Modeling seismic noise by normal mode summation

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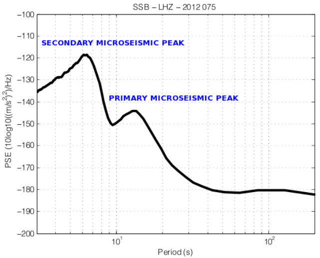
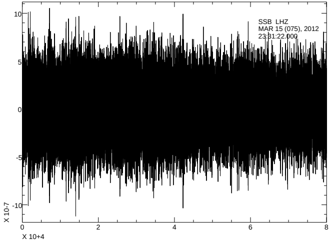
Secondary microseismic noise theory

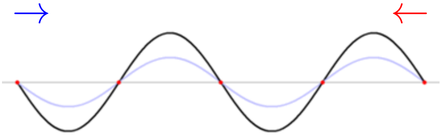




GOAL

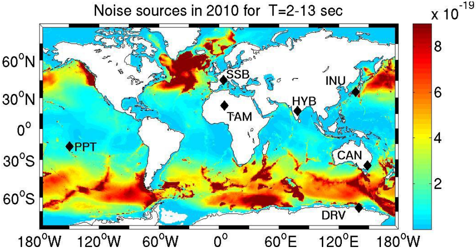
modeling the secondary microseisms peak

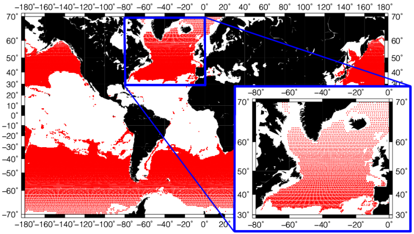
Theory by Longuet-Higgins (1950):
"two progressive waves of the same wave-length travelling in the opposite direction generate a second-order pressure variation which is not attenuated with depth"



— **the mean pressure** varies with twice the frequency of the ocean waves.

Discretization of seismic noise sources on a grid





OCEAN WAVE MODEL :
(WAVEWATCH III^R, Ardhuin et al., 2011)

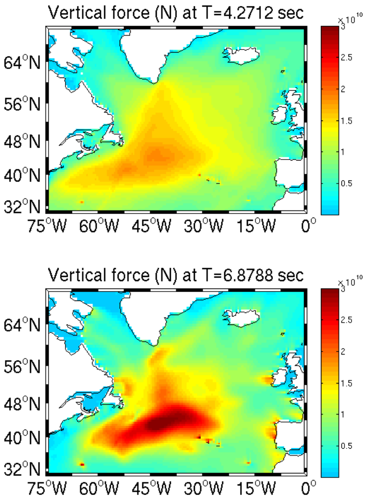
- grid step of 0.5 degree;
- 6-hourly wind analyses from the European Centre for Medium-Range Weather Forecasts;
- coastal reflections of ocean waves.

SOURCES DISCRETIZATION:

- spherical distribution of 50610 point sources;
- grid step of 50 *km*;
- **vertical force with random phase**

Vertical force associated with microseisms

Ocean wave model ⇒ **pressure just below the surface of the ocean** produced by the ocean wave-wave interaction ⇒ **vertical force** applied just below the ocean surface

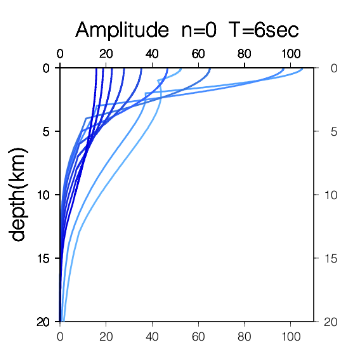


Effect of ocean depth on the normal mode computation

$T = 6$ *sec*

${}_0\mathrm{U}_l$

ocean depth $1 - 10$ *km*:



