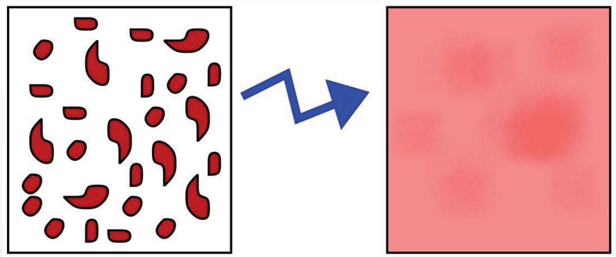
Optimizing 2D non periodic homogenization for the elastic waves for strong spatial variations of the dominant wavelength cases.

Philippe CANCE

Laboratoire de Planétologie et de Géodynamique de Nantes, CNRS, France

LPGNantes, 2012/05/22

Homogenization recipe



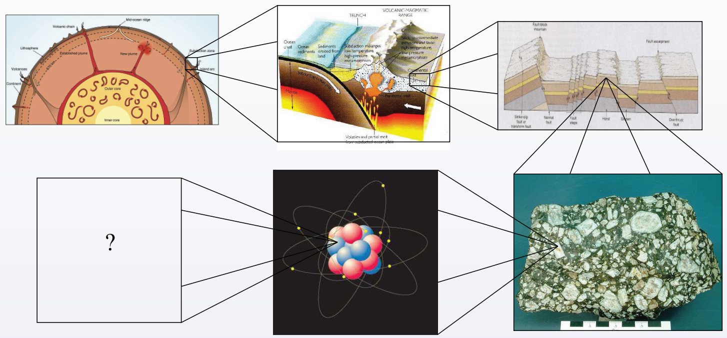
"Ingredients"

- 1 heterogeneous medium defined at a microscopic scale
- 2 scales : microscopic and macroscopic
- 1 problem which could be transformed to be solved at the macroscopic level only

"Products"

- 1 effective medium defined at the macroscopic scale only
- The associated effective problem to be solved at the macroscopic scale

Homogenization in seismology



Full waveform modelling

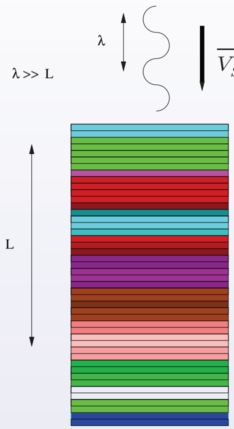
$$\rho \partial_{tt} u - \nabla . \sigma = f$$

$$\sigma = \mathbf{c} : \frac{\nabla u +^t \nabla u}{2}$$

- assumption : there is a minimum wavelength $\lambda_m \sim \frac{V_m}{f_c}$ for the wavefield
- Heterogeneities $\ll \lambda_m$
- Relation $(\rho, \mathbf{c})$ (fine scale)$\leftrightarrow$ $(\rho^*, \mathbf{c}^*)$ (effective) ?

Homogenization and averages

(Backus 1962)



$$\frac{n}{{V^*_{SV}}^2} = \frac{1}{V^2_{S,1}} + \frac{1}{V^2_{S,2}} + ... + \frac{1}{V^2_{S,n}}$$

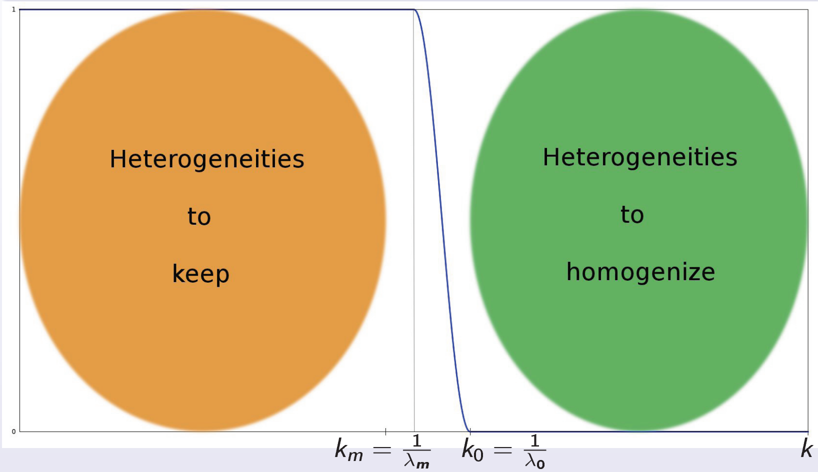
$$n{V^*_{SH}}^2 = V^2_{S,1} + V^2_{S,2} + ... + V^2_{S,n}$$

