Full waveform modeling of the earth's mantle at the global scale: from normal modes to SEM

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Long period seismograms by normal mode summation

1) Spherically symmetric Earth (1D)

$$u(\mathbf{x},t) = \mathrm{Re}\{\sum_k A_k^0(\Delta)e^{i\omega_k t}\}$$

Source excitation

distance

Eigenfrequency (complex)

2) 3D Earth - First order perturbation theory

Step 1: high frequency approximation

$$u(\mathbf{x},t) = \mathrm{Re}\{\sum A_k^0(\Delta)e^{i(\omega_k+\delta\hat{\omega}_k)t}\}$$

$$\delta\hat{\omega}_k = \frac{1}{2\pi}\int_\gamma \delta\omega(s)\,ds$$

Eigenfrequency shift

3) "Path average approximation (PAVA)"

$$u(\mathbf{x},t) = \mathrm{Re}\{\sum_k A_k^0(\Delta+\delta\Delta)e^{i(\omega_k+\delta\hat{\omega}_k)t}\}$$

$$\delta\Delta = \frac{aU}{l+1/2}(\delta\hat{\omega}-\delta\tilde{\omega}) \qquad \delta\tilde{\omega}_k = \frac{1}{\Delta}\int_0^\Delta \delta\omega(s)\,ds$$

Minor arc average

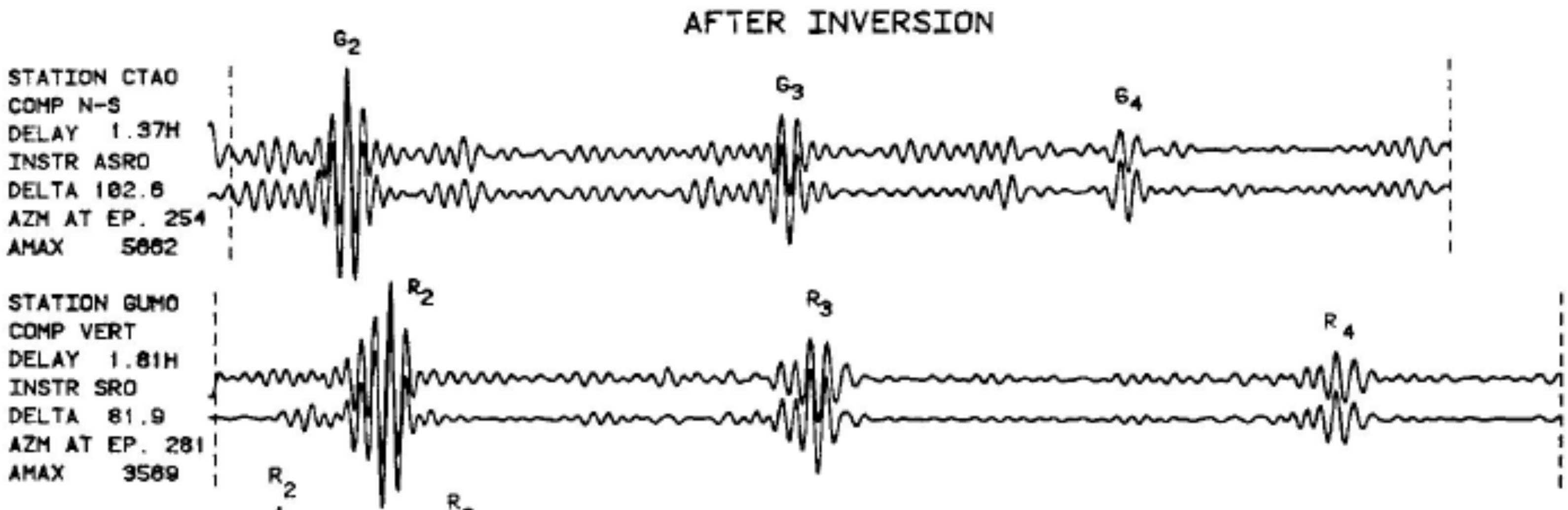
Great circle average

-> Introduced by Woodhouse and Dziewonski (1984)

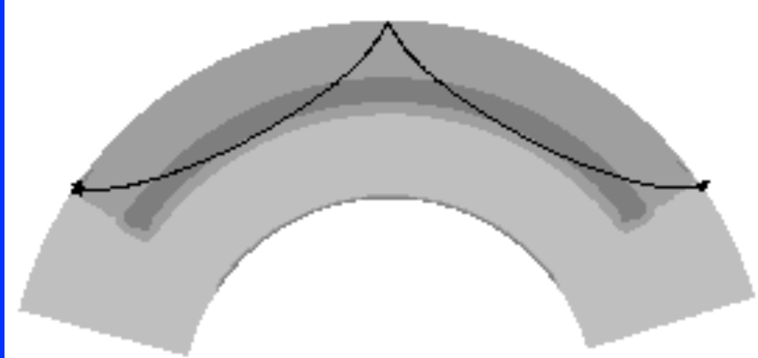
-> Equivalent to surface wave PAVA approximation (Mochizuki, 1986; Romanowicz, 1987)

Time domain waveform inversion in global seismology

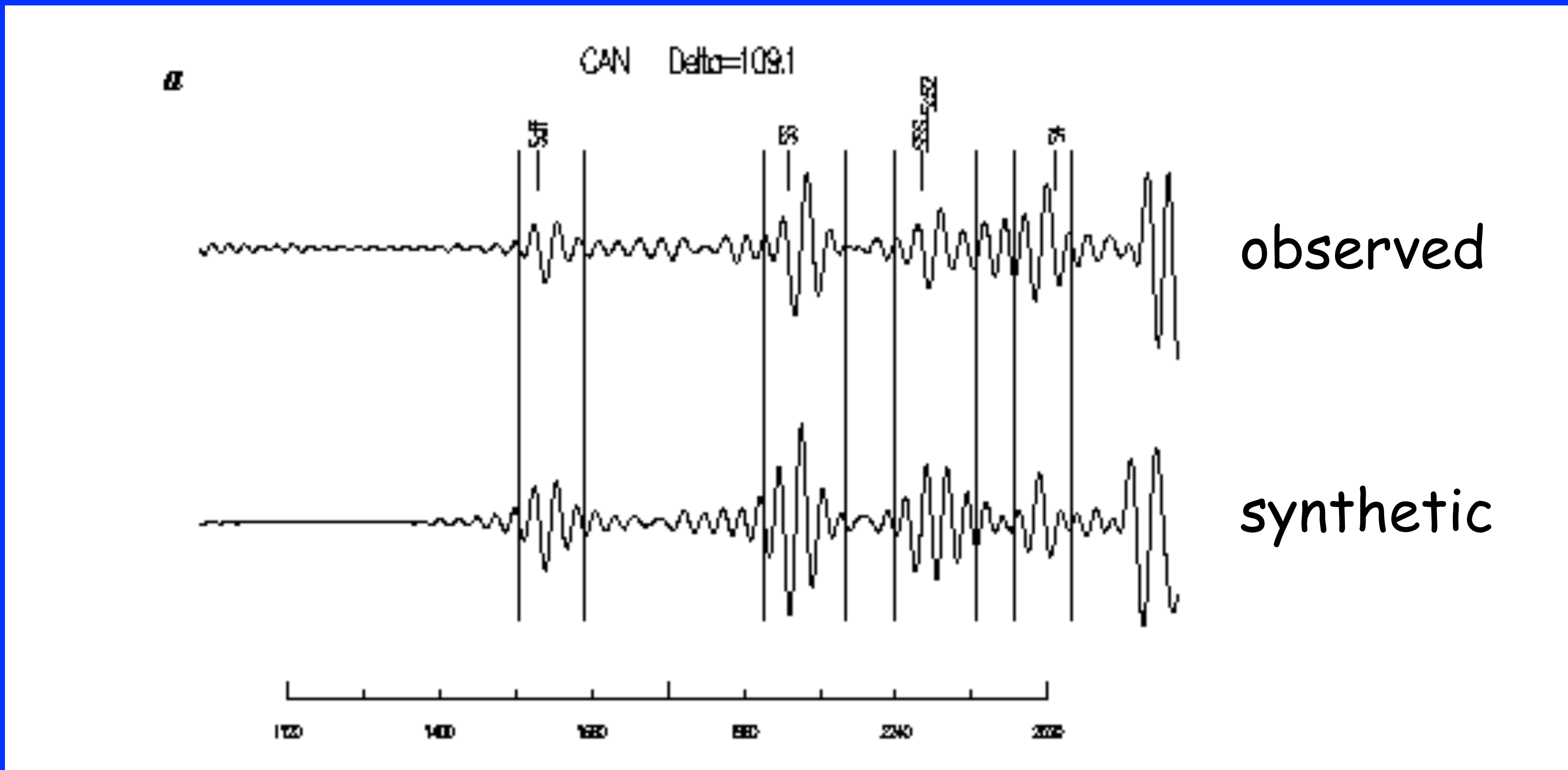
- Woodhouse and Dziewonski (1984)



- Normal mode theory
- Path AVerage Approximation (PAVA)-> 1D sensitivity kernels
- *Later, complement with body wave travel times (ray theory) to access lower mantle structure*