Effect of ocean depth on the normal mode computation

$T = 6\ sec$

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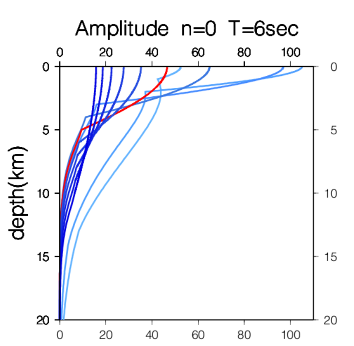
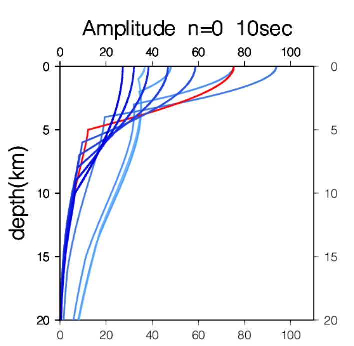
ocean depth $1 - 10\ km$:



$T = 10\ sec$

${}_0\mathrm{U}_l$

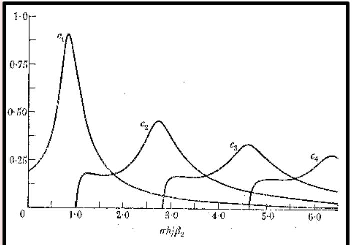
ocean depth $1 - 10\ km$:



Effect of the bathymetry in an half space

L-H analytical computation vs Normal mode computation

Longuet-Higgins' analytical computation (1950)



(from Longuet-Higgins, 1950)

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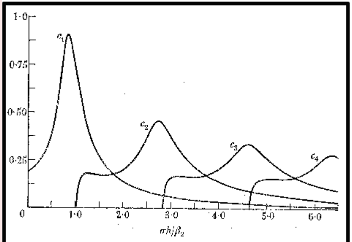

(from Longuet-Higgins, 1950)

Normal mode computation
T=3-12 sec - ocean: 1-10 km

Effect of the bathymetry in an half space

L-H analytical computation vs Normal mode computation

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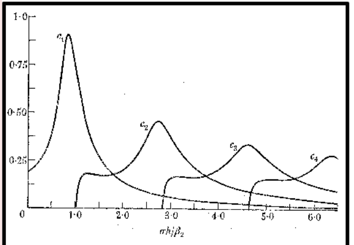
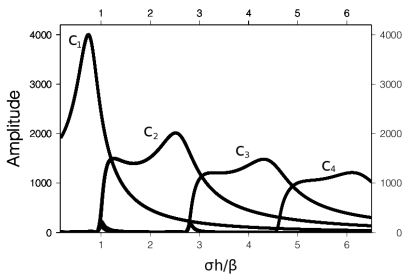


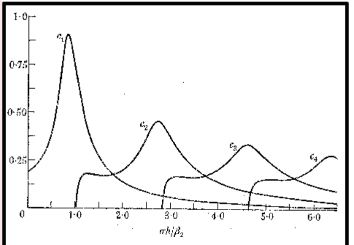
(from Longuet-Higgins, 1950)

Normal mode computation
T=3-12 sec - ocean: 1-10 km



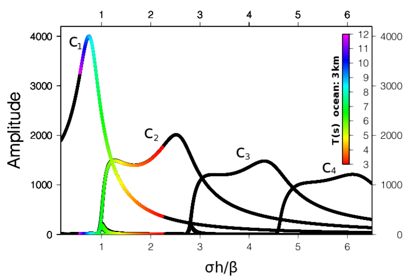
Amplification of 3 km depth

Longuet-Higgins' analytical computation (1950)



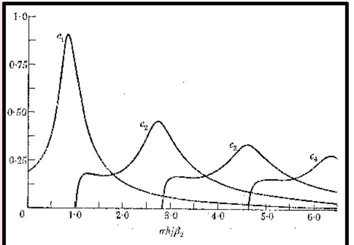
(from Longuet-Higgins, 1950)

Normal mode computation
T=3-12 sec - ocean: 1-10 km



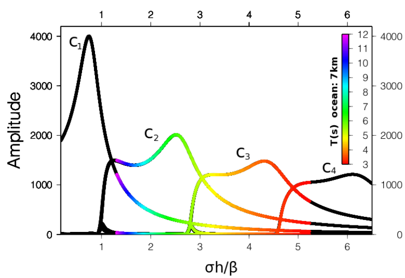
Amplification of 7 km depth

Longuet-Higgins' analytical computation (1950)



(from Longuet-Higgins, 1950)

