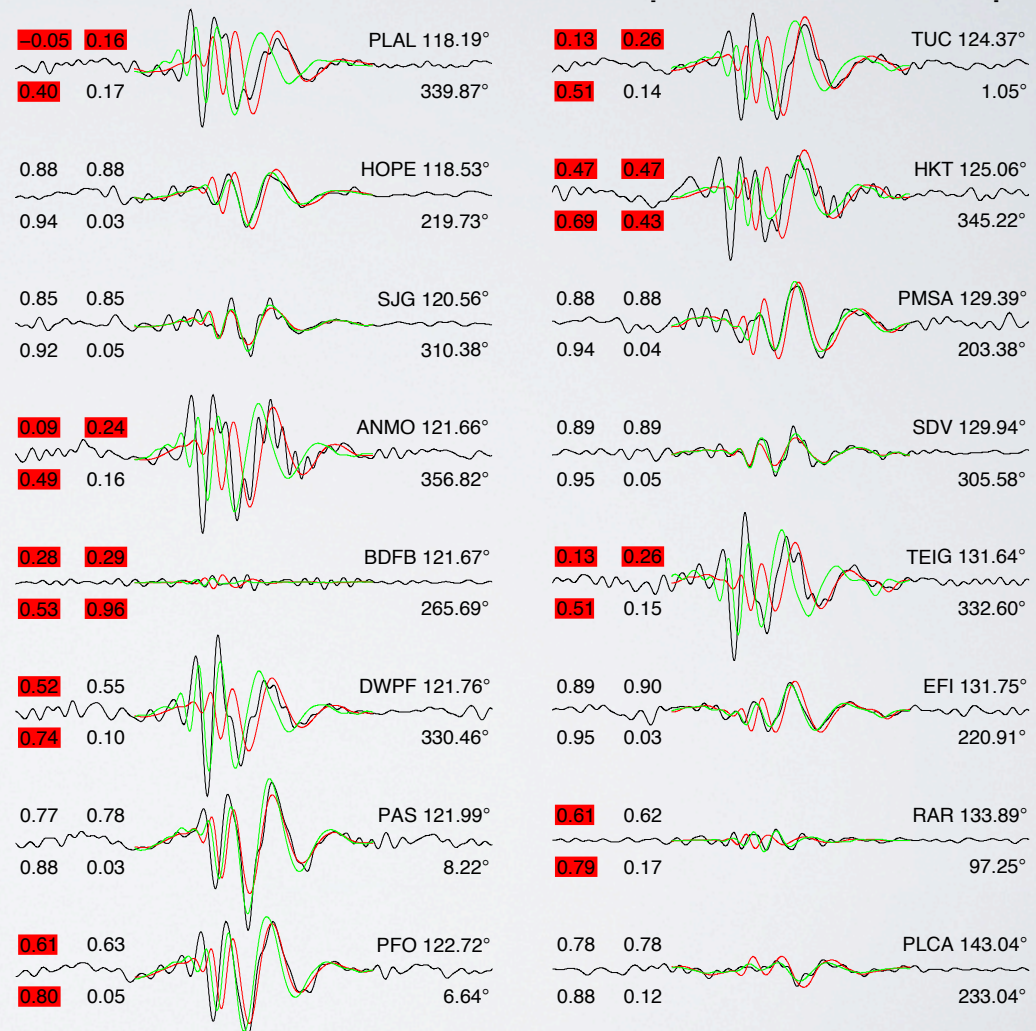
INDIA EQ - EFFECT OF 3D STRUCTURE ON WAVEFORMS



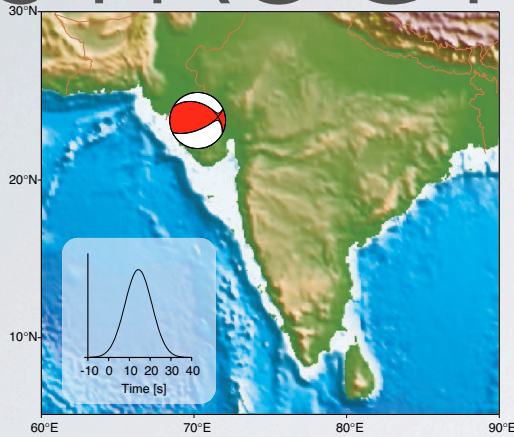
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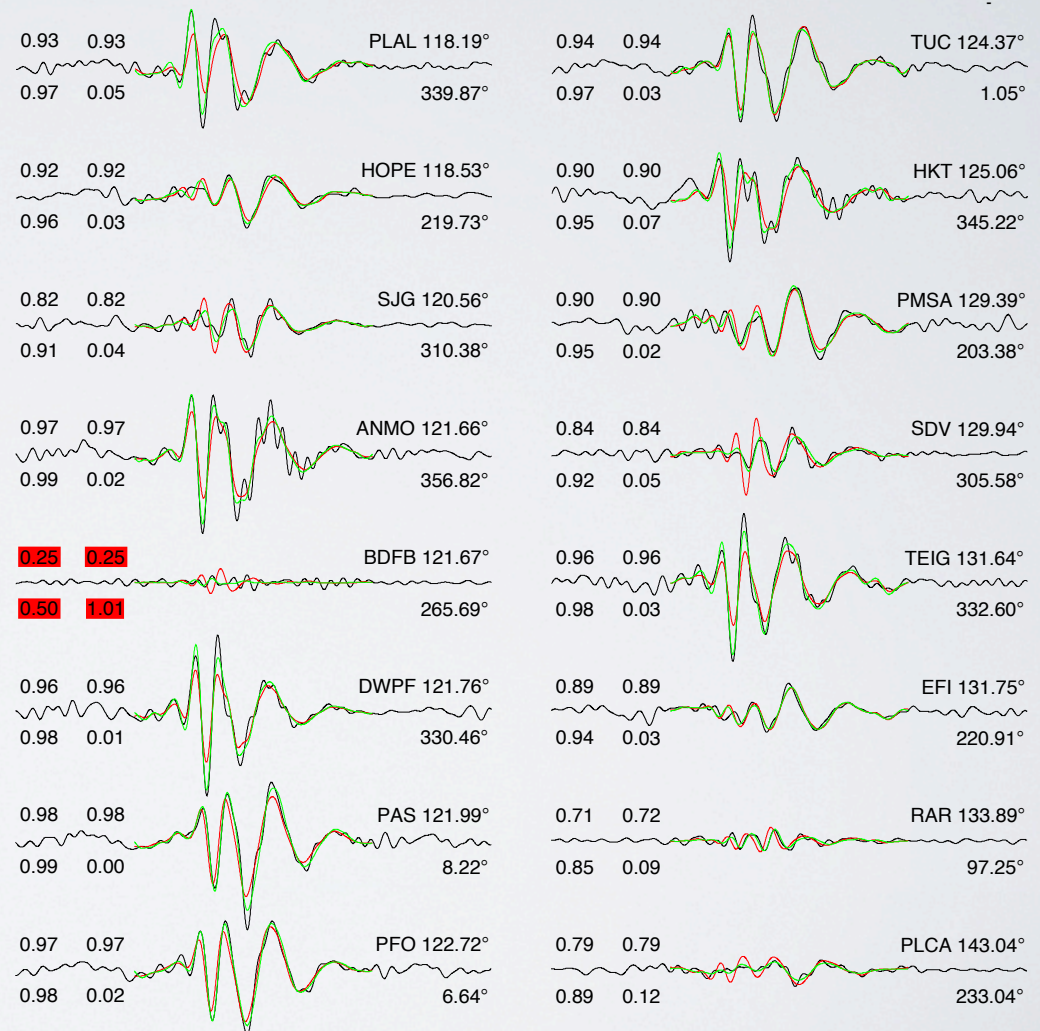
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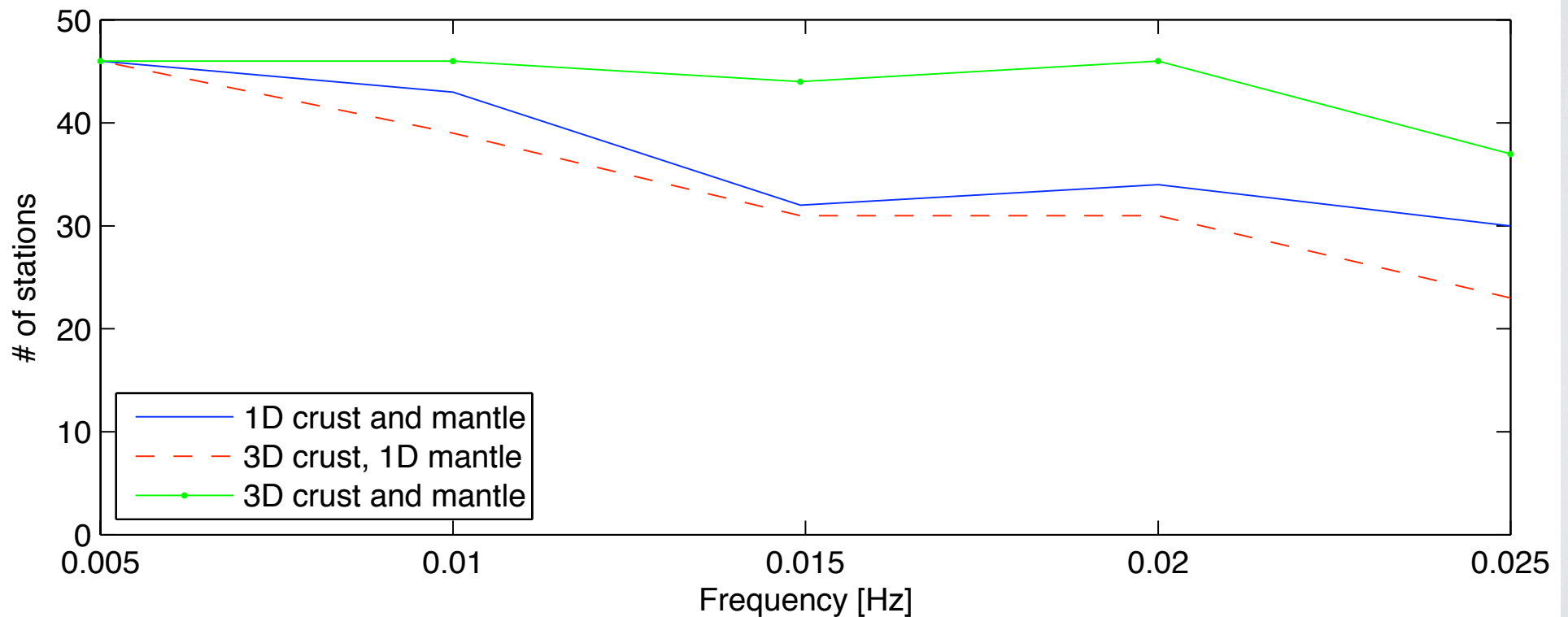
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“GOOD RECORDS”