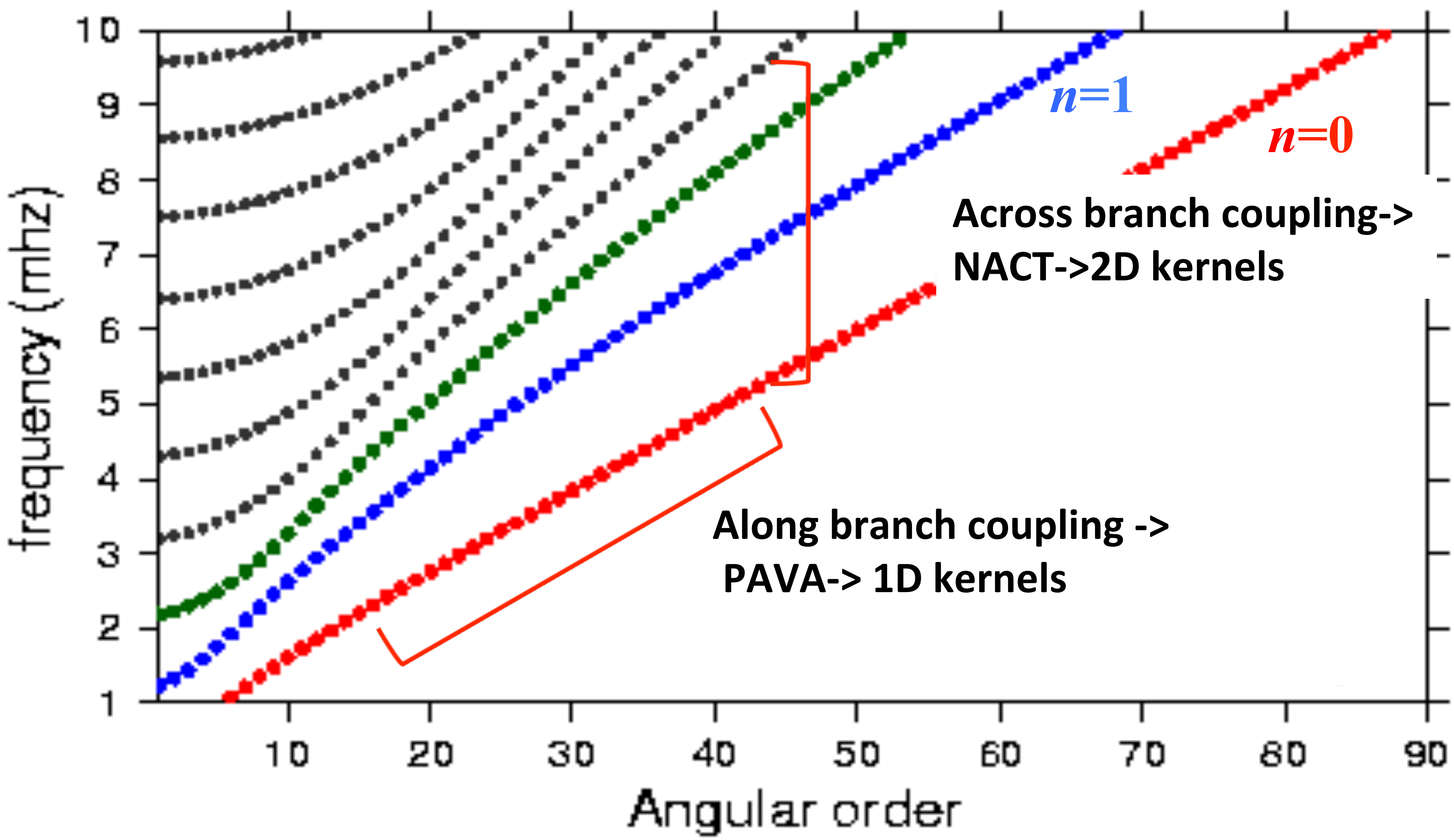


Full Waveform Tomography of the whole mantle



- To include body waveforms with the "correct" sensitivity Concentrated along the raypath, one needs to include across branch coupling (Li and Romanowicz, 1995; Marquering et al., 1996)

Toroidal modes : $_n T_l$



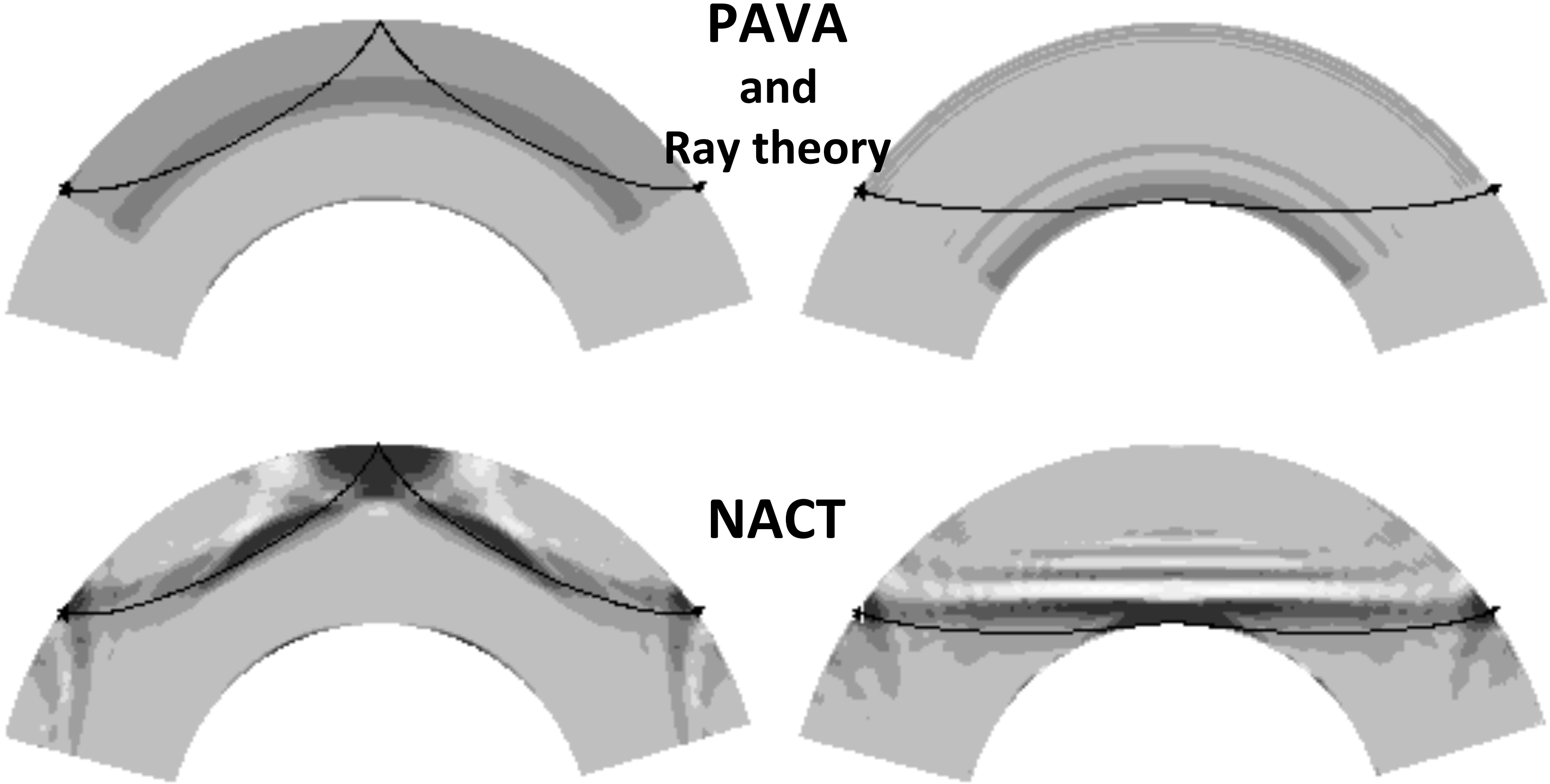
l: angular order, horizontal nodes

n: overtone number, vertical nodes

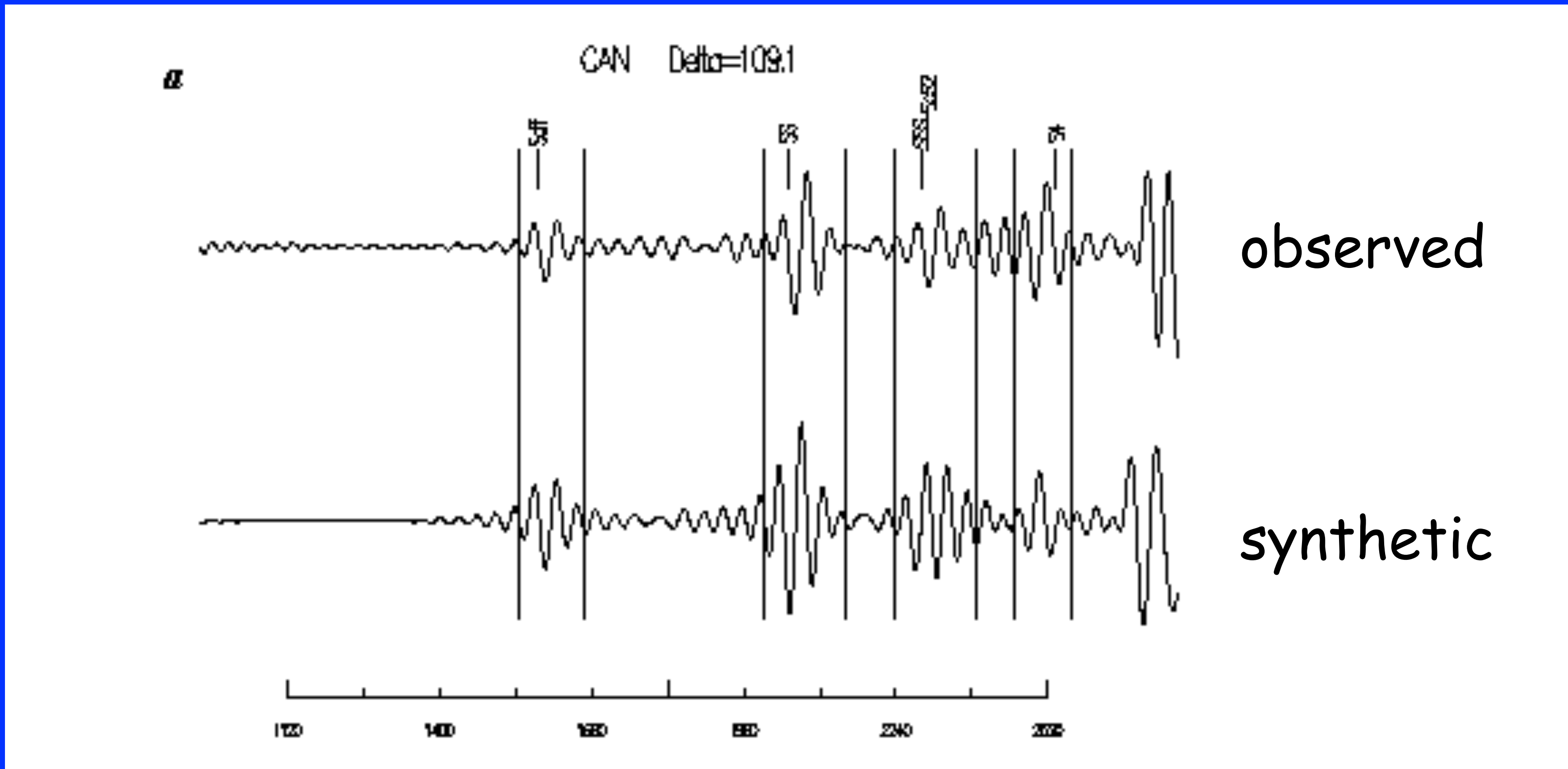
4) Non-linear asymptotic coupling theory (NACT)-> 2D Kernels in the vertical plane *(Li and Romanowicz, 1995)*

$$u(\mathbf{x},t) = \mathrm{Re}\{\sum_k A_k^0(\Delta + \delta\Delta)e^{i(\omega_k + \delta\hat{\omega}_k)t} +$$

Across-branch
Coupling terms
Asymptotic Born



Full Waveform Tomography of the whole mantle



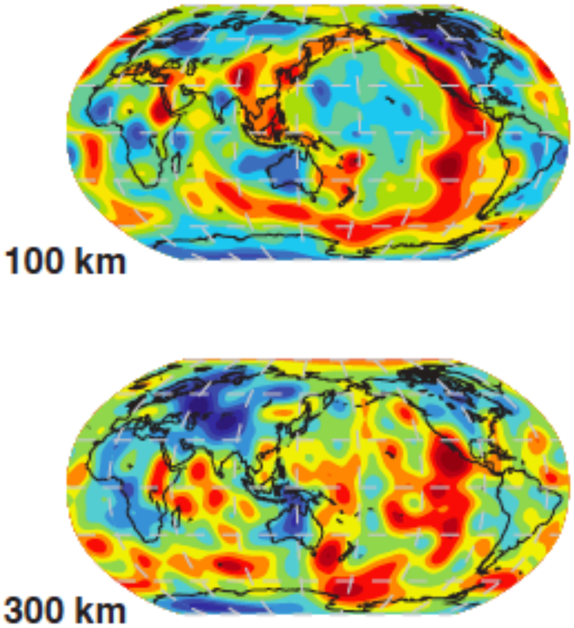
- NACT: Surface waves, overtones (T>80s), body waves (T>32 s)