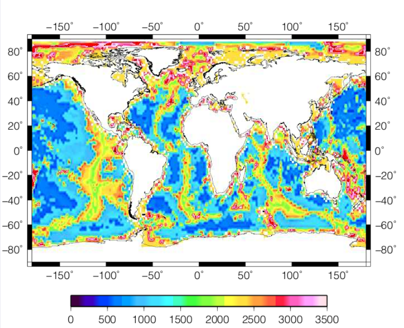
Normal mode computation
T=3-12 sec - ocean: 1-10 km



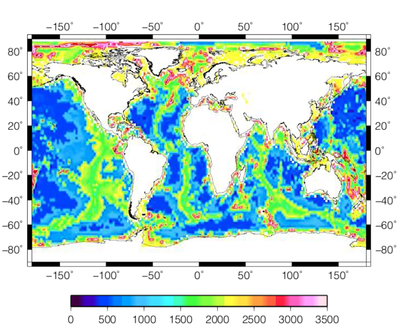
Map of amplification factor

Map of the amplification coefficient - T=6 sec

HALF SPACE



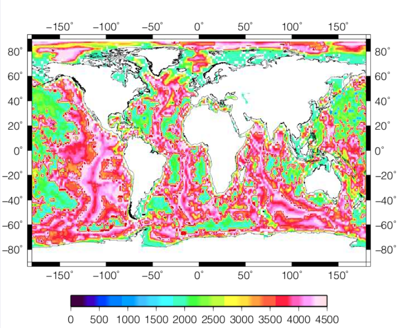
PREM



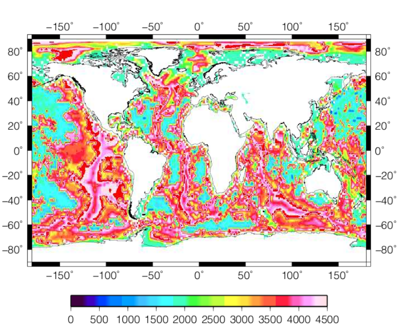
Map of amplification factor

Map of the amplification coefficient - T=10 sec

HALF SPACE

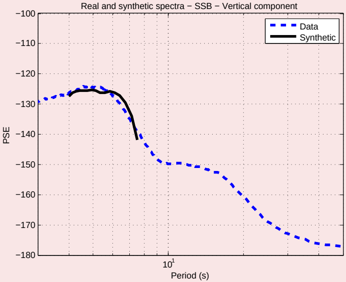


PREM

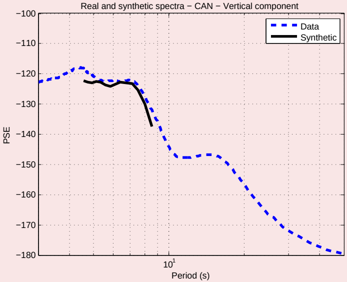


CODE: MINOS program (Gilbert, Woodhouse and Masters) modified by Y. Capdeville and NMS program realized by Y. Capdeville.

Station SSB



Station CAN



Conclusions

We are able to model the amplitude of the secondary microseism using normal mode summation:

- simulating the pressure at the surface of the ocean by point vertical forces with random phase;
- calculating the vertical force from the pressure product by the ocean wave-wave interaction;
- introducing the effect of the bathymetry in the computation and using a more realistic model than an half space.

Future prospectives

Short term activities

- evaluate
  - the regional and global scale sources;
  - the effect of the sediment on noise modeling;
  - the effect of the coastal reflection as function of geographical region;
- modeling of the fundamental mode, overtones and body waves;

Long term activities

- investigating the effect of the 3D Earth structure on seismic noise modeling (spectral element technique).