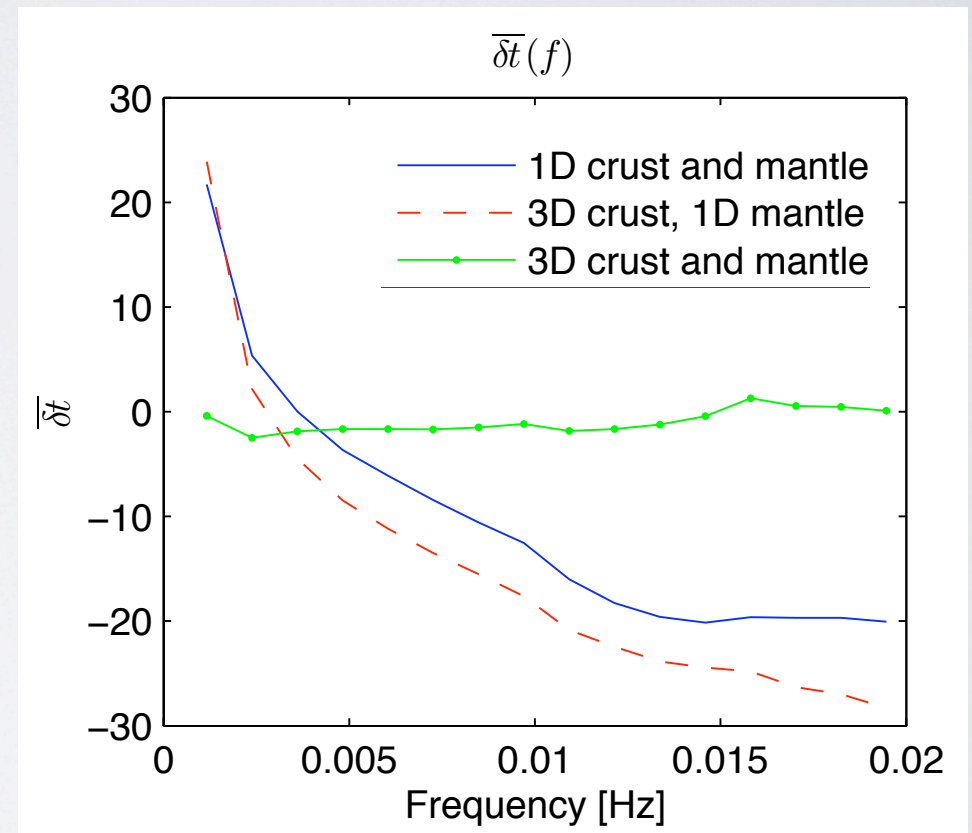
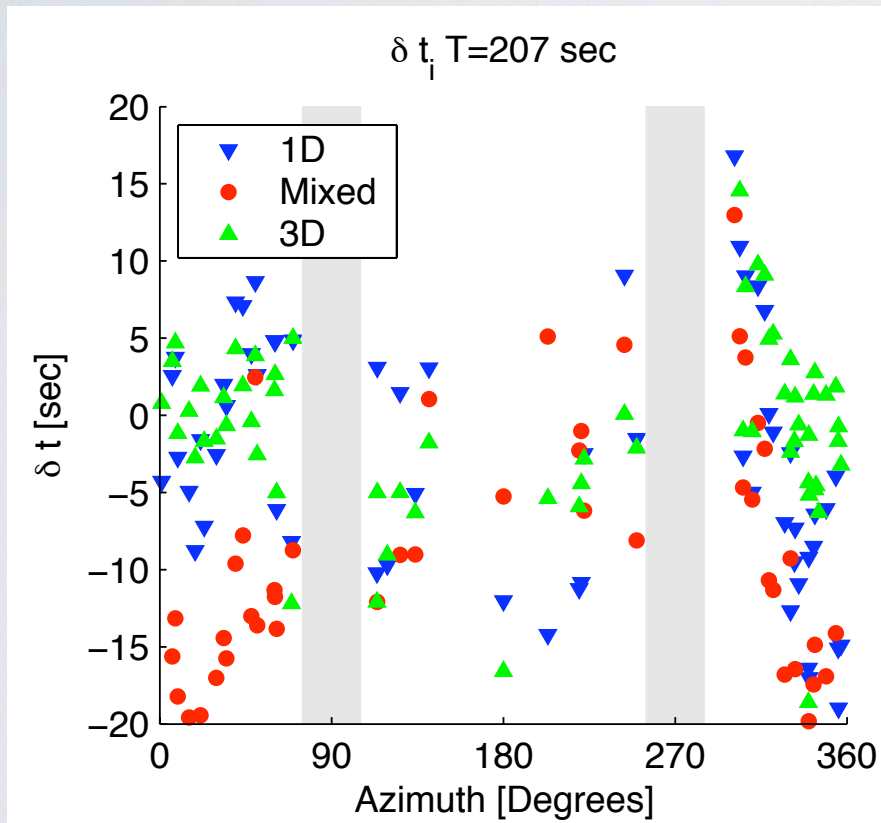
Number of stations where the original seismogram can be reconstructed “sufficiently well” by a simple transfer function



TIME SHIFTS

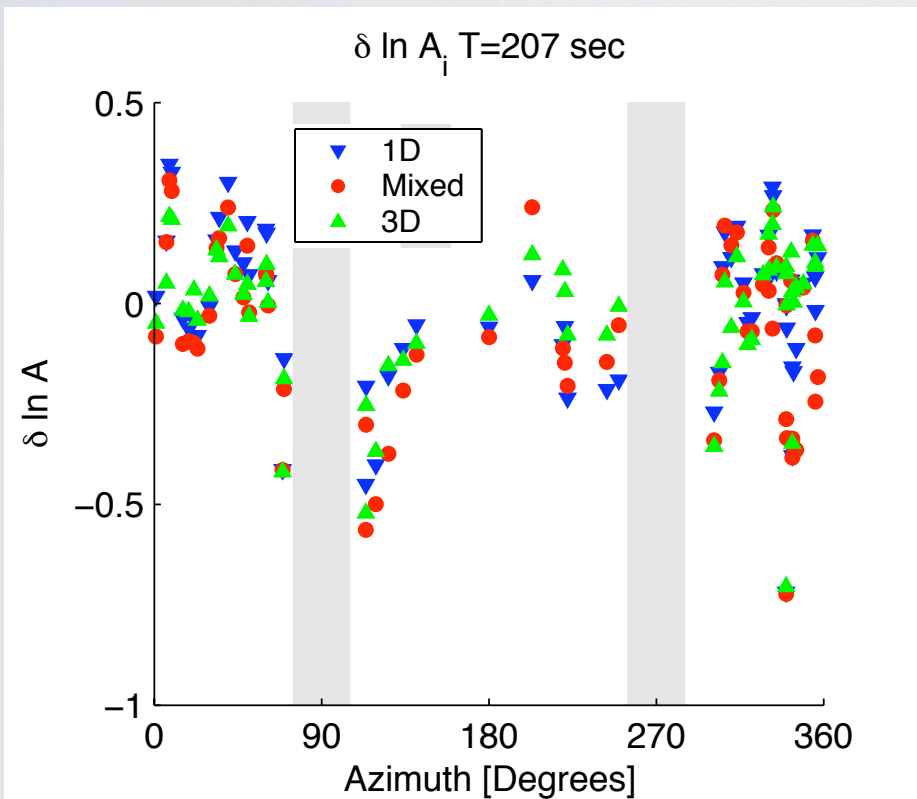
Time shift at each station
(207 seconds)

Time shifts averaged
over all stations

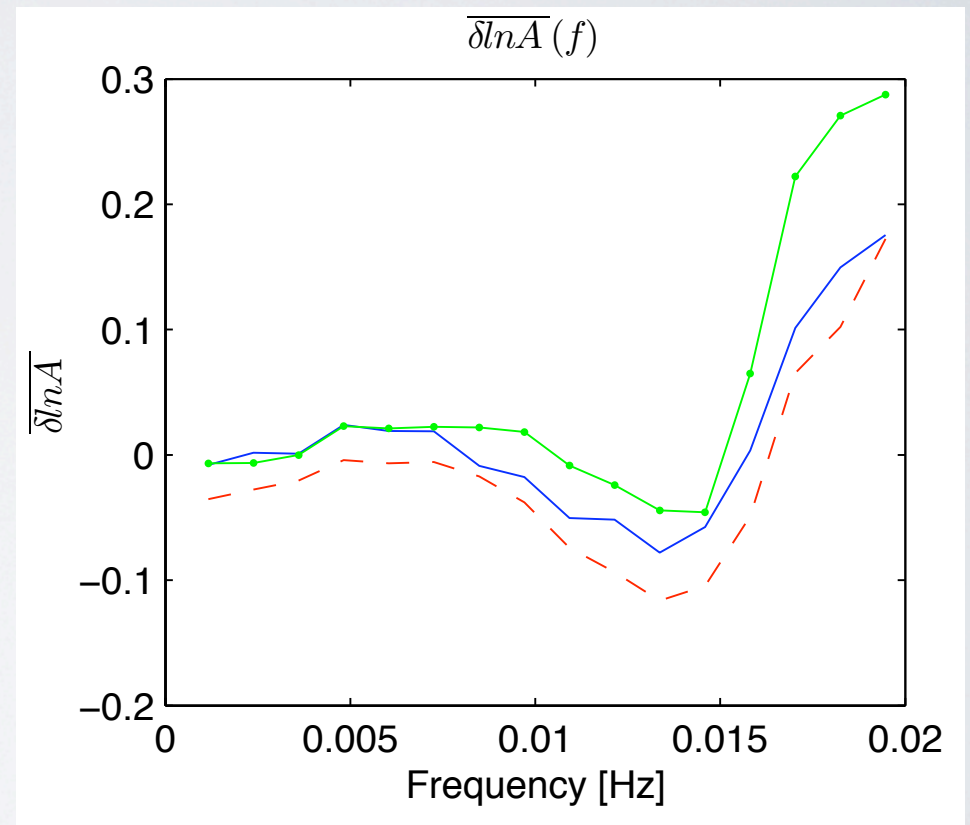


AMPLITUDES

Amplitude anomaly at each station (207 seconds)



Amplitude anomalies averaged over all stations



QUESTIONS

For many purposes 1D Earth models are good approximations to the real Earth. In what period range do we need to worry about 3D effects?

1D global models are almost always not sufficient, [but frequency dependent corrections to the phase, using current phase velocity maps, are very helpful to about 50 - 80 seconds (see also Ferreira, Woodhouse 2006)].

Current (degree 20 models) get us to 50 seconds [see also work by Bozdag, Trampert].

How can we incorporate 3D seismograms into source inversions?

TIME REVERSAL IMAGING

$$A m = d$$

$$A^T A m = A^T d$$

$$m = (A^T A)^{-1} A^T d$$

$$m \sim A^T d$$

A : Green's functions

A^T : Adjoint of A ($A_{ij} = A^T_{ji}$)

m : model parameters

d : data

- Computing A is expensive (we don't want to do it!!)
- Wave equation is self adjoint
- Use regular wave-propagation codes to propagate data and get model parameters
- Use gradient based methods to iteratively determine m
- It can be shown that to evaluate the moment density tensor, one should monitor the adjoint strain.