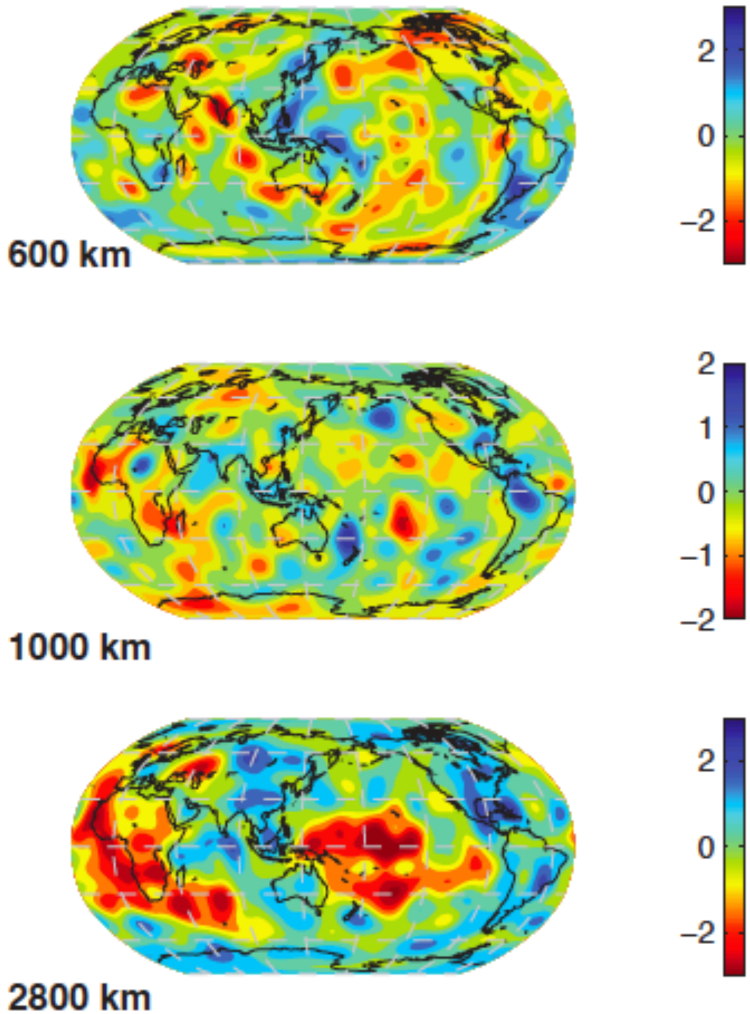
-Misfit function: Windowing to allow weighing of wavepackets, in order to equalize amplitudes.

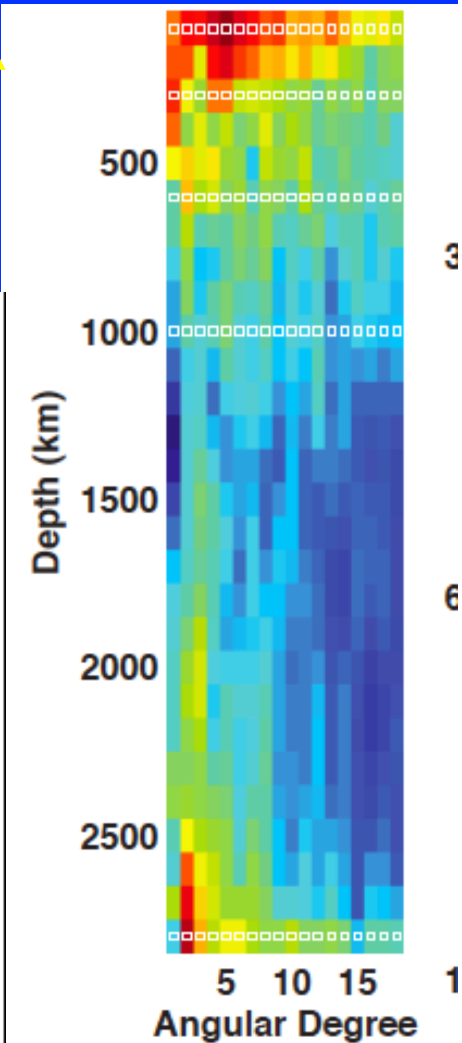
-Several generations of whole mantle shear velocity models,
-Including radial anisotropy, attenuation

SAW24B16

Spectrum of heterogeneity







Mégnin and Romanowicz, 2000

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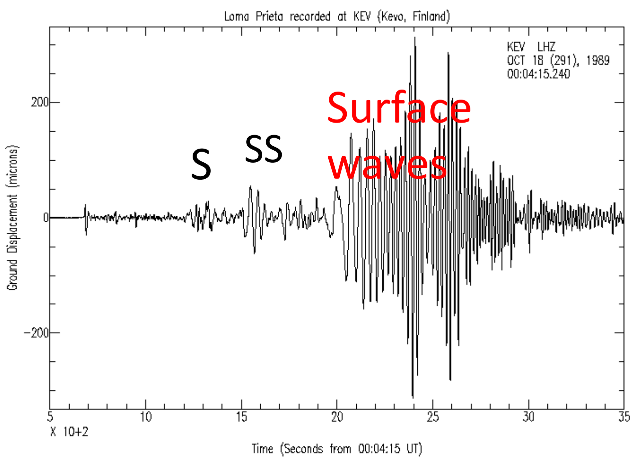
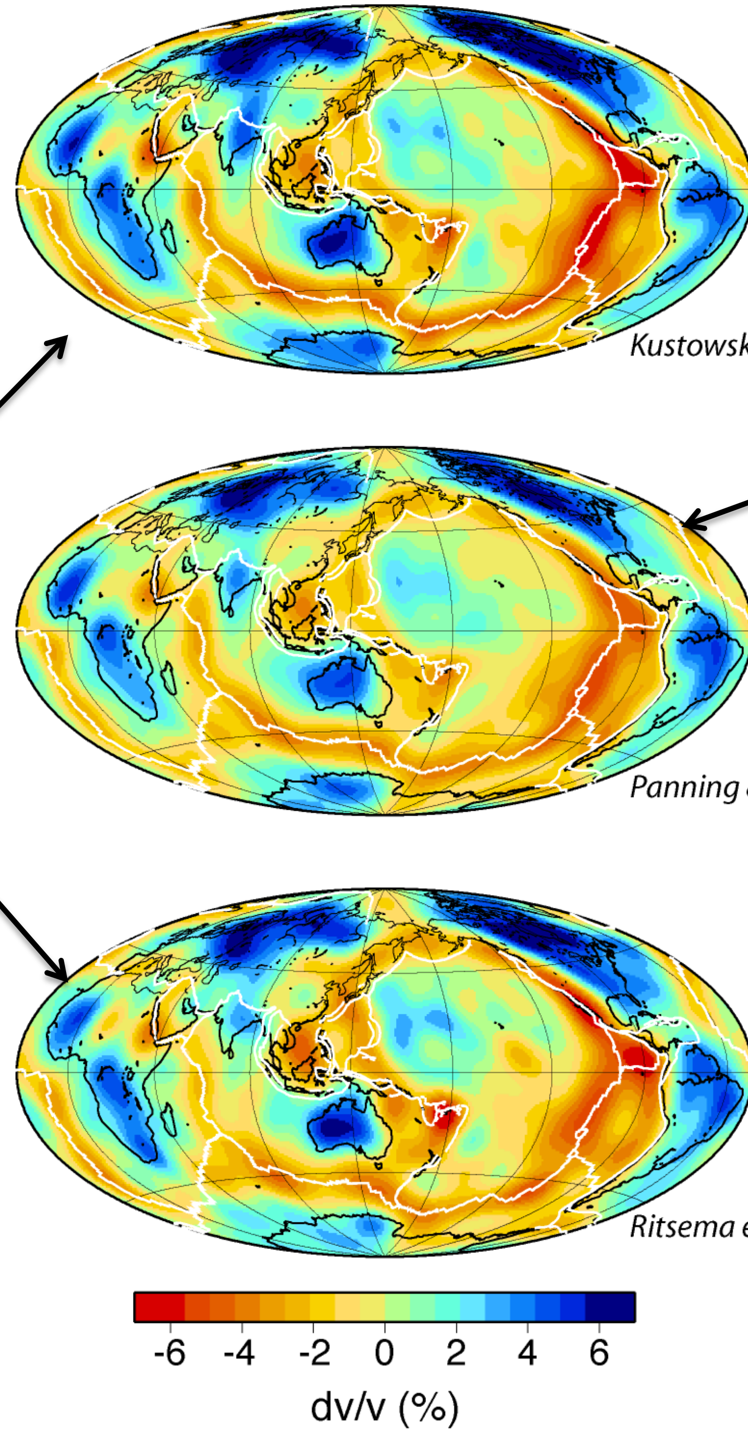
Isotropic Vs
Depth = 100 km

- Long period S, SS, ScS, SKS…travel times
- Surface waves and overtones
- Normal Mode splitting
- High frequency Approx.