THANK YOU
FOR YOUR ATTENTION

Appendix

Vertical force associated with microseisms 1/2

1- From power spectral density of the vertical displacement:

$$PSD = \int_0^{2\pi} \int_0^{\pi} \frac{2\pi c_1^2}{\rho_s^2 \beta^5} \times \underbrace{\overbrace{\rho_w^2 g^2 f_s E^2(f) I(f)}^{\text{EQUIVALENT PRESSURE}}}_{[Pa^2 \cdot m^2 \cdot s]]} \times \frac{exp^{\frac{-2\pi f_s \Delta}{QU}}}{R_E \sin \alpha} \times R^2 \sin \Phi' d\lambda' d\Phi'$$

2- we can calculate the vertical pressure generate by ocean waves:

$$P = \sqrt{\rho_w^2 g^2 E^2(f) I(f) df}$$

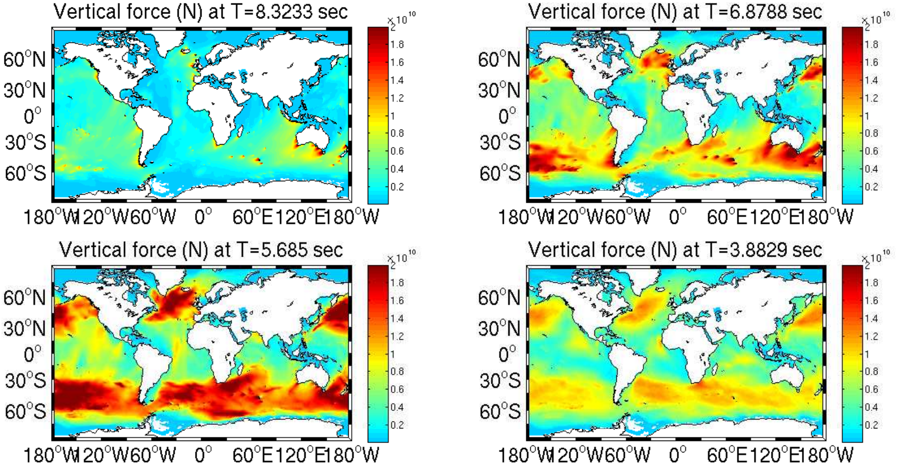
3- and then the vertical force:

$$\boxed{\boldsymbol{F = P \times \left(R^2 \sin \Phi' d\lambda' d\Phi'\right)}} \qquad [N]$$

for each frequency, latitude and longitude.

Vertical force associated with microseisms 2/2

Ocean wave model ⇒ **pressure at the surface of the ocean** produced by the ocean wave-wave interaction ⇒ **vertical force** applied at the surface



Synthetic seismogram by normal mode summation 1/4

The synthetic seismogram can be also written doing a separation of the **spheroidal** and the **toroidal** modes:

$$\boxed{u = {}_n\mathrm{U}_l Y_l^m(\Theta,\Phi)\hat{e_r} + {}_n\mathrm{V}_l \nabla_1 Y_l^m(\Theta,\Phi) + {}_n\mathrm{W}_l(\hat{e_r} \times \nabla_1) Y_l^m(\Theta, Phi)}$$

where $Y_l^m(\Theta,\Phi)$ are the spherical harmonics.

It is more useful to expand the (r,Θ,Φ) components of a vector in *canonical base* using the generalized spherical harmonics $Y_l^{Nm}(\Theta,\Phi)$ (Phinney and Burridge, 1973):

$$\vec{u} = \begin{cases} \gamma_l \Omega_0^l ({}_n\mathrm{V}_l - i\,{}_n\mathrm{W}_l) Y_l^{-m} & \rightarrow u^- \\ \gamma_l\, {}_n\mathrm{U}_l Y_l^{0m} & \rightarrow u^0 \\ \gamma_l \Omega_0^l ({}_n\mathrm{V}_l + i\,{}_n\mathrm{W}_l) Y_l^{+m} & \rightarrow u^+ \end{cases}$$

where: $N = (+,0,-)$, $\gamma_l = \sqrt{\frac{2l+1}{4\pi}}$ and $\Omega_0^l = \sqrt{\frac{l(l+1)}{2}}$.