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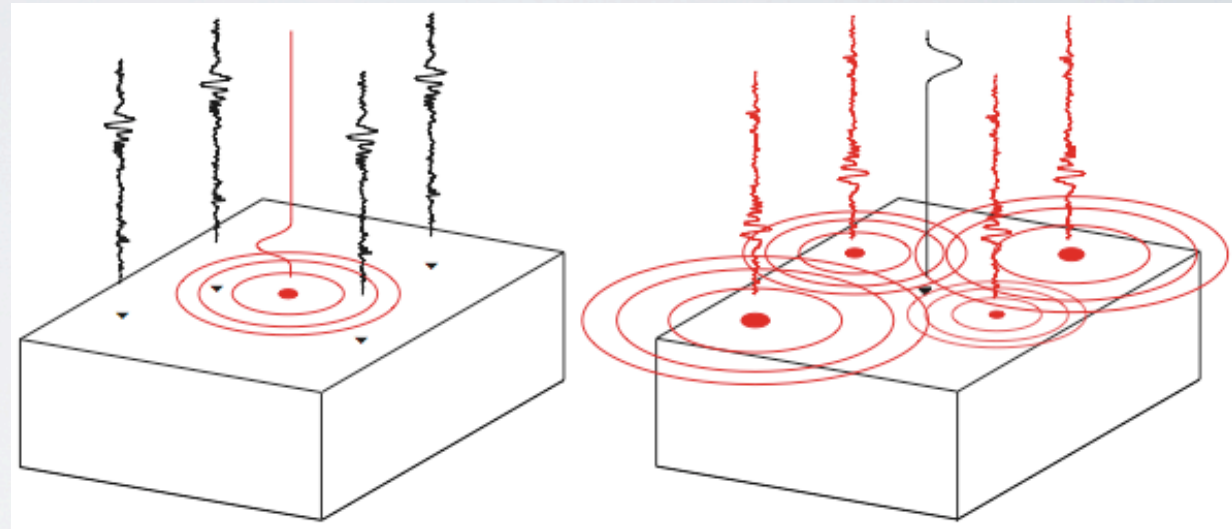
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Forward

Time reversed

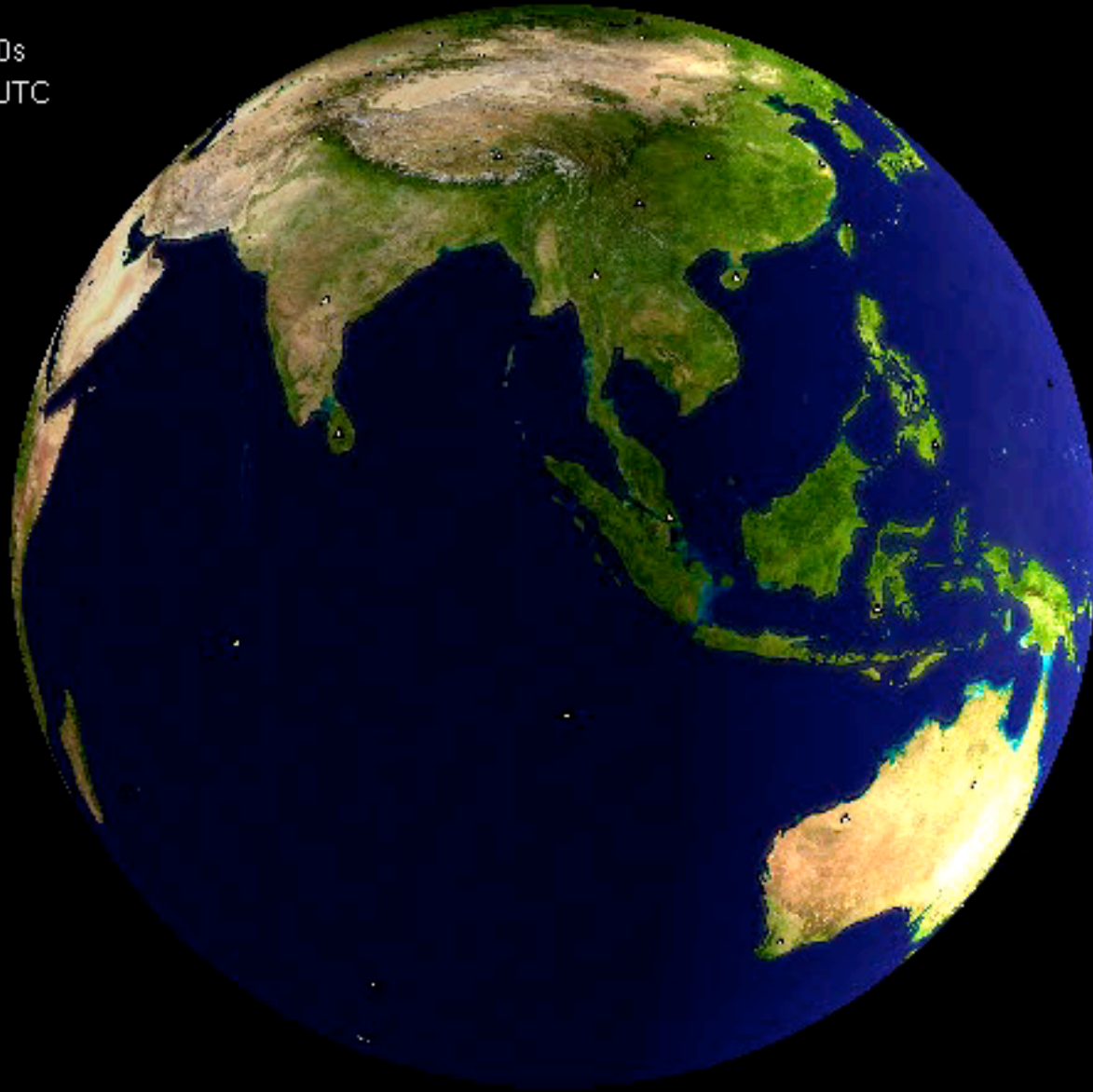


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FORWARD WAVE FIELD

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■ time= 13.0s
00:59:06 UTC



EXPERIMENTS

160 global stations

3 component records

Duration: 100 minutes (R1)

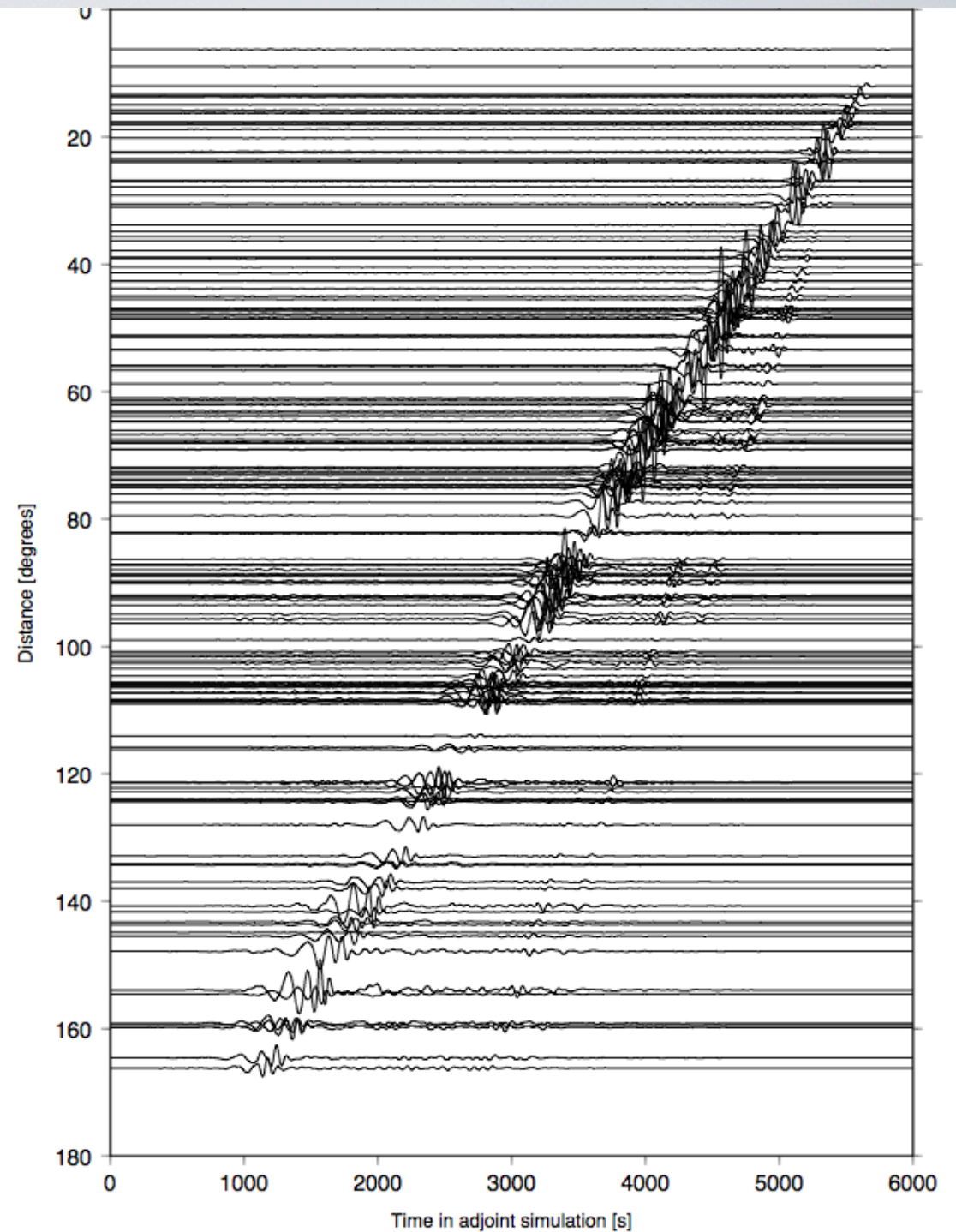
Bandpass 40-500 seconds

SEM 3D wave propagation

S20rts (3D mantle) +
Crust2.0 (3D crust)