Harvard

Kustowski et al. 2006

Time domain Waveforms ~20,000

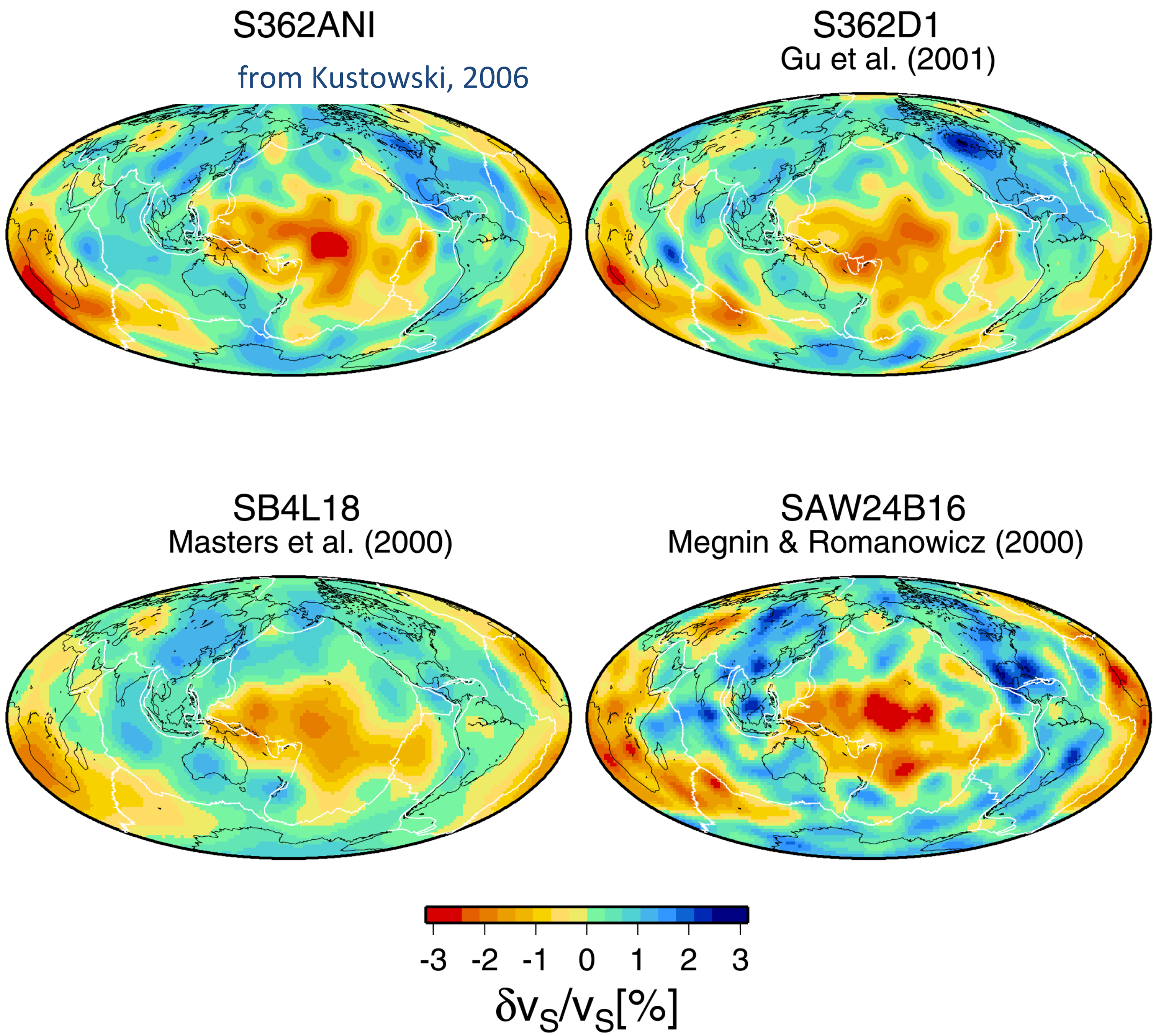
Berkeley

Panning & Romanowicz 2006

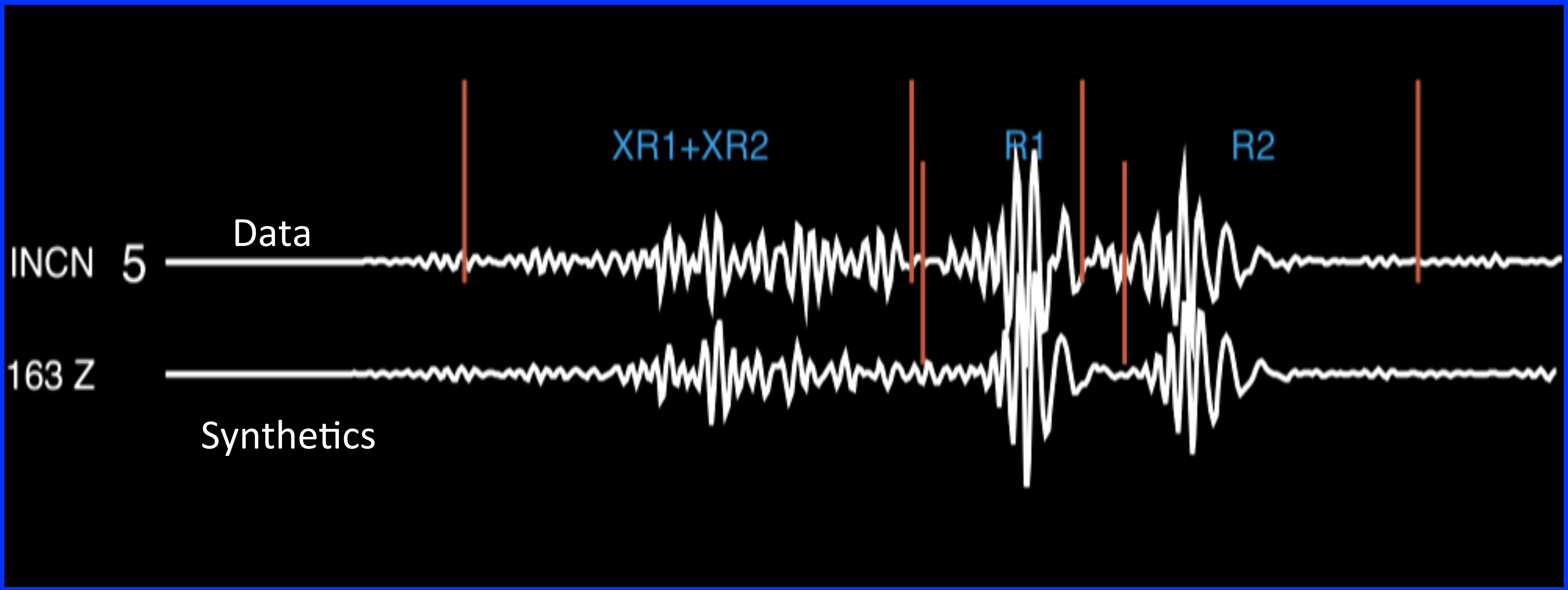
Caltech

Ritsema et al. 1999

Isotropic Vs
Depth = 2800 km



Full Waveform Tomography using SEM:



Replace mode synthetics by numerical synthetics computed using the Spectral Element Method (SEM)

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Challenges for SEM based global waveform tomography

- *Computation time:*
  - One event periods > 60s: 4 hours on 32 cores
  - Need several hundred events, many iterations
- *The earth's crust:*
  - Strongly heterogeneous
  - Thin low velocity layers → slows down the computation
  - Crustal structure is not perfectly known at the global scale
  - Strong non-linearity→ cycle slips

Our strategy

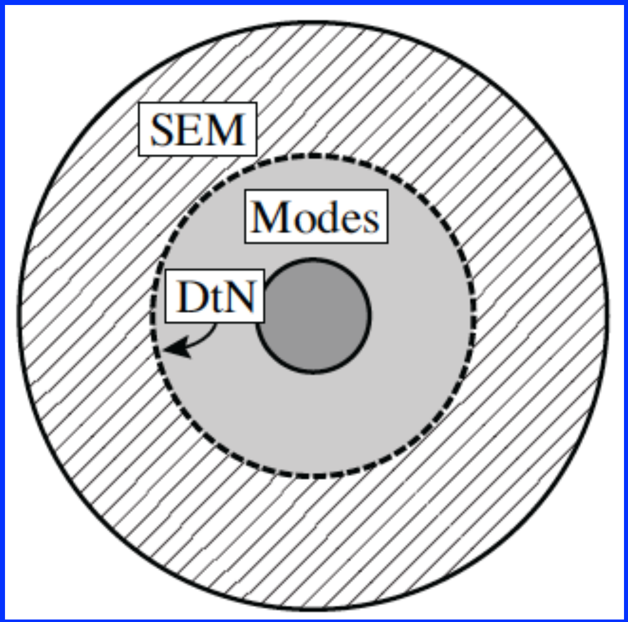
- Take “modest steps” and in the process learn something about the earth
- Start at long periods (T> 60s)
- Progressively add waveforms as observed and predicted phases line up
- Compute forward wavefield precisely using C-SEM
- Compute inverse Hessian kernels approximately (NACT)
- Use “homogenized”, smooth crustal model appropriate for the period range considered

I-HYBRID INVERSION APPROACH

At each iteration:

1-Forward modeling step

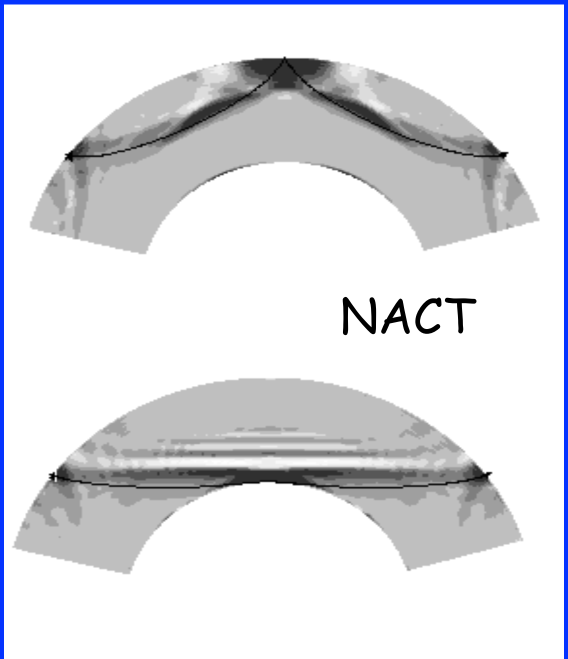
Use coupled spectral element method of Capdeville et al. (2003) to accurately forward model wave propagation through the 3D Earth



Γ1= Normal modes in 1D
Γ2 = Spectral element method

2-Inverse step

Use approximate Hessian calculated in NACT. Much faster than adjoint!



Li and Romanowicz, 1995

II-NACT kernels

- Based on asymptotic mode coupling theory
- updated with each update of the model (PAVA term includes multiple forward scattering)
- We use the Hessian rather than a gradient method
- Can account for attenuation effects accurately

Adjoint kernels

- Computed numerically
- single scattering approximation
- Conjugate Gradient method
- Attenuation approximated

III-Smooth homogeneized crustal model

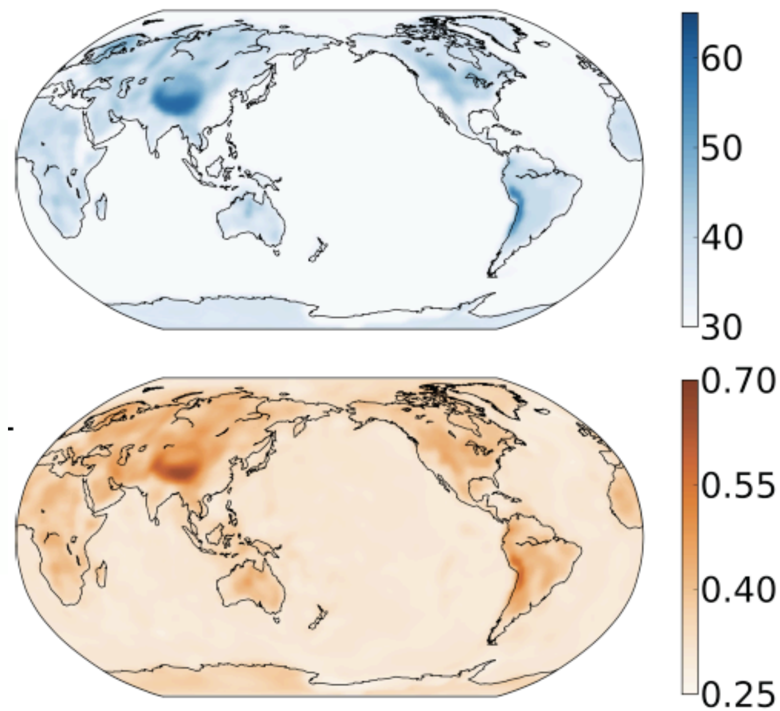
Equivalent smooth anisotropic layer (Backus, 1962)

Two generations of models:

- SEMum – 60 km constant Moho (*Lekic and Romanowicz, 2011*)

- SEMum2- variable >30 km

- In both cases:
  - Start with Clipped, filtered Crust2.0
  - Fit global dataset of dispersion maps (*Ritzwoller et al., 2002*) using Monte Carlo



Top: Smooth Moho of SEMum2 (km); **Bottom**: SEM (C=0.4) time step (s)