Synthetic seismogram by normal mode summation 2/4

Instrumental vector: Receiver vector located in (r_r, Θ_r, Φ_r)

$$\begin{cases} v^- = \frac{1}{\sqrt{2}}(v_\Theta + iv_\Phi) \\ v^0 = v_r \\ v^+ = \frac{1}{\sqrt{2}}(-v_\Theta + iv_\Phi) \end{cases}$$

<u>**Receiver term:**</u> ${}_n\mathrm{R}_l(r_r, \Theta_r, \Phi_r) = \vec{u} \cdot \vec{v} = u^0 v^0 - u^+ v^- - u^- v^+$

	SPHEROIDAL	MODES			
N=-1	${}_n\mathrm{R}_l^-(r_r, \Theta_r, \Phi_r) =$	$\sqrt{\frac{2l+1}{4\pi}} \frac{\sqrt{l(l+1)}}{2}$	$(v_\Theta - iv_\Phi)$	${}_n\mathrm{V}_l$	$Y_l^-(\Theta_r, \Phi_r)$
N=0	${}_n\mathrm{R}_l^0(r_r, \Theta_r, \Phi_r) =$	$\sqrt{\frac{2l+1}{4\pi}}$	v_r	${}_n\mathrm{U}_l$	$Y_l^0(\Theta_r, \Phi_r)$
N=+1	${}_n\mathrm{R}_l^+(r_r, \Theta_r, \Phi_r) =$	$\sqrt{\frac{2l+1}{4\pi}} \frac{\sqrt{l(l+1)}}{2}$	$(-v_\Theta - iv_\Phi)$	${}_n\mathrm{V}_l$	$Y_l^+(\Theta_r, \Phi_r)$
	TOROIDAL	MODES			
N=-1	${}_n\mathrm{R}_l^-(r_r, \Theta_r, \Phi_r) =$	$\sqrt{\frac{2l+1}{4\pi}} \frac{\sqrt{l(l+1)}}{2}$	$(-v_\Phi - iv_\Theta)$	${}_n\mathrm{W}_l$	$Y_l^-(\Theta_r, \Phi_r)$
N=0	${}_n\mathrm{R}_l^0(r_r, \Theta_r, \Phi_r) =$			0	
N=+1	${}_n\mathrm{R}_l^+(r_r, \Theta_r, \Phi_r) =$	$\sqrt{\frac{2l+1}{4\pi}} \frac{\sqrt{l(l+1)}}{2}$	$(v_\Phi - iv_\Theta)$	${}_n\mathrm{W}_l$	$Y_l^+(\Theta_r, \Phi_r)$

Synthetic seismogram by normal mode summation 3/4

Point force: $f(\vec{r}) = F\delta(\vec{r} - \vec{r_0})$ in $\vec{r_0} = (r_s, \Theta_s, \Phi_s)$

$$\begin{cases} F^- = \frac{1}{\sqrt{2}}(F_\Theta + iF_\Phi) \\ F^0 = F_r \\ F^+ = \frac{1}{\sqrt{2}}(-F_\Theta + iF_\Phi) \end{cases}$$

Source term:

$${}_n\mathrm{S}_l(r_s, \Theta_s, \Phi_s) = \int_{V_E} f(\vec{r})\,{}_nu_l^*(\vec{r})dV = F \cdot {}_nu_l^*(\vec{r_0}) = F^-u^- + F^0u^0 + F^+u^+$$

	SPHEROIDAL	**MODES**			
N=-1	${}_n\mathrm{S}_l^-(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(F_\Theta + iF_\Phi)$	${}_n\mathrm{V}_l$	$Y_l^-(\Theta_s, \Phi_s)$
N=0	${}_n\mathrm{S}_l^0(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}$	F_r	${}_n\mathrm{U}_l$	$Y_l^0(\Theta_s, \Phi_s)$
N=+1	${}_n\mathrm{S}_l^+(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(-F_\Theta + iF_\Phi)$	${}_n\mathrm{V}_l$	$Y_l^+(\Theta_s, \Phi_s)$
	TOROIDAL	**MODES**			
N=-1	${}_n\mathrm{S}_l^-(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(-F_\Phi + iF_\Theta)$	${}_n\mathrm{W}_l$	$Y_l^-(\Theta_s, \Phi_s)$
N=0	${}_n\mathrm{S}_l^0(r_s, \Theta_s, \Phi_s) =$			0	
N=+1	${}_n\mathrm{S}_l^+(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(F_\Phi + iF_\Theta)$	${}_n\mathrm{W}_l$	$Y_l^+(\Theta_s, \Phi_s)$