Several focal mechanisms

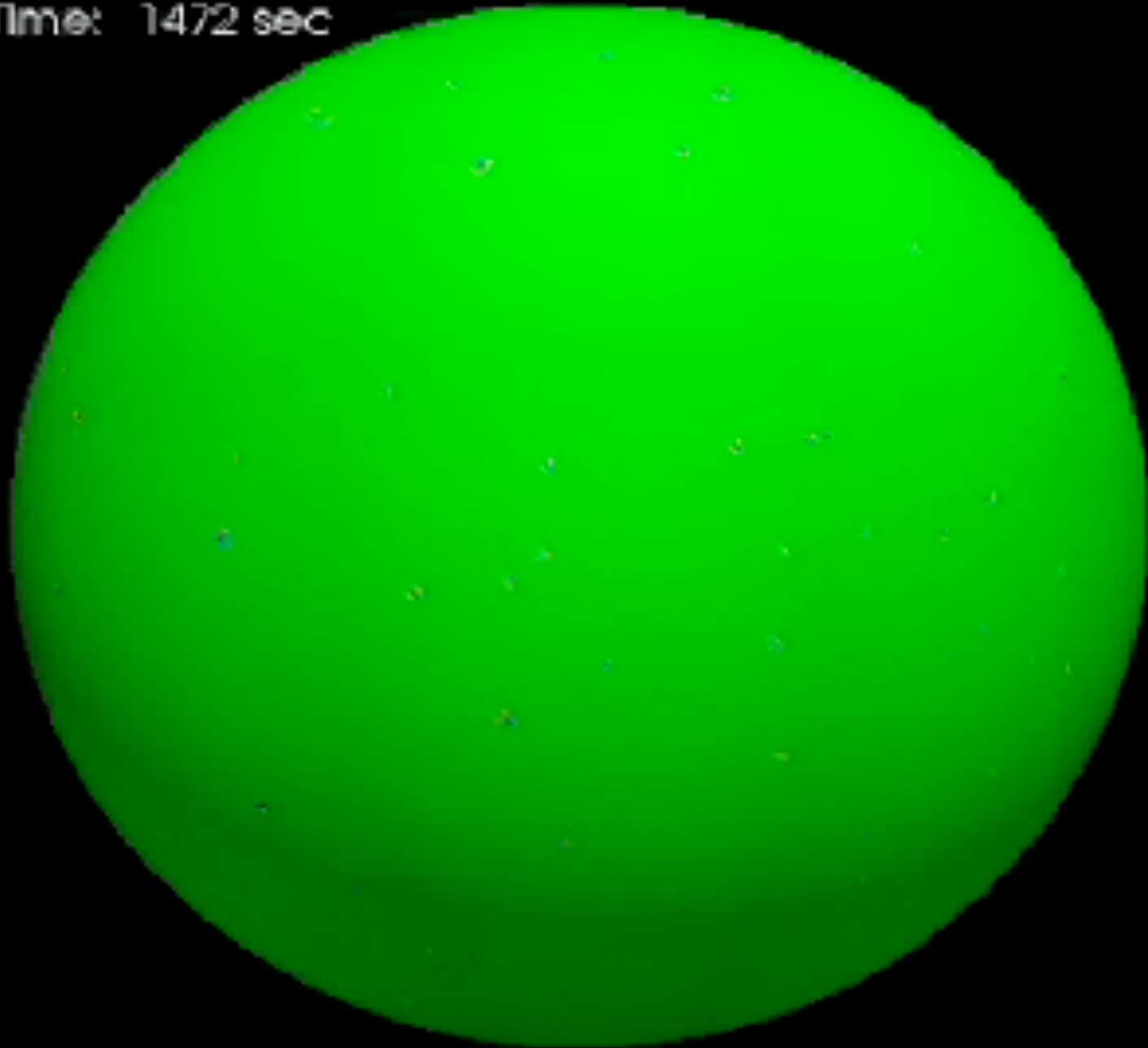
Zero starting model



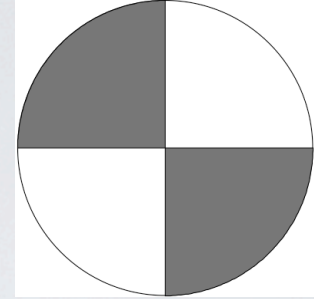
TIME REVERSED

TIME REVERSED

Time: 1472 sec



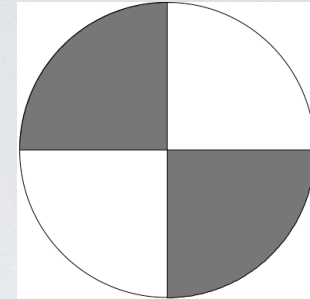
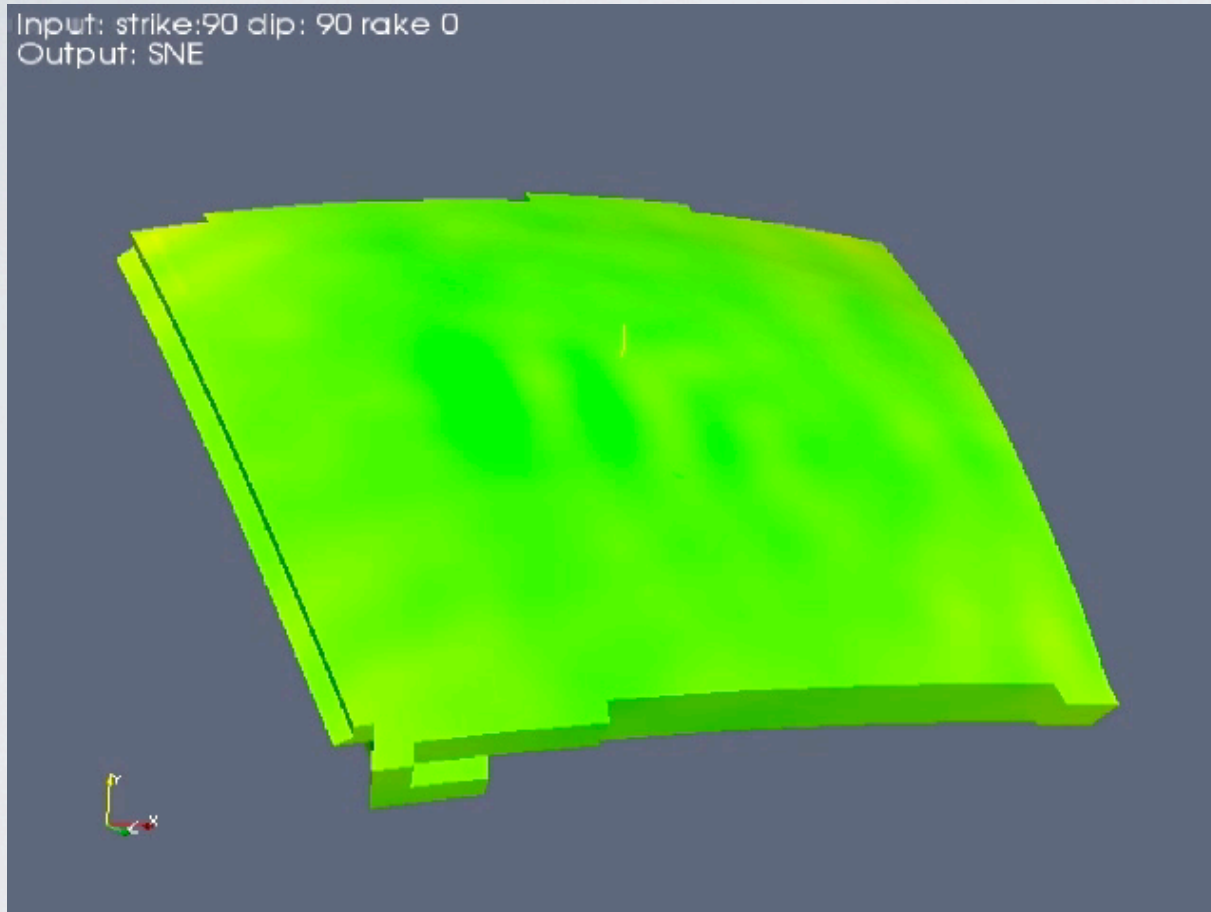
TIME REVERSAL FOR POINT SOURCE



$$\begin{vmatrix} M_{\theta\theta} & M_{\theta\varphi} & M_{\theta r} \\ M_{\theta\varphi} & M_{\varphi\varphi} & M_{\varphi r} \\ M_{\theta r} & M_{\varphi r} & M_{rr} \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

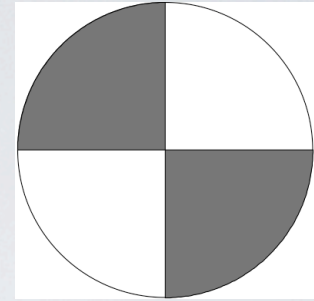
TIME REVERSAL FOR POINT SOURCE



$M_{\theta\theta}$	$M_{\theta\varphi}$	$M_{\theta r}$
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$M_{\theta r}$	$M_{\varphi r}$	M_{rr}

0	1	0
1	0	0
0	0	0

TIME REVERSAL FOR POINT SOURCE

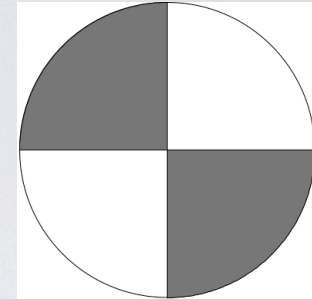
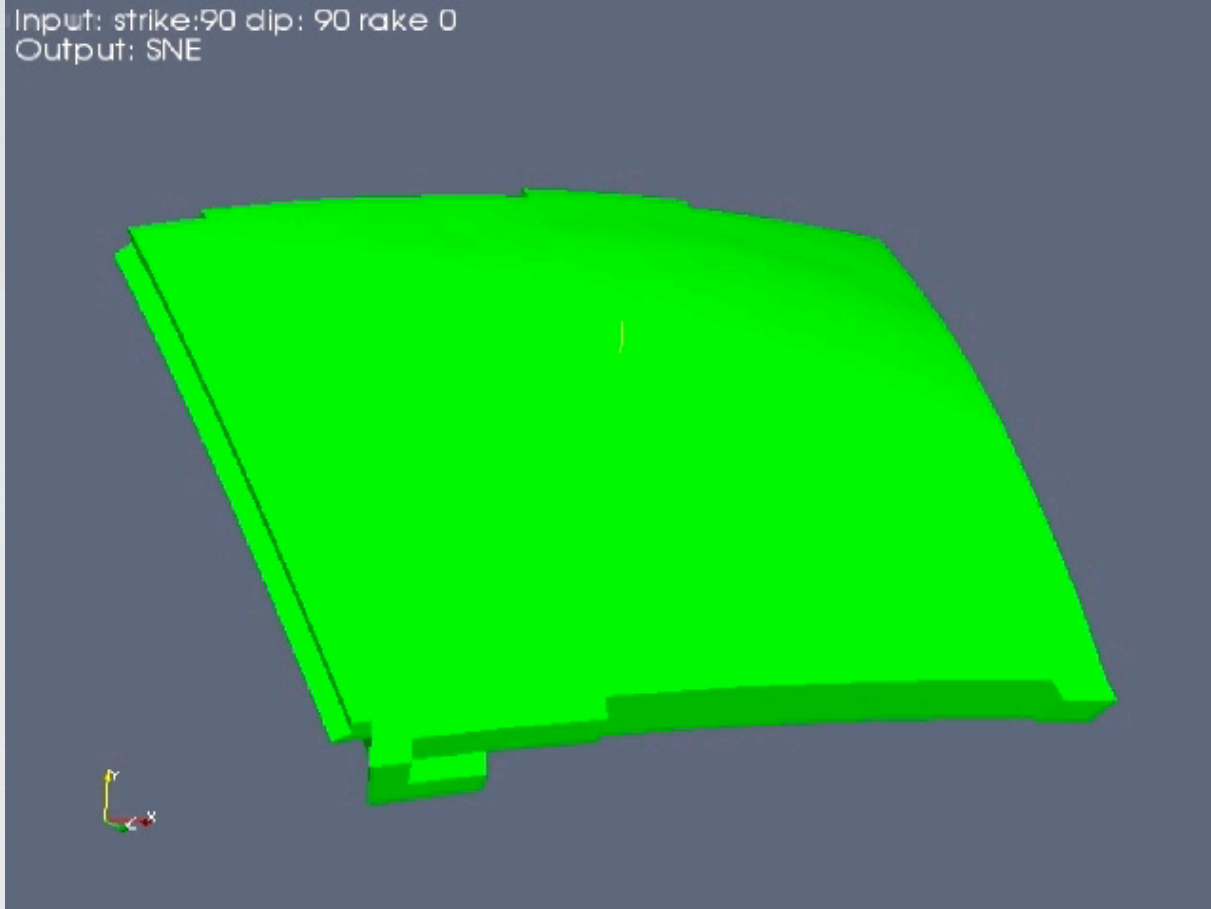


$M_{\theta\theta}$	$M_{\theta\varphi}$	$M_{\theta r}$
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$M_{\theta r}$	$M_{\varphi r}$	M_{rr}

0	1	0
1	0	0
0	0	0

TIME REVERSAL FOR POINT SOURCE

Input: strike:90 dip: 90 rake 0
Output: SNE

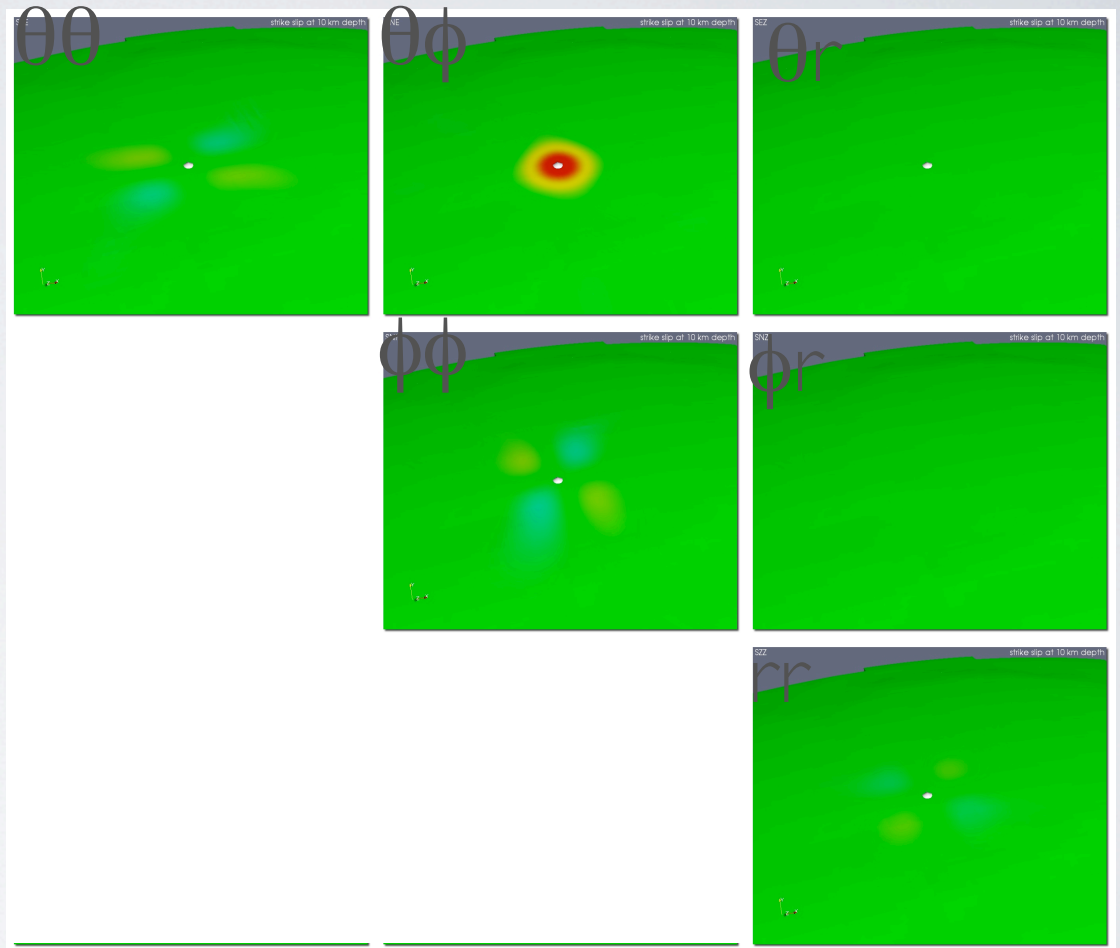


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RESULTS, SHALLOW POINT SOURCE