=> SEM time step prolonged 4 times

1^{st} generation upper mantle model: SEMum

- Full waveforms, T> 60 s, 200 events

- Replace realistic crust by a homogenized, smooth radially anisotropic crustal model
  - Uniform thickness of 60 km
  - Made to fit a global surface wave group velocity dispersion data set (20-60 s)

- Radially anisotropic model
  - Vs (isotropic shear velocity)
  - $\xi = (Vsh/Vsv)^2$

- Upper mantle only:
  - Lower mantle from existing tomographic model SAW24B16

- Start with 1D model
- As iterations progress:
  - Progressively add waveforms as model improves
  - Add 3D radial anisotropy at 3rd iteration
  - Refine model parametrization
    - -> 642 to 2562 spherical spline nodes in Vs
    - -> 162 to 642 nodes for ξ
  - Recalculate kernels at each iteration (non-linear)
- After 10 iterations-> SEMum

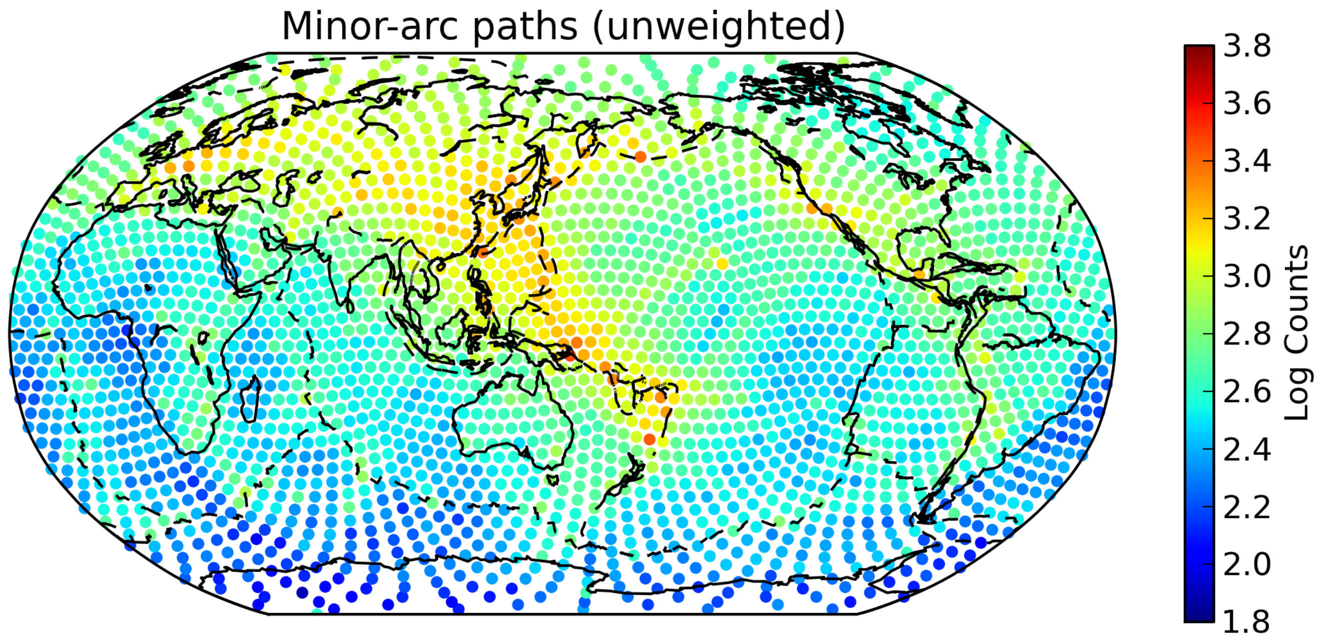
2^{nd} generation model SEMum2

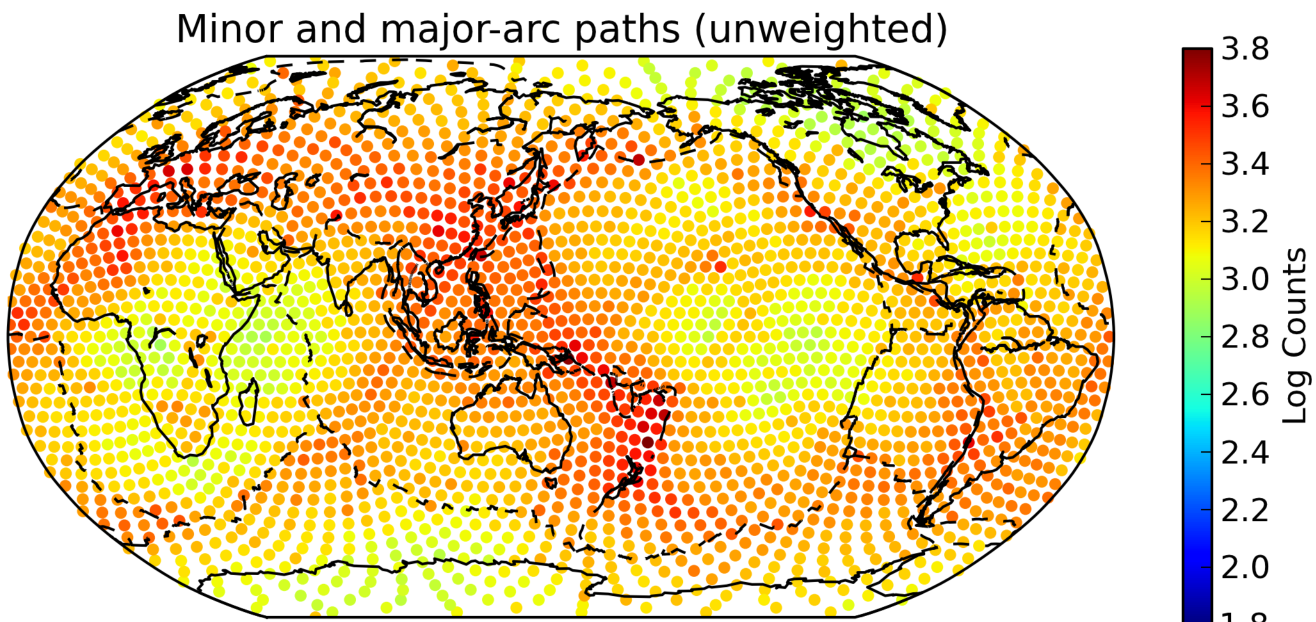
- Replace 60 km crust by variable Moho homogeneized crust (designed to fit same group velocity dataset)
- Introduce modified crustal corrections to account for strong non-linearity in NACT
- 2 iterations beyond SEMum