Synthetic seismogram by normal mode summation 4/4

VERTICAL Point force: $f(\vec{r}) = F\delta(\vec{r} - \vec{r}_0)$ in $\vec{r}_0 = (r_s, \Theta_s, \Phi_s)$

$$\begin{cases} F^- = \frac{1}{\sqrt{2}}(F_\Theta + iF_\Phi) \\ F^0 = F_r \\ F^+ = \frac{1}{\sqrt{2}}(-F_\Theta + iF_\Phi) \end{cases}$$

Source term:

$${}_n\mathrm{S}_l(r_s, \Theta_s, \Phi_s) = \int_{V_E} f(\vec{r})\,{}_n u_l^*(\vec{r})dV = F \cdot {}_n u_l^*(\vec{r}_0) = F^- u^- + F^0 u^0 + F^+ u^+$$

	SPHEROIDAL	MODES			
N=-1	${}_n\mathrm{S}_l^-(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(F_\Theta + iF_\Phi)$	${}_n\mathrm{V}_l$	$Y_l^-(\Theta_s, \Phi_s)$
N=0	${}_n\mathrm{S}_l^0(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}$	F_r	${}_n\mathrm{U}_l$	$Y_l^0(\Theta_s, \Phi_s)$
N=+1	${}_n\mathrm{S}_l^+(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(-F_\Theta + iF_\Phi)$	${}_n\mathrm{V}_l$	$Y_l^+(\Theta_s, \Phi_s)$
	TOROIDAL	MODES			
N=-1	${}_n\mathrm{S}_l^-(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(F_\Theta + iF_\Phi)$	${}_n\mathrm{W}_l$	$Y_l^-(\Theta_s, \Phi_s)$
N=0	${}_n\mathrm{S}_l^0(r_s, \Theta_s, \Phi_s) =$			0	
N=+1	${}_n\mathrm{S}_l^+(r_s, \Theta_s, \Phi_s) =$	$\sqrt{\frac{2l+1}{4\pi}}\frac{\sqrt{l(l+1)}}{2}$	$(-F_\Theta + iF_\Phi)$	${}_n\mathrm{W}_l$	$Y_l^+(\Theta_s, \Phi_s)$

Validation of NMS theory using a force: Mount St Helens landslide (1981) 1/2

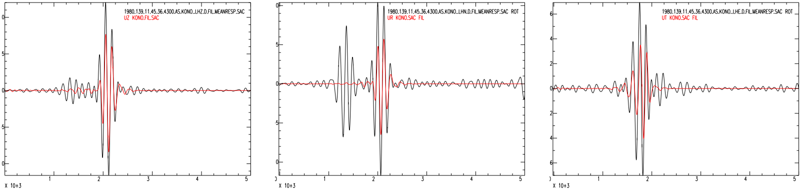
__CODE:__ **MINOS program** (Gilbert, Woodhouse and Masters) modified by Y. Capdeville and **NMS program** realized by Y. Capdeville. This code was validated to be used only with moment tensor.

__PURPOSE:__ simulate an event using a force as source to validate the theoretical/analytical computation of normal modes summation:
- **vertical force:** no earthquake or landslide data.
- **horizontal force:** Mount St Helens landslide (Kanamori & Given, 1982):

Horizontal force direction: $S5^\circ W$
Amplitude of the force: $10^{13} N$

__Earth model__ **used in the computation:** PREM
bandpass filter between 50 s and 200 s

Validation of NMS theory using a force: Mount St Helens landslide (1981) 2/2



$\rightarrow$ The amplitude not so well retrieved $\rightarrow$ $1D$ model

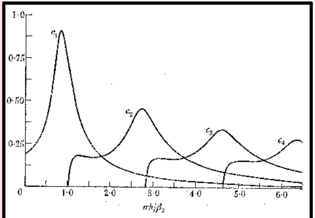
$\rightarrow$ It is necessary increase n to simulate the others peaks (here only $n = 30$)

The phase of the synthetic seismograms is correct and the the main peak is retrieved in the right position.

Amplification due to the bathymetry using PREM model

Effect of the bathymetry: PREM model

Longuet-Higgins' analytical computation (1950)



(from Longuet-Higgins, 1950)

Normal mode computation
T=3-12 sec - ocean: 1-10 km