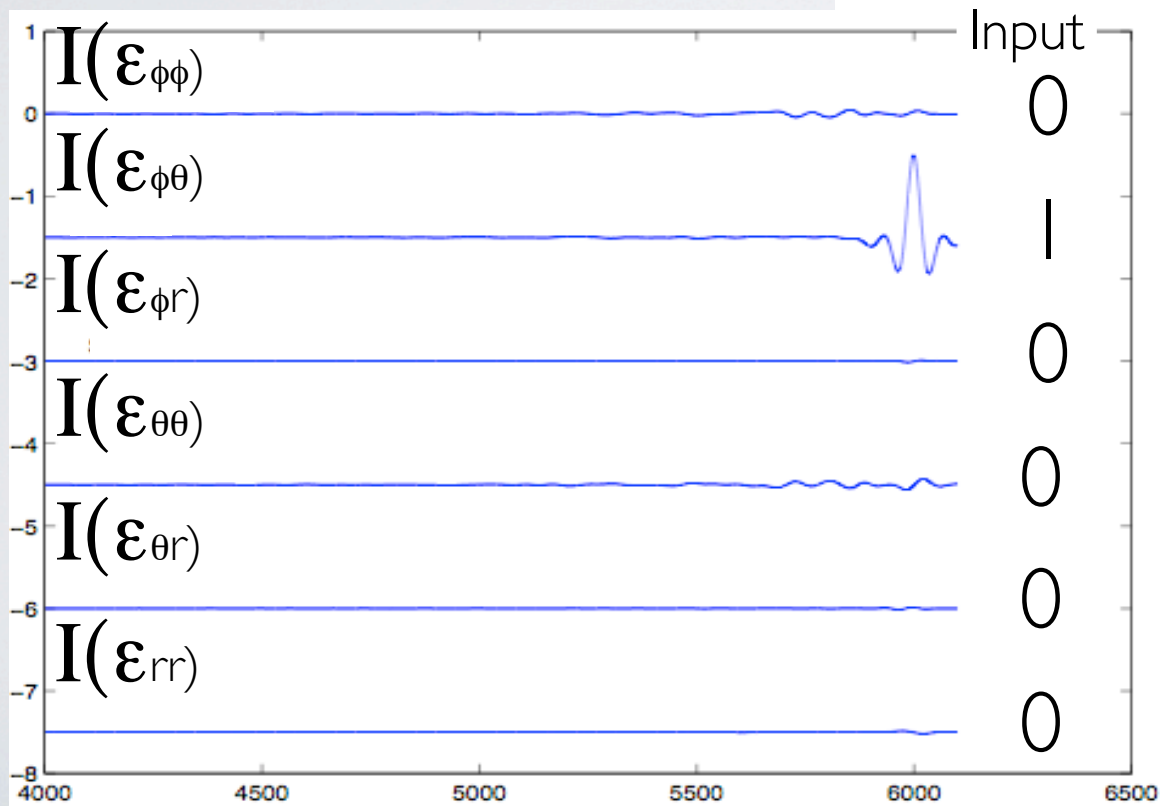
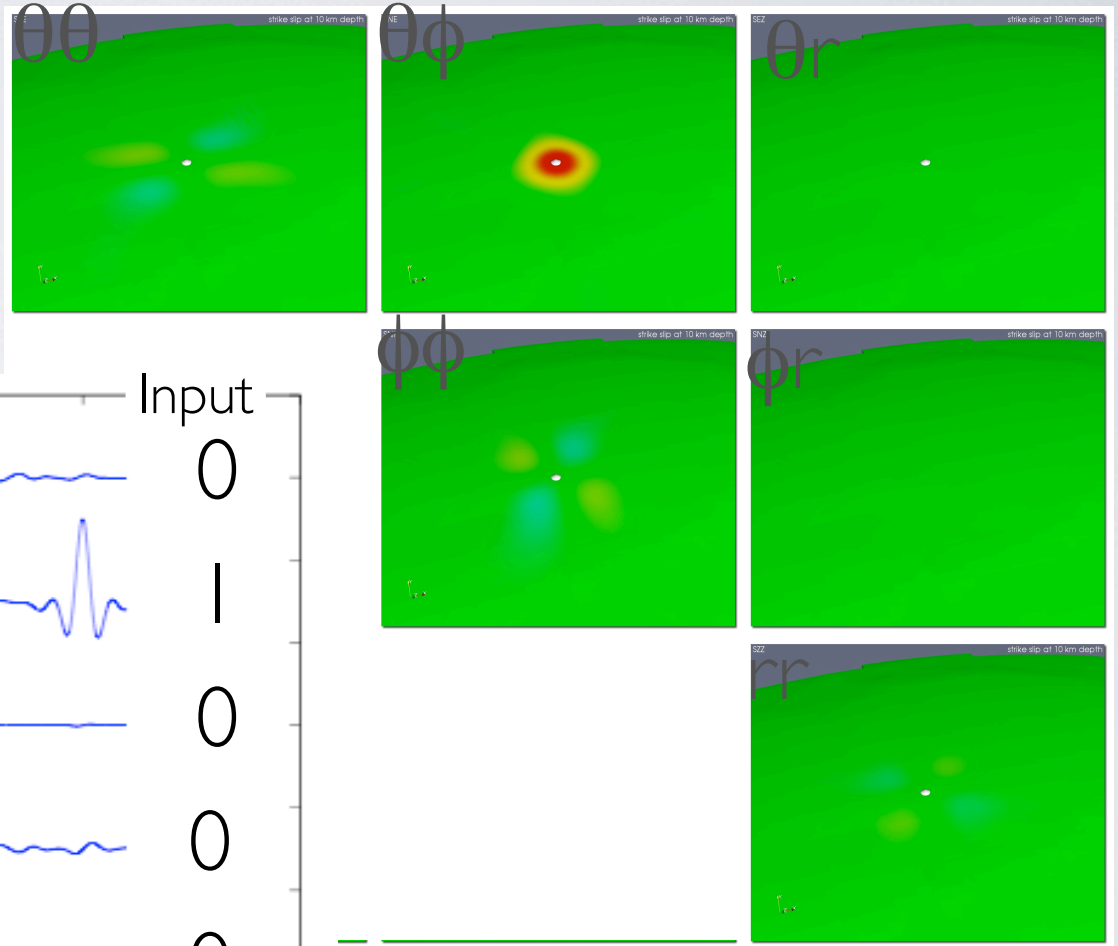
Localization in space



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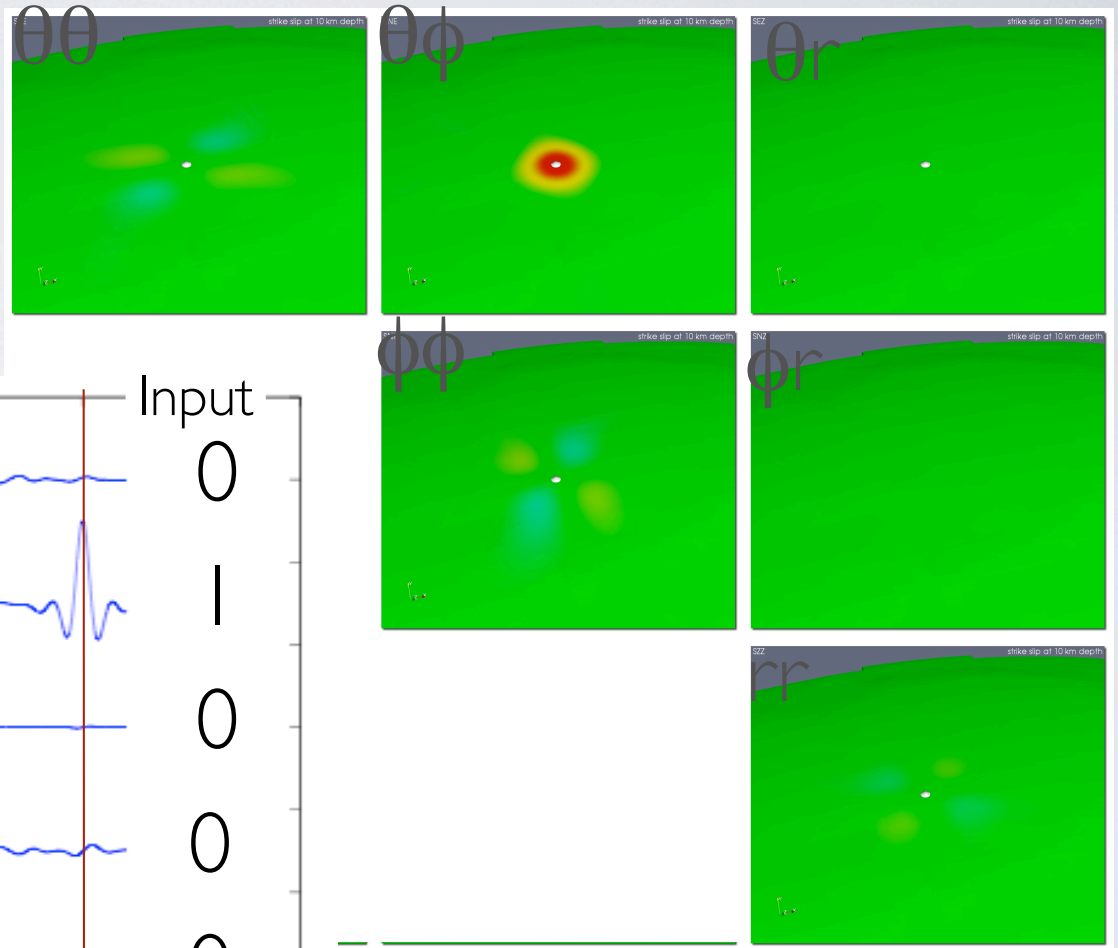
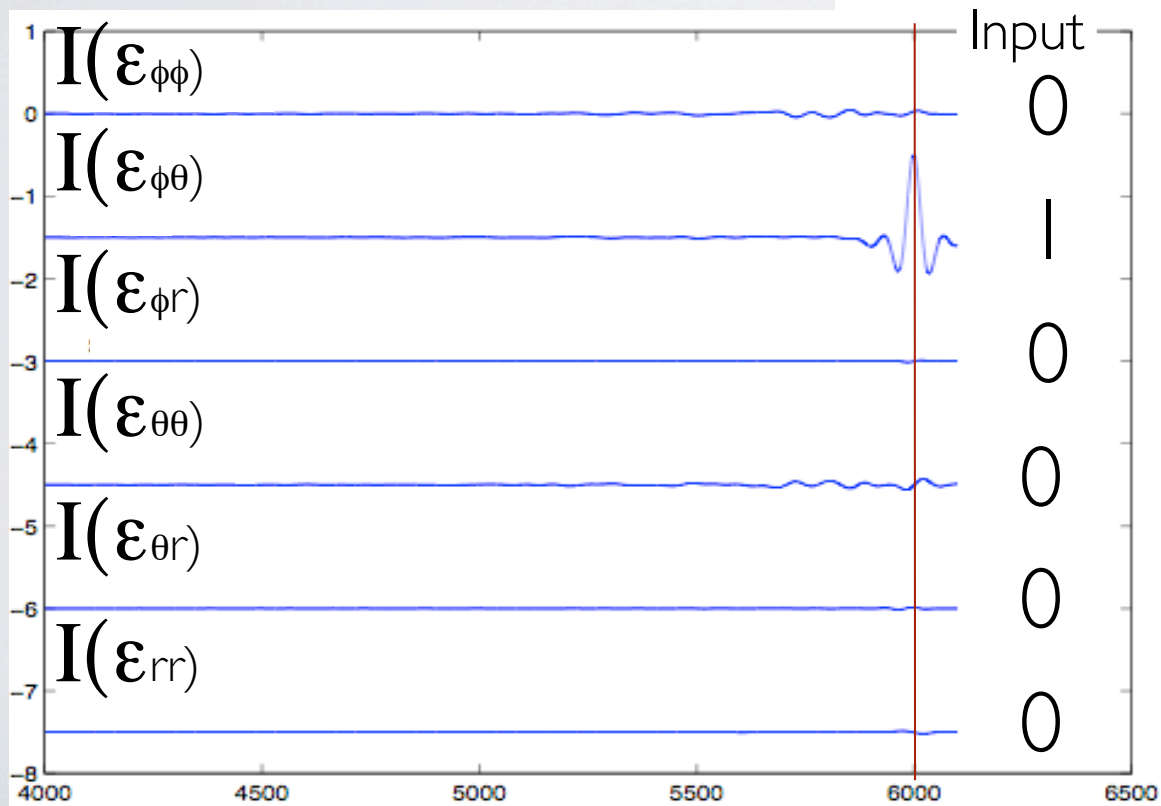
Localization in time