  Additional waveforms get included

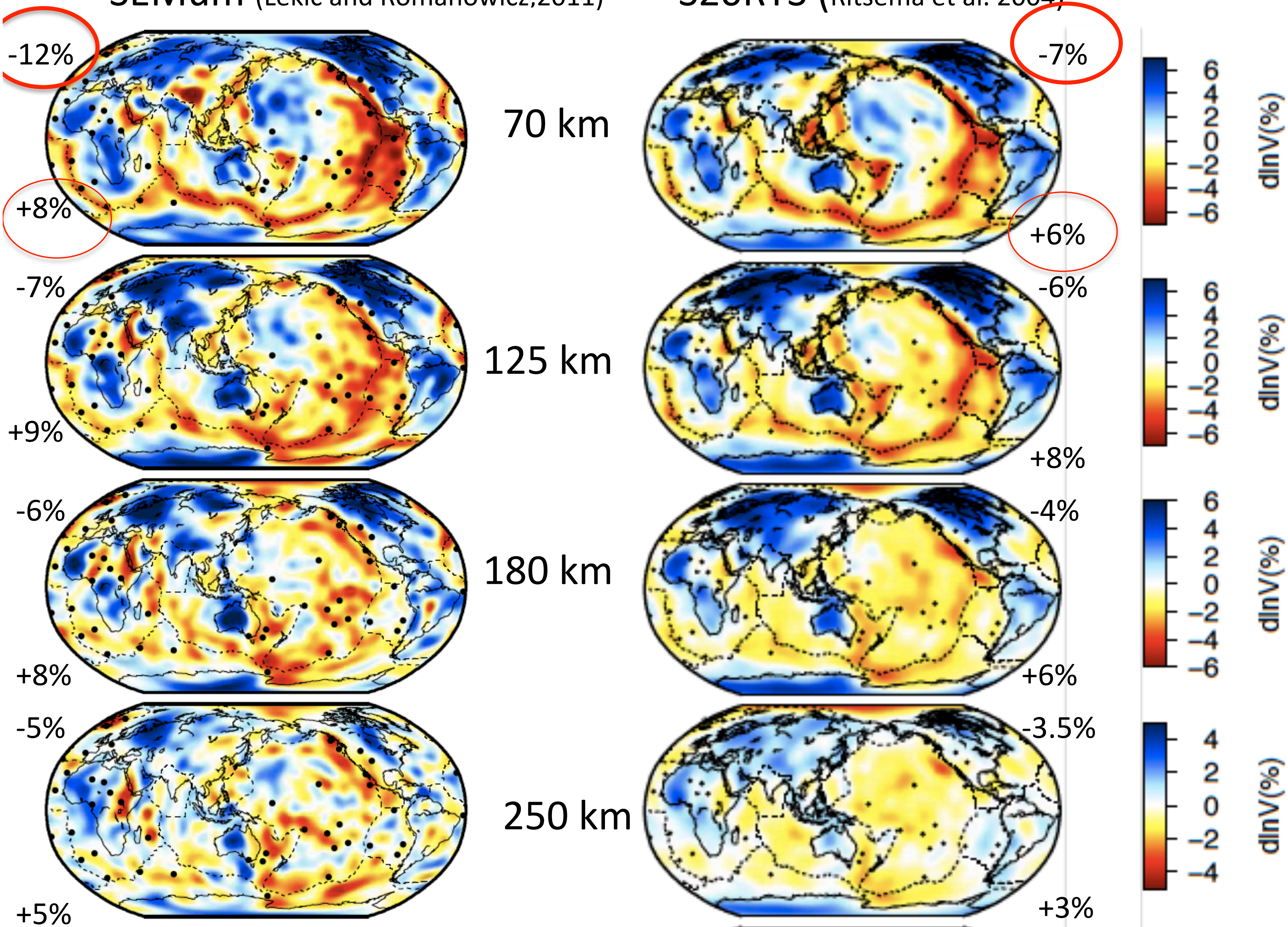
French et al., 2012

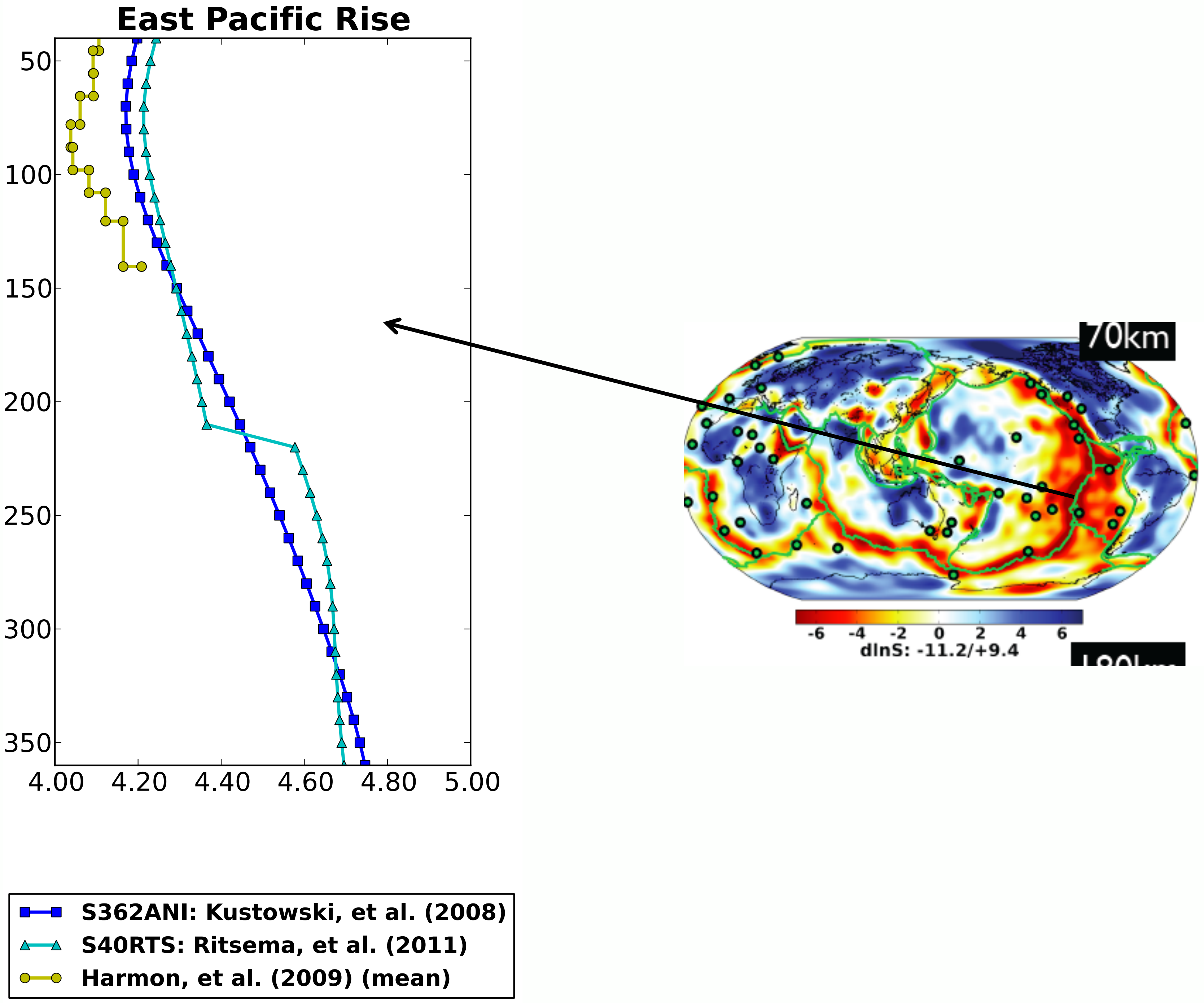
SEMum2:
204 events
~ 100,000
wavepackets

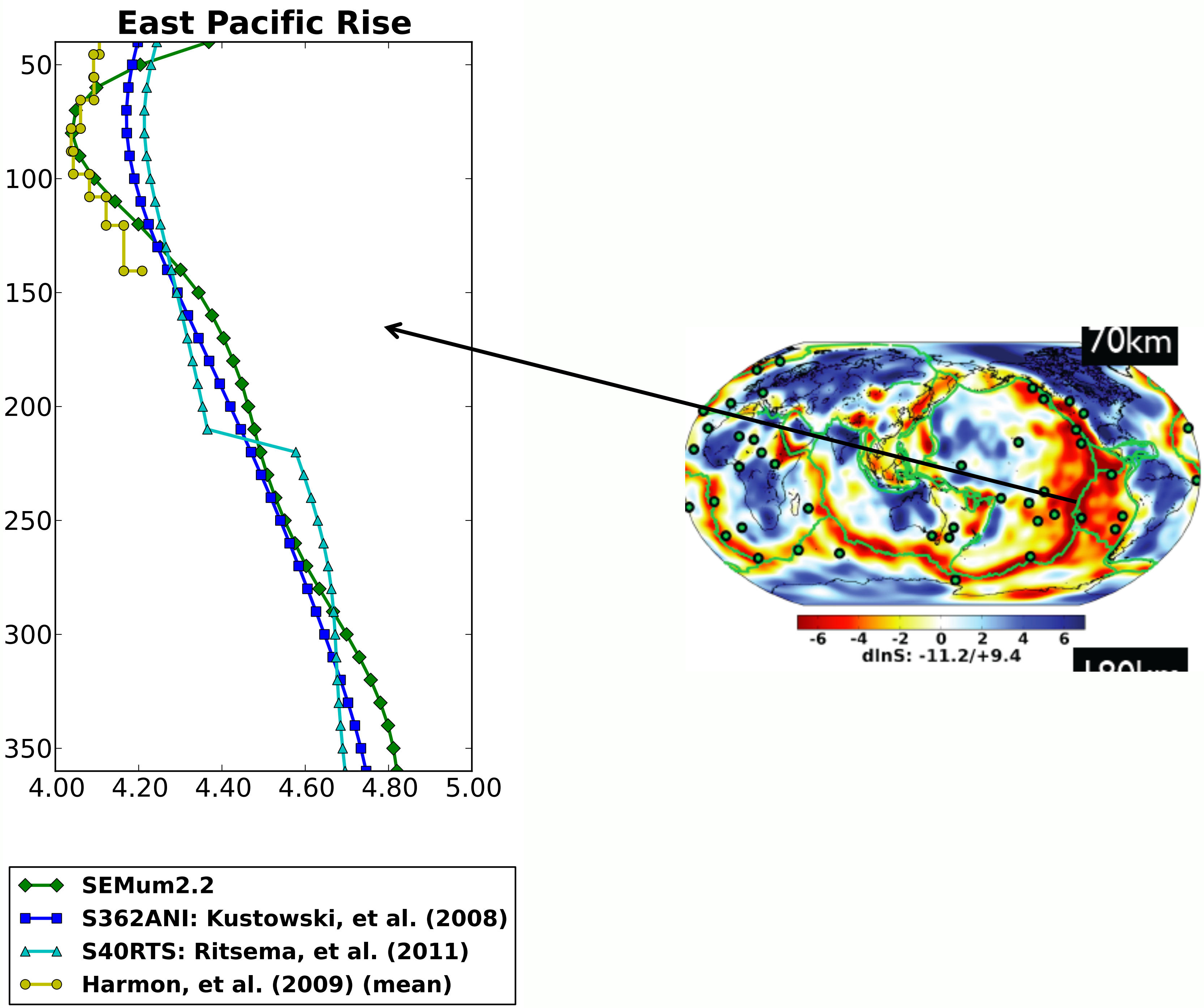




SEMum (Lekic and Romanowicz,2011) S20RTS (Ritsema et al. 2004)




East Pacific Rise
50
100
150
200
250
300
350
4.00
4.20
4.40
4.60
4.80
5.00
S362ANI: Kustowski, et al. (2008)
S40RTS: Ritsema, et al. (2011)
Harmon, et al. (2009) (mean)
70km
-6
-4
-2
0
2
4
6
dlnS: -11.2/+9.4

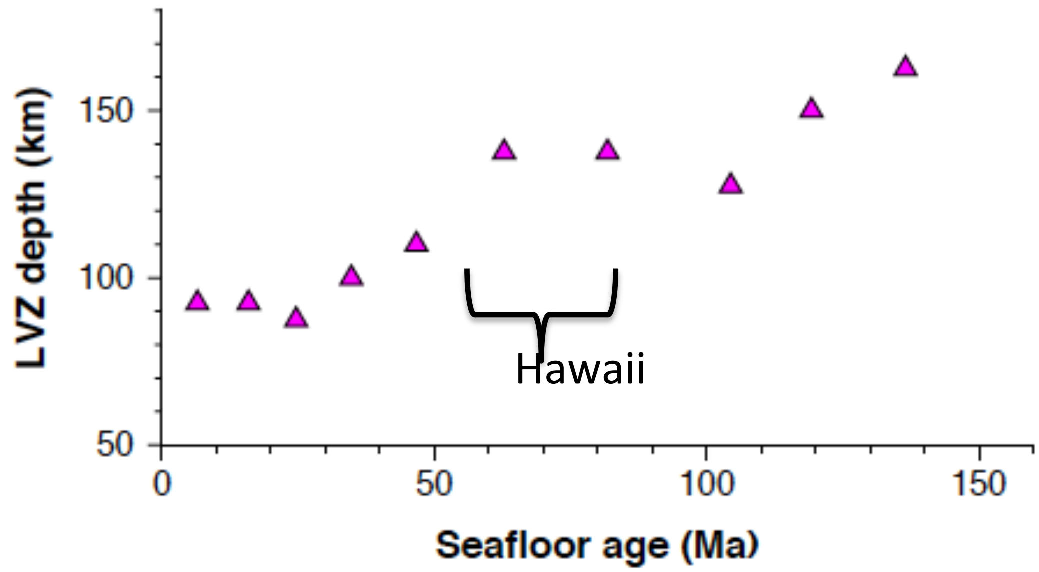


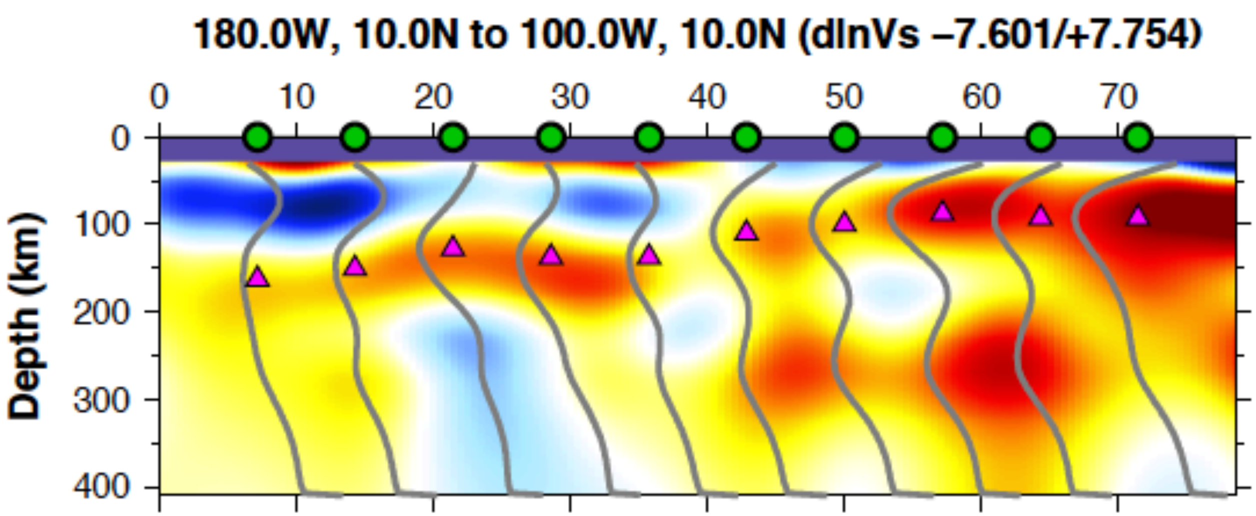
East Pacific Rise
50
100
150
200
250
300
350
4.00
4.20
4.40
4.60
4.80
5.00
SEMum2.2
S362ANI: Kustowski, et al. (2008)
S40RTS: Ritsema, et al. (2011)
Harmon, et al. (2009) (mean)
70km
-6
-4
-2
0
2
4
6
dlnS: -11.2/+9.4