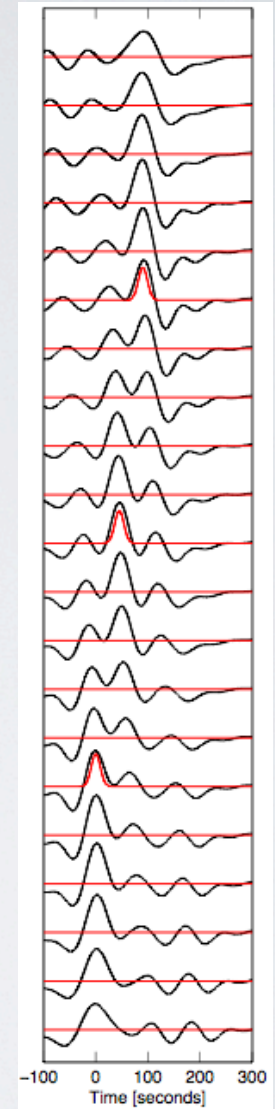
RESULTS, SHALLOW POINT SOURCE

Localization in space

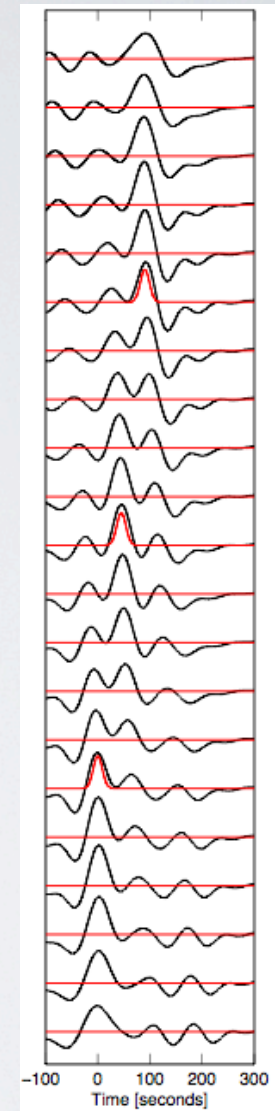
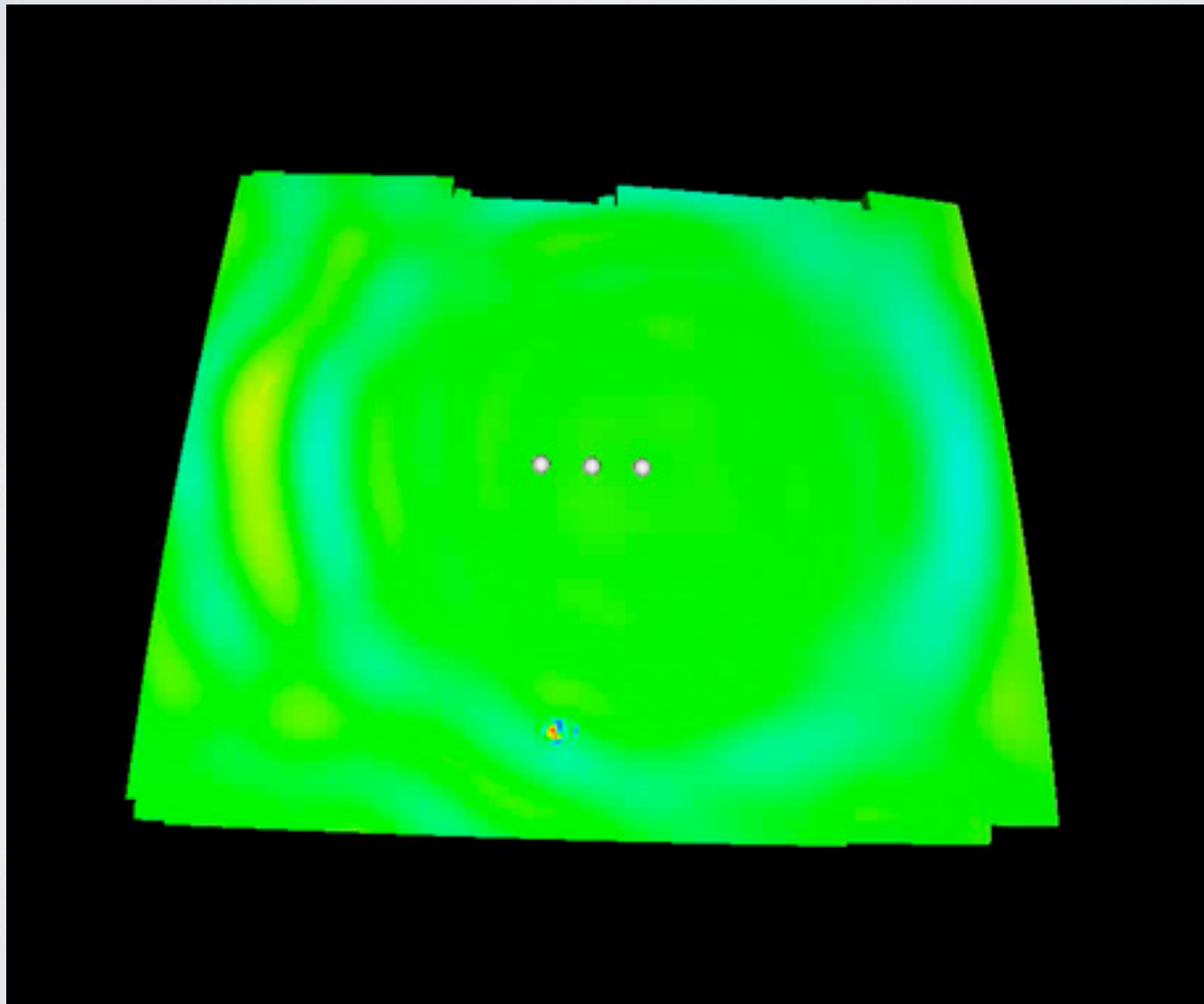
Localization in time



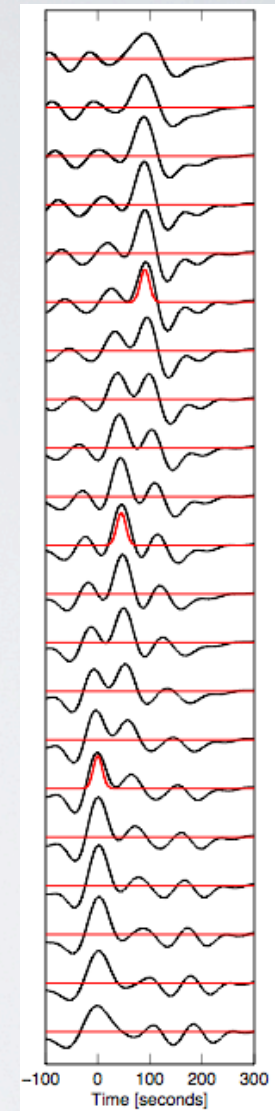
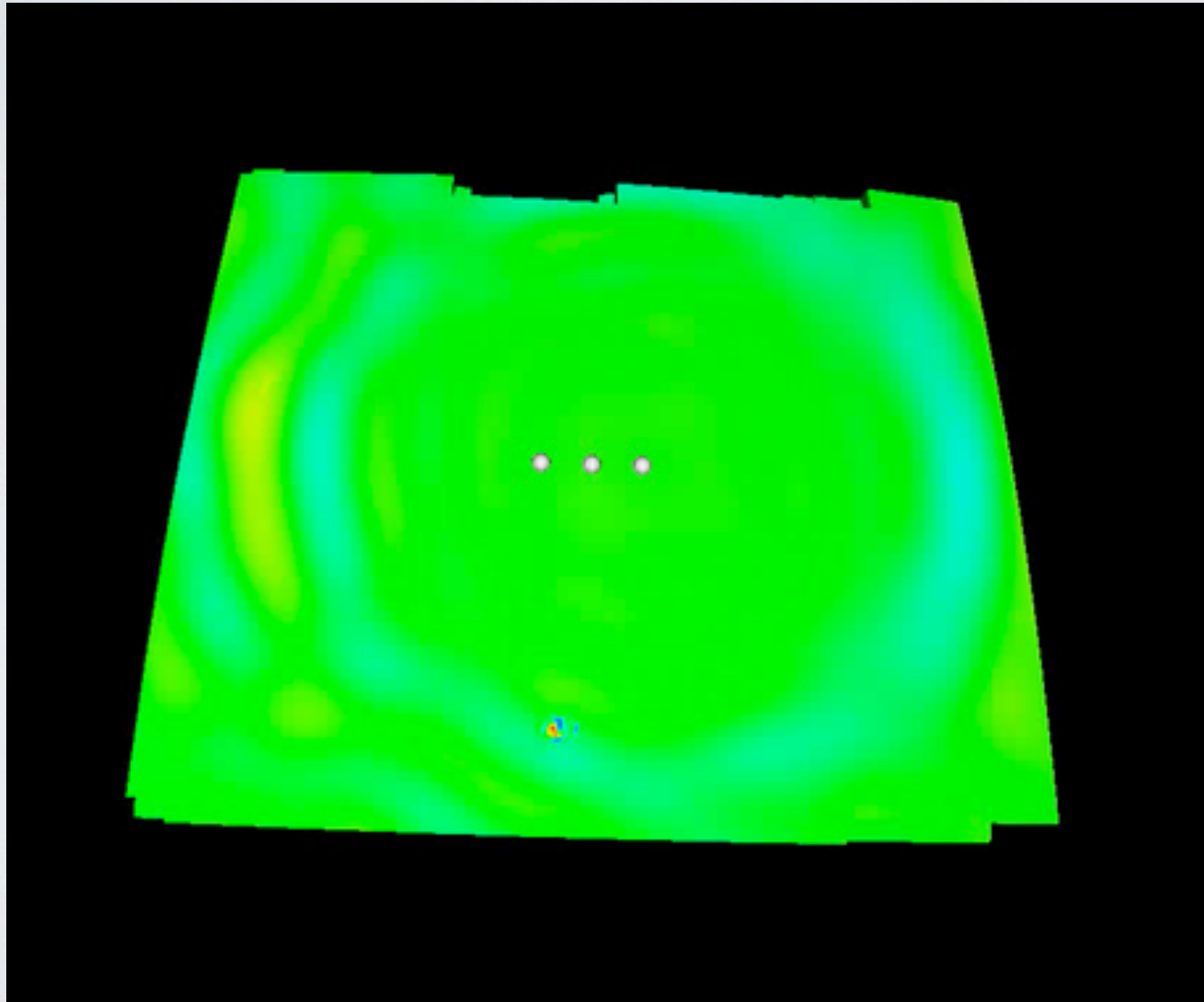
3 POINT SOURCES



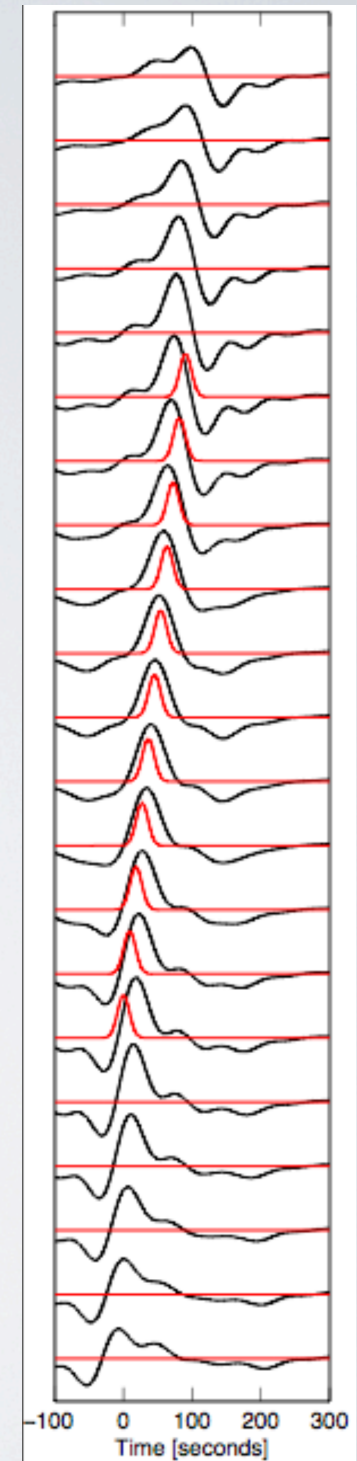
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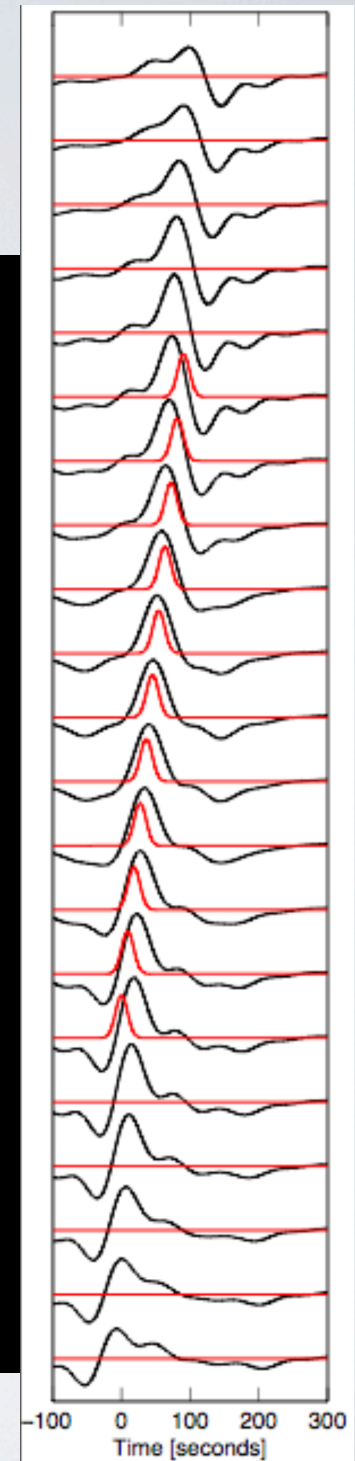
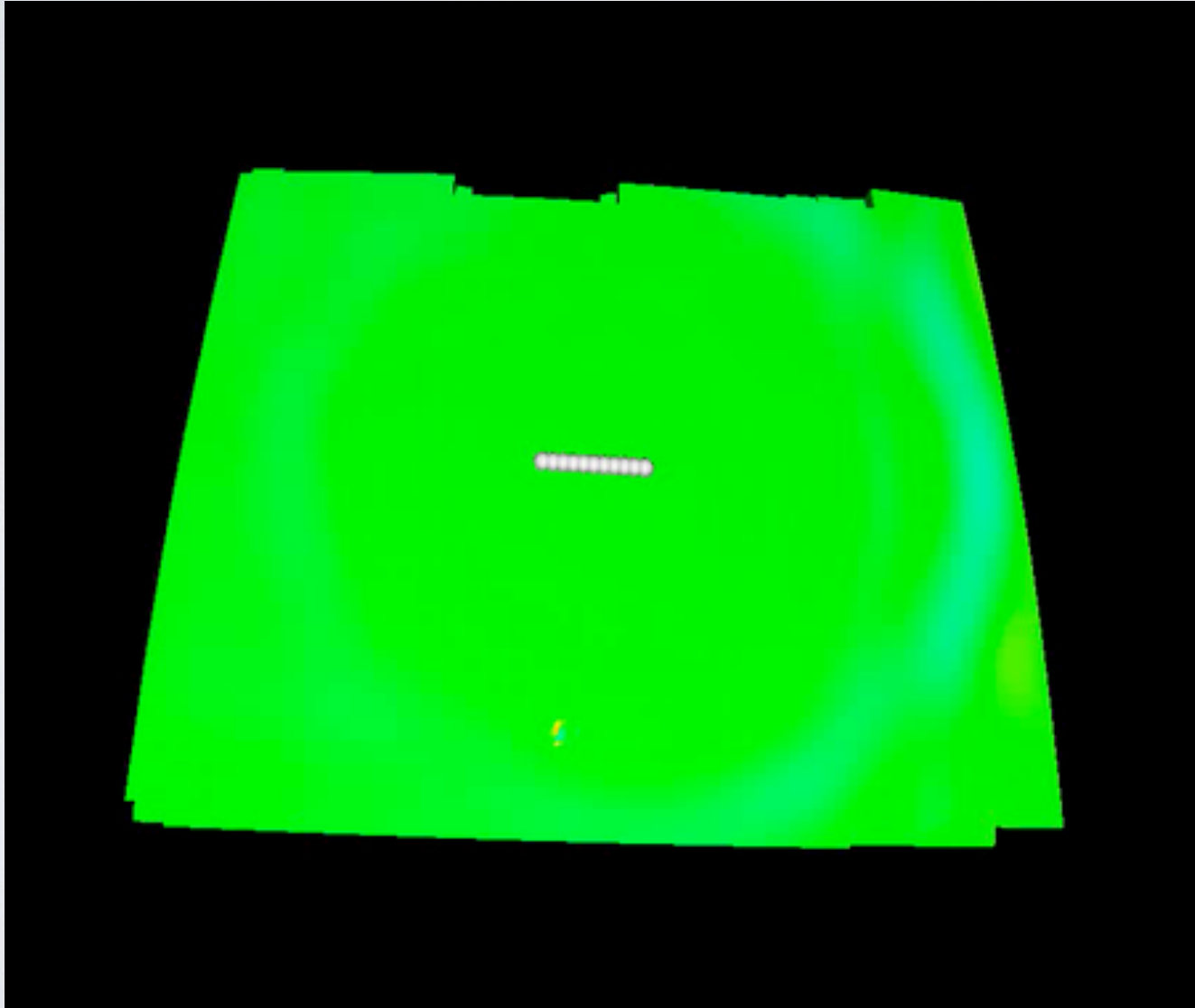
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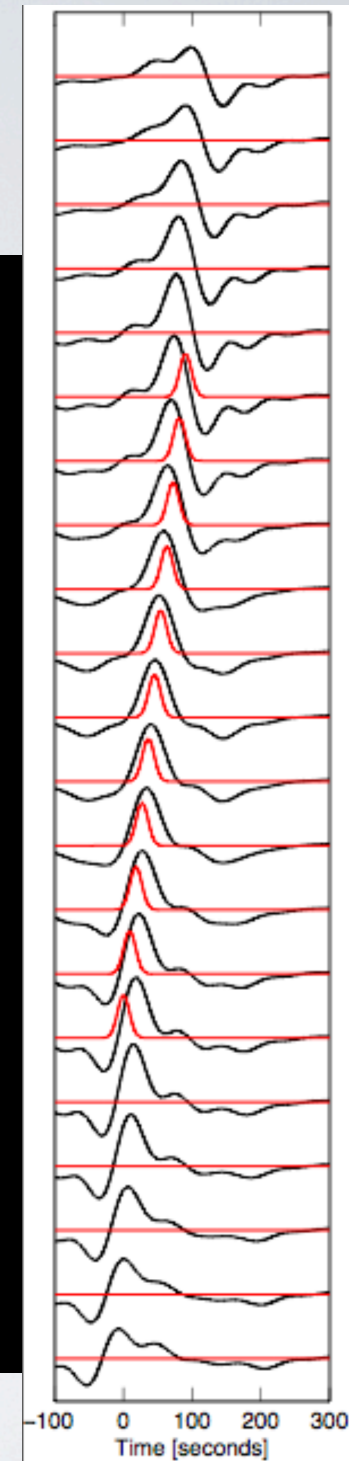
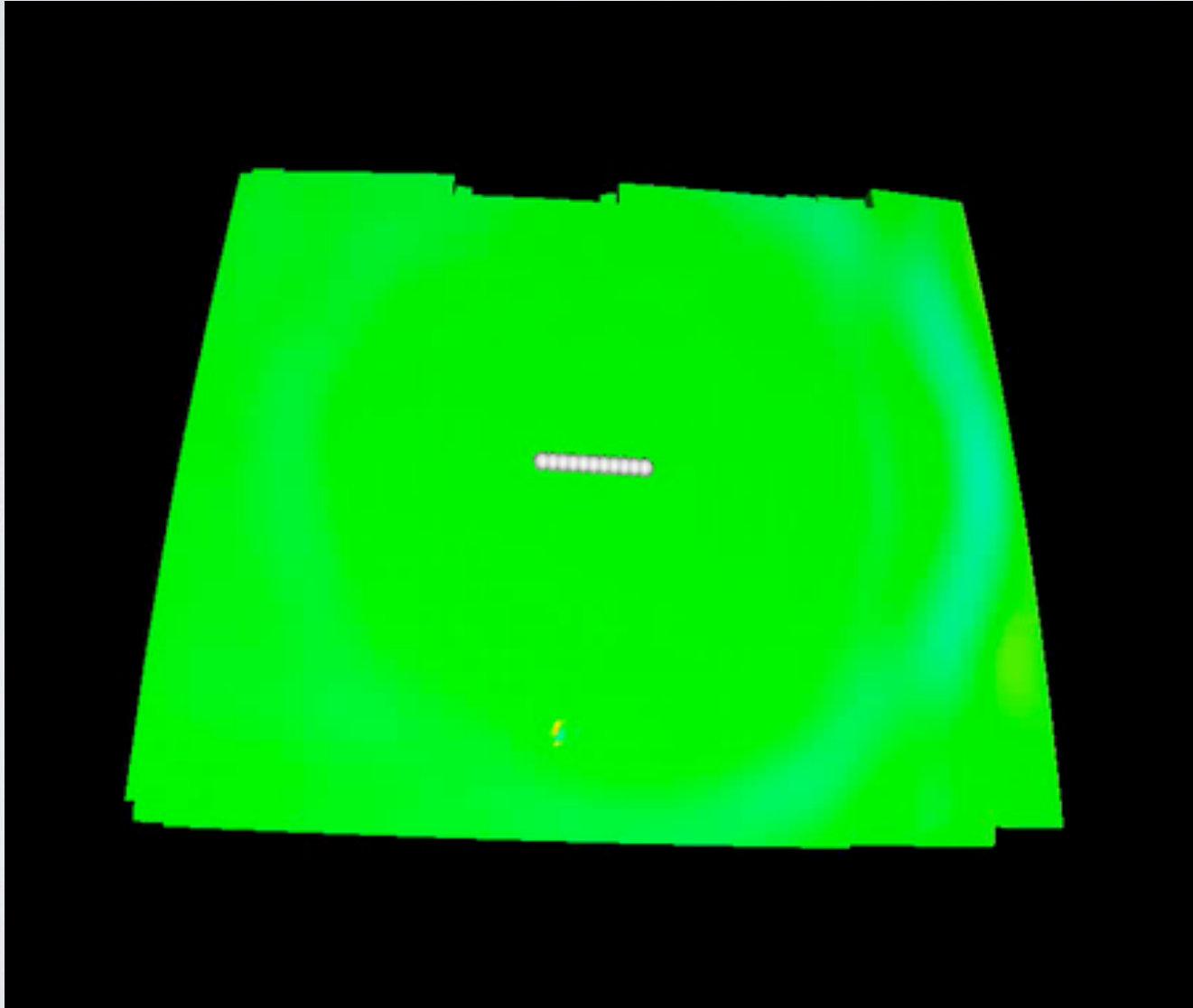
“FINITE” SOURCE