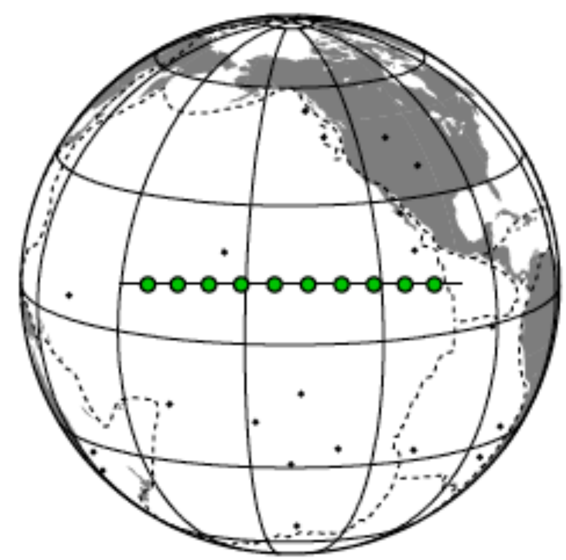
SEMum2.2 model structure

Pacific LVZ structure

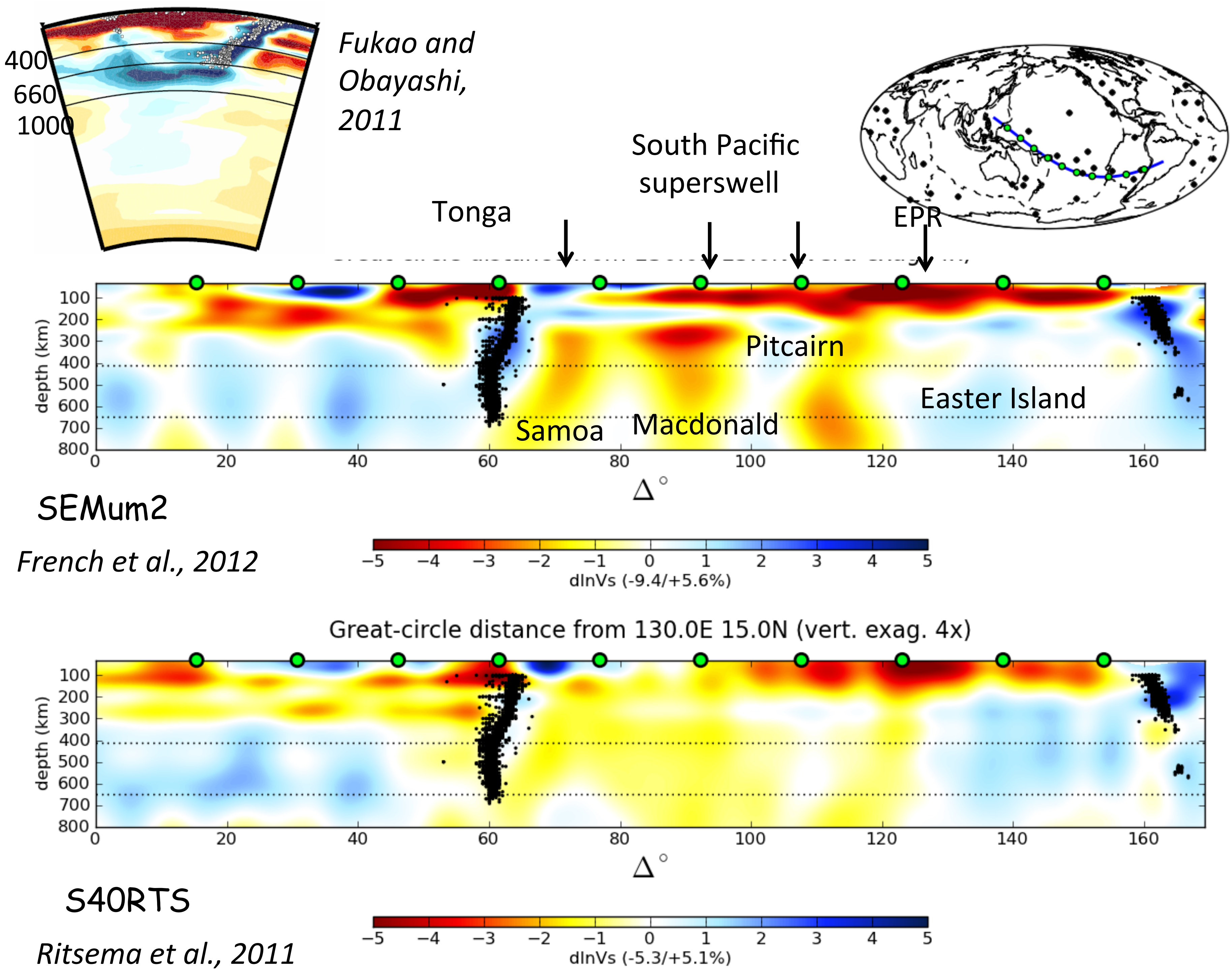
Relationship with seafloor age

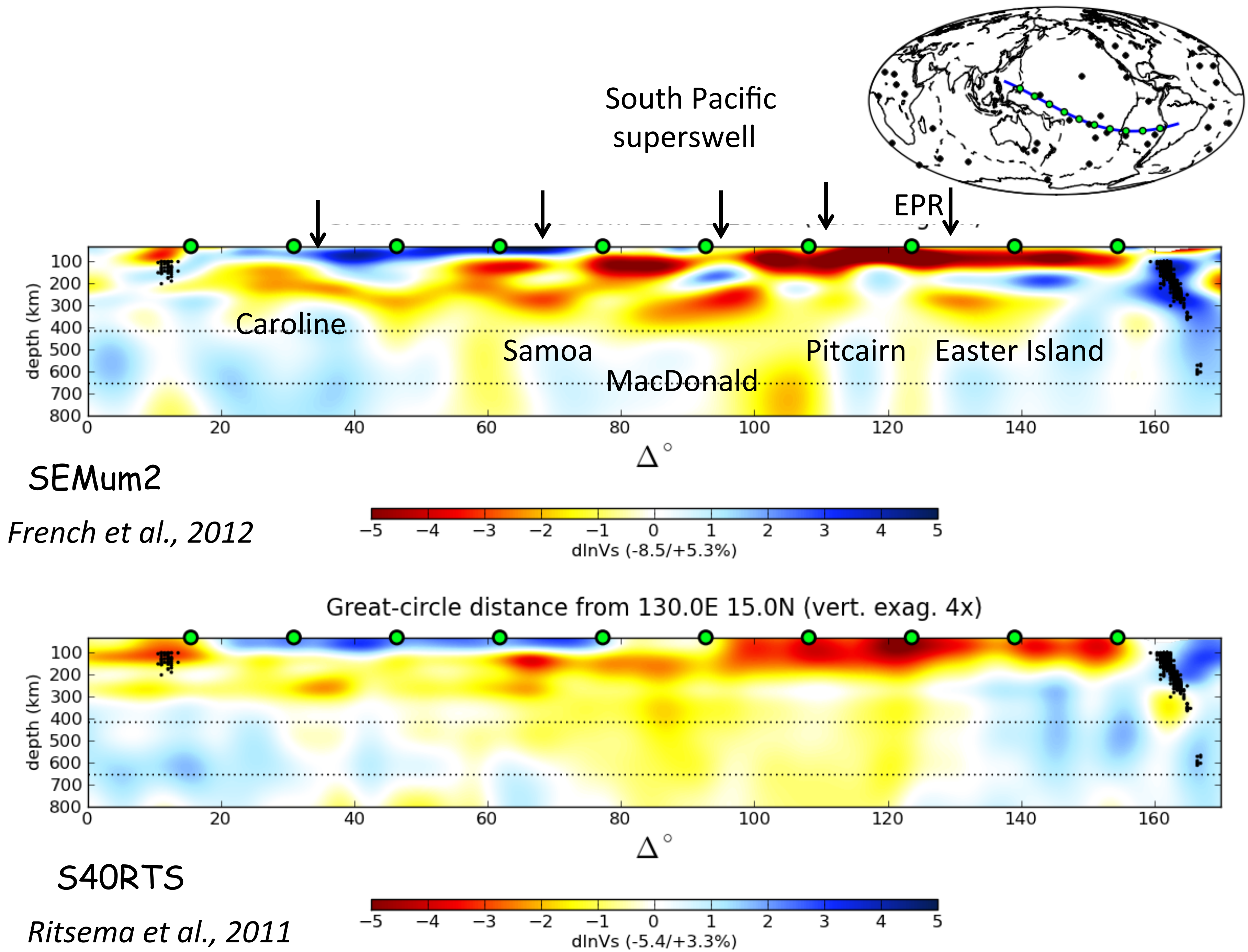




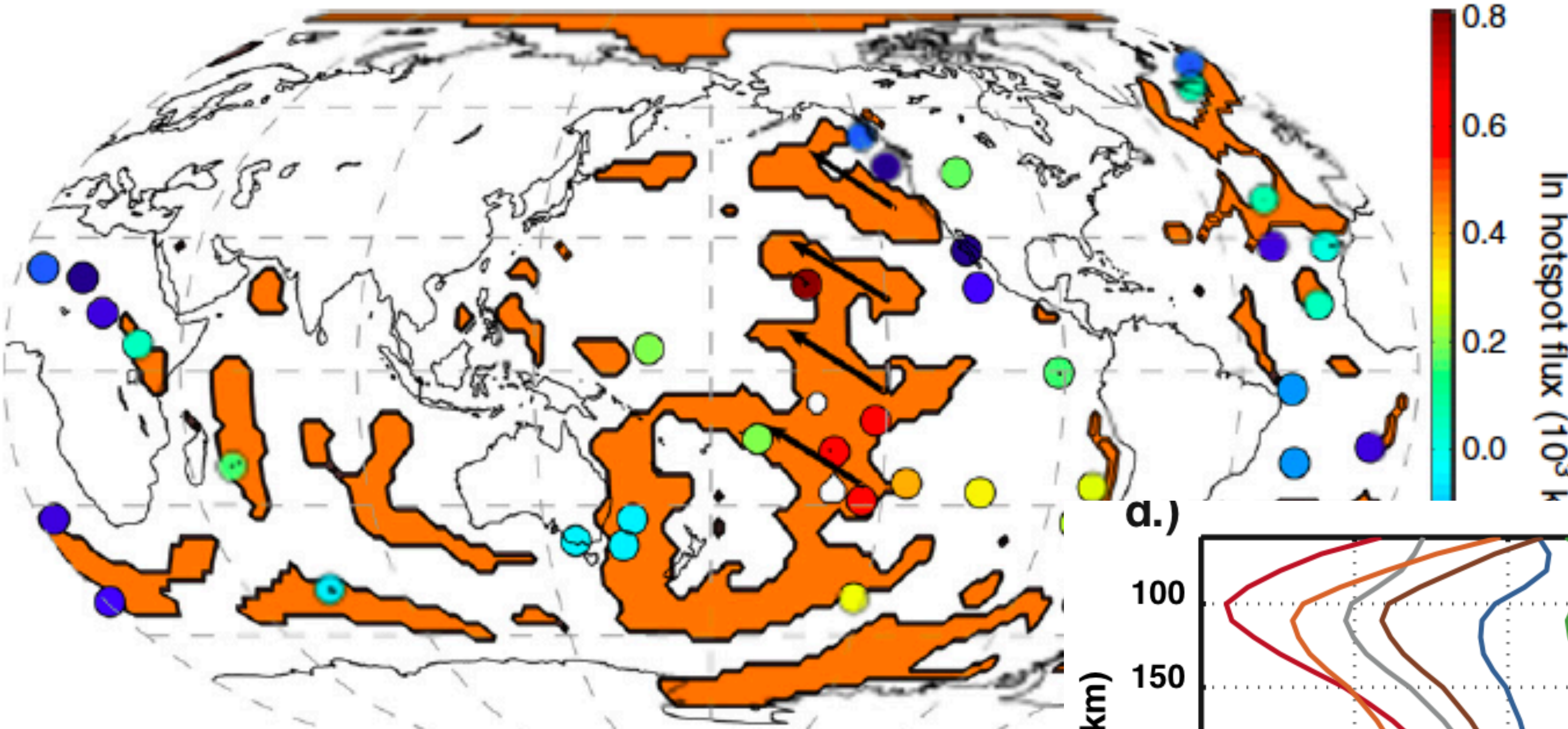


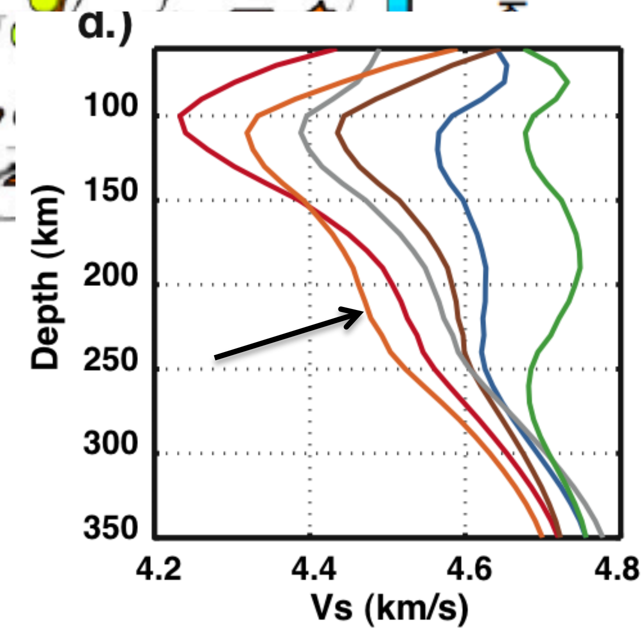
(Ages from Muller, et al. 2009, G³)


400
660
1000
Fukao and Obayashi, 2011
South Pacific superswell
Tonga
EPR
100
200
300
400
500
600
700
800
depth (km)
Pitcairn
Easter Island
Samoa
Macdonald
0
20
40
60
80
100
120
140
160
Δ°
SEMum2
French et al., 2012
−5 −4 −3 −2 −1 0 1 2 3 4 5
dlnVs (-9.4/+5.6%)
Great-circle distance from 130.0E 15.0N (vert. exag. 4x)
S40RTS
Ritsema et al., 2011
dlnVs (-5.3/+5.1%)


South Pacific
superswell
EPR
Caroline
Samoa
MacDonald
Pitcairn
Easter Island
depth (km)
Δ°
SEMum2
French et al., 2012
dlnVs (-8.5/+5.3%)
Great-circle distance from 130.0E 15.0N (vert. exag. 4x)
S40RTS
Ritsema et al., 2011
dlnVs (-5.4/+3.3%)

Geographic extent of oceanic region OR2 in clustering analysis of SEMum with N=6, and the location of major hotspots

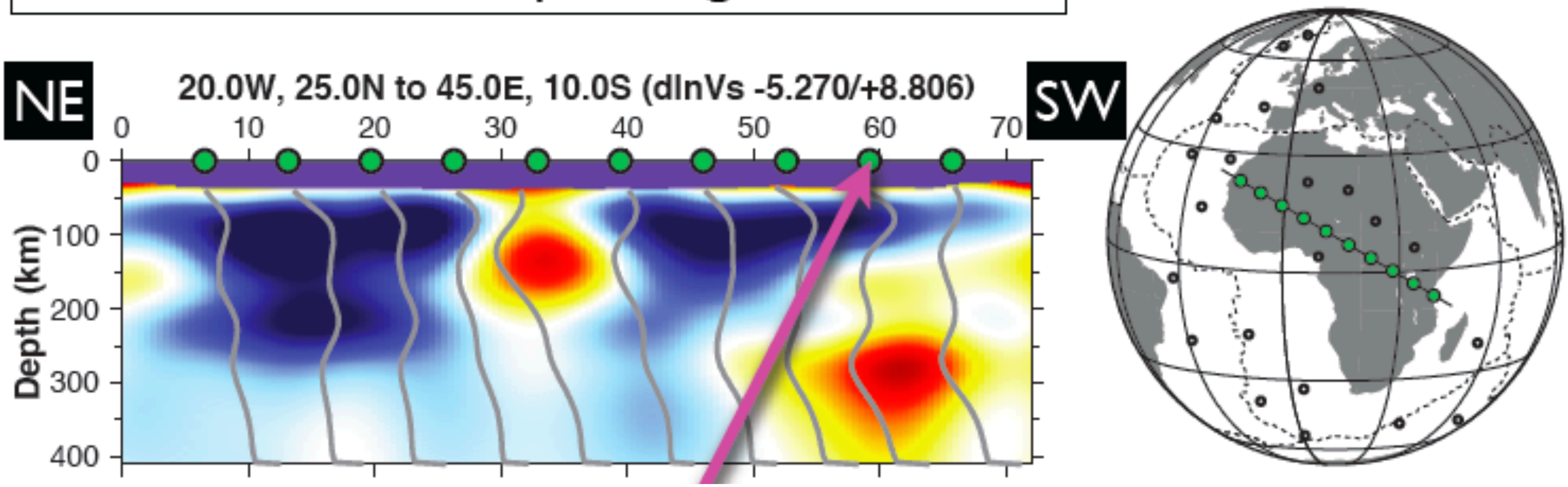




Lekic and Romanowicz, 2011

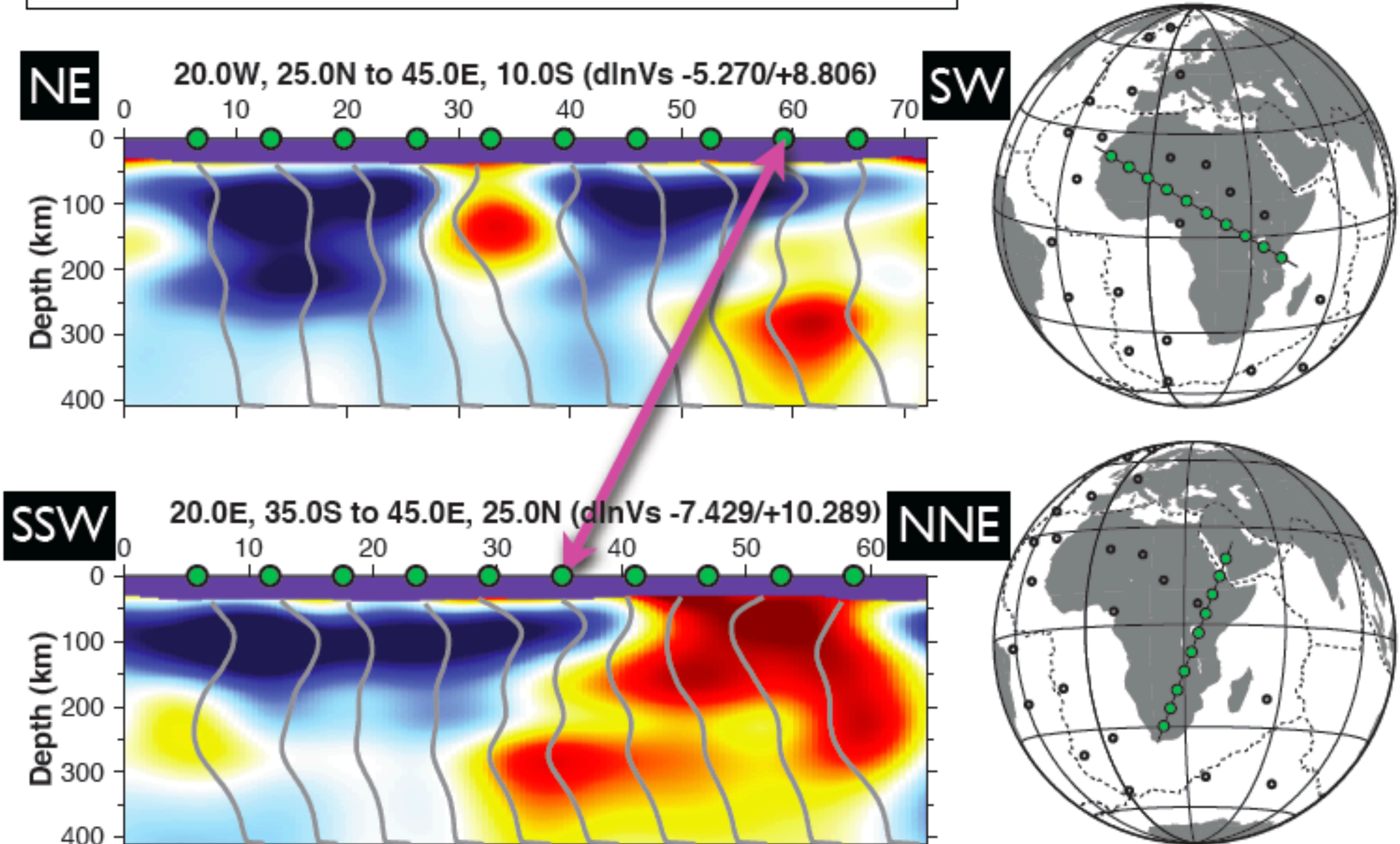
SEMum2.2 model structure

Continental example region: **Africa**



SEMum2.2 model structure

Continental example region: **Africa**



Conclusions

- SEMum2 shows similar large scale features in good agreement with previous global models developed using "approximate" theory
- Subduction zones better resolved, approaching resolution of recent P models (e.g. Fukao and Obayashi, 2011)
- Continental roots are well marked, max. depth 200-250 km
- SEMum2 exhibits significantly stronger low velocity regions:

Upper mantle low velocity zone

- Well developed and strong LVZ in the oceans
- Depth of velocity minimum increases with age
- Velocity minimum in agreement with local study at EPR (in depth and strength)
- Bottom of LVZ well marked in general
- Deeper zone of low velocities (~250 km depth) forming “streaks” that appear to align with APM in Pacific
- Between 300 -800 km, columnar low velocity features associated with hotspots, but not necessarily with single hotspot, and can be offset horizontally

Outlook

- Our modeling confirms common wisdom that more exact theory can resolve low velocity structures better

- Only beginning – next steps:
  - Perform source perturbations
  - Shorter periods (40 s) ->
    - Lower mantle
    - Higher resolution
  - Upper mantle attenuation (*Jamie Barron's poster)*