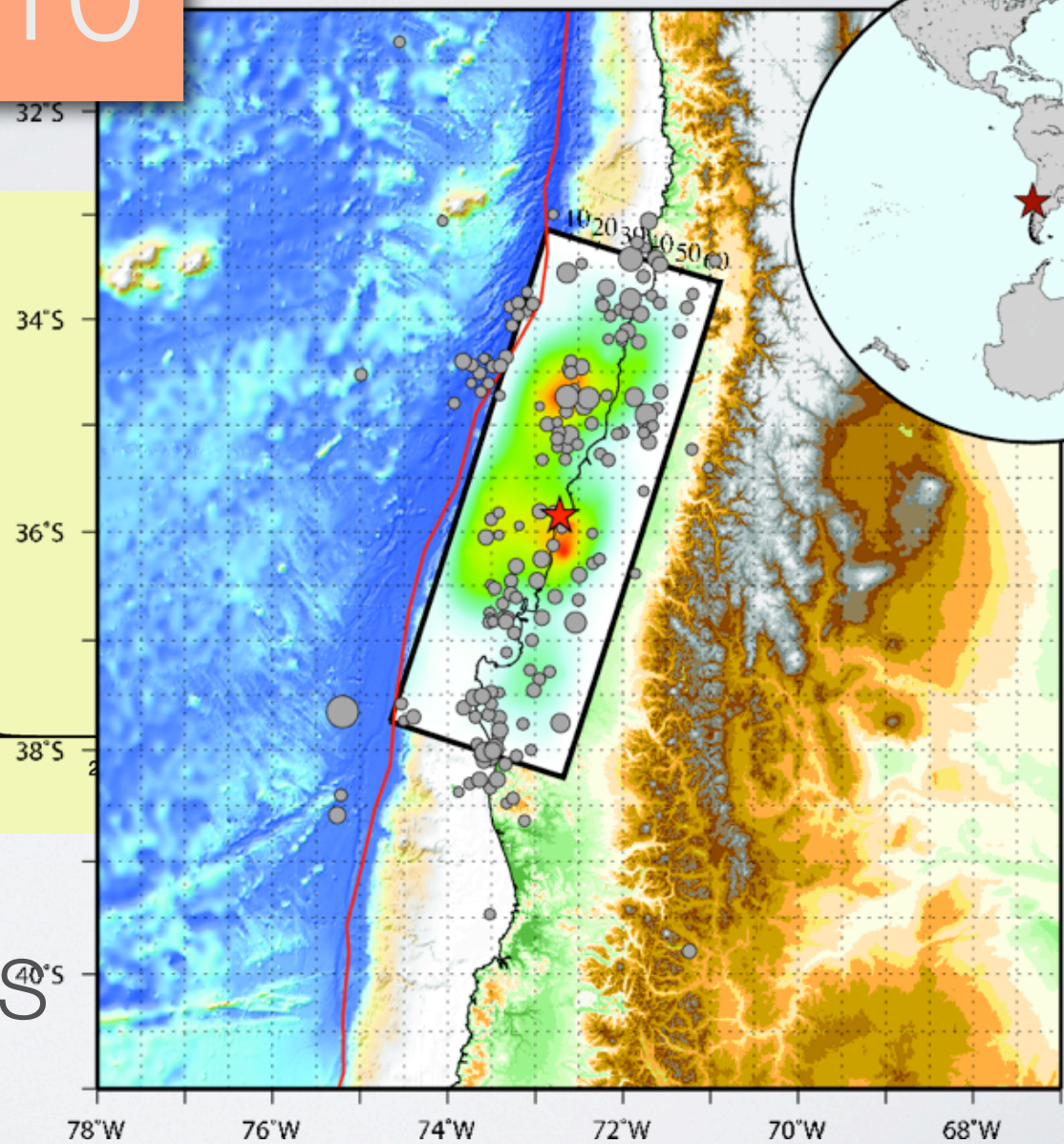
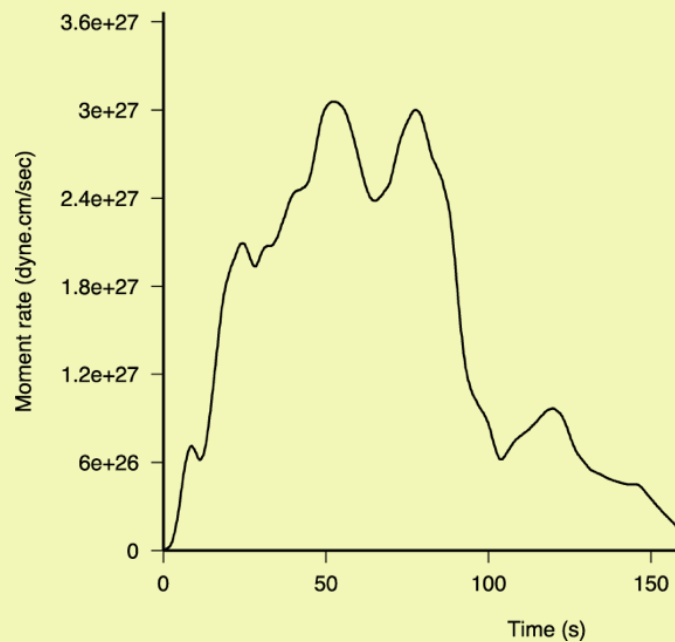
“FINITE” SOURCE



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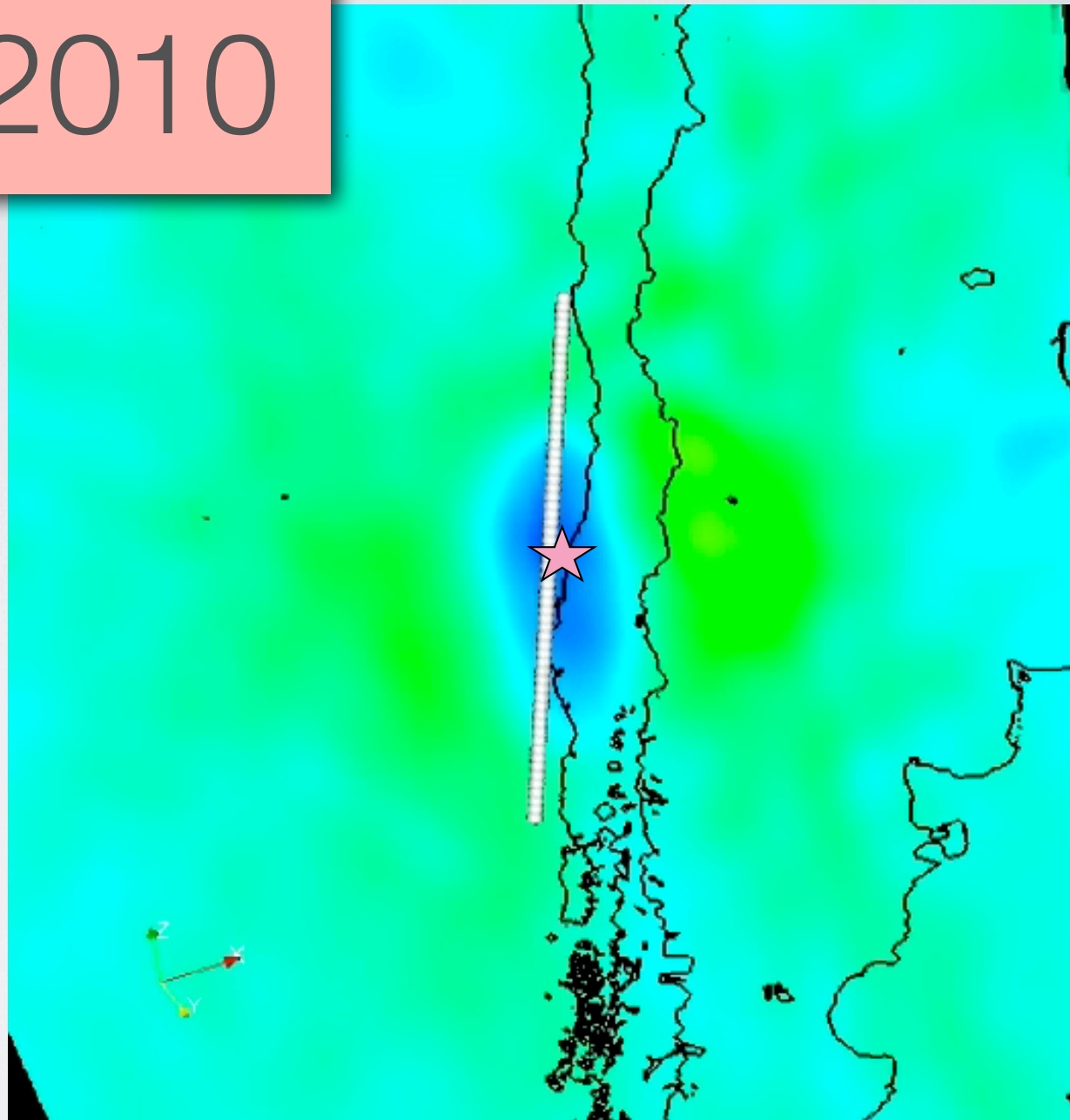


CHILE 2010



Gavin Hayes, USGS

Chile 2010



CONCLUSIONS

For many purposes 1D Earth models are good approximations to the real Earth. In what period range do we need to worry about 3D effects?

How can we incorporate 3D seismograms into source inversions?

Adjoint methods may be a “cheap” way to incorporate 3D synthetic seismograms into finite source inversion, allowing us to use a “new” dataset to constrain source models (see also Kim, Liu Tromp 2011)

Time reversal imaging can be used as a method to visualize the source process without any parametrization. This can aid in interpretation of finite source models derived by traditional methods, but care should be taken in interpreting the images by themselves (see also Larmat et al 2006).

COMMENTS

Only by using many different datasets, having different trade-offs between source parameters, can we obtain reliable finite source models. For M_w 6-7.5 earthquakes point source models can be important integral constraints on finite source models.

Current (degree 20) models may be sufficiently good for source inversions using surface waves of 50 seconds and longer.

Full 3D synthetic seismograms are important for:

- point source inversions of small events
- finite fault inversions of large events, using more data than just direct P and S waves.