(the density is constant)

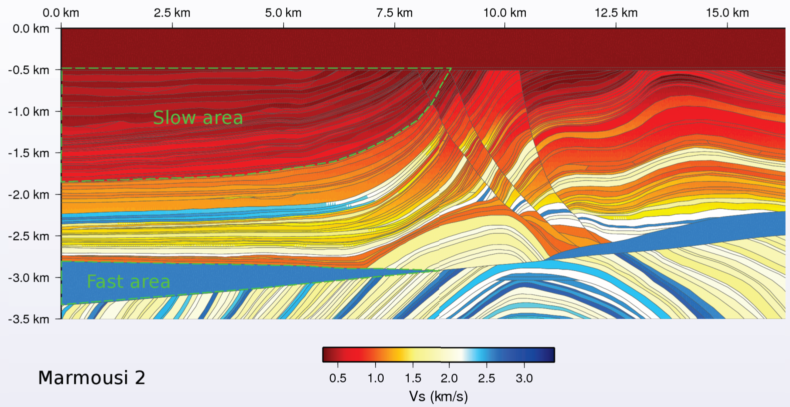
not linear, not a simple filtering operator

Fundamental tools for non periodic homogenization

- $\varepsilon = \frac{\lambda}{\lambda_m} < \varepsilon_0 = \frac{\lambda_0}{\lambda_m} << 1$ (ℓ-periodic case : $\varepsilon_0 = \frac{\ell}{\lambda_m}$)
- macroscopic variables ($\mathbf{x}$) and microscopic variables ($\mathbf{y} = \frac{\mathbf{x}}{\varepsilon}$)
- Scale separator : spatial filter F (convolution) : $F(u) = u * w_m$



Problem : high spatial variabilitiy of λ_m



Marmousi 2

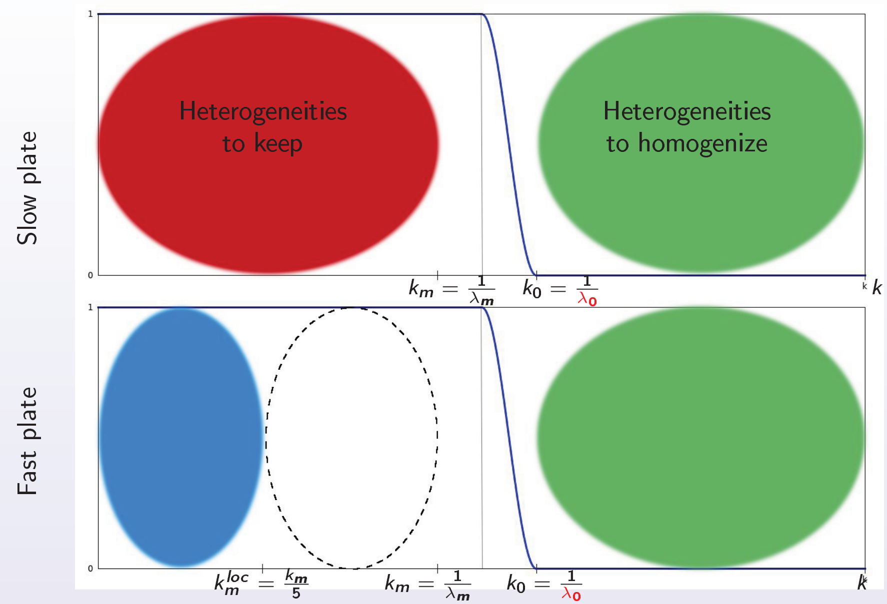
Slow area

$\lambda_m^{loc} = \lambda_m, \quad \varepsilon_0^{loc} = \varepsilon_0 = \frac{\lambda_0}{\lambda_m}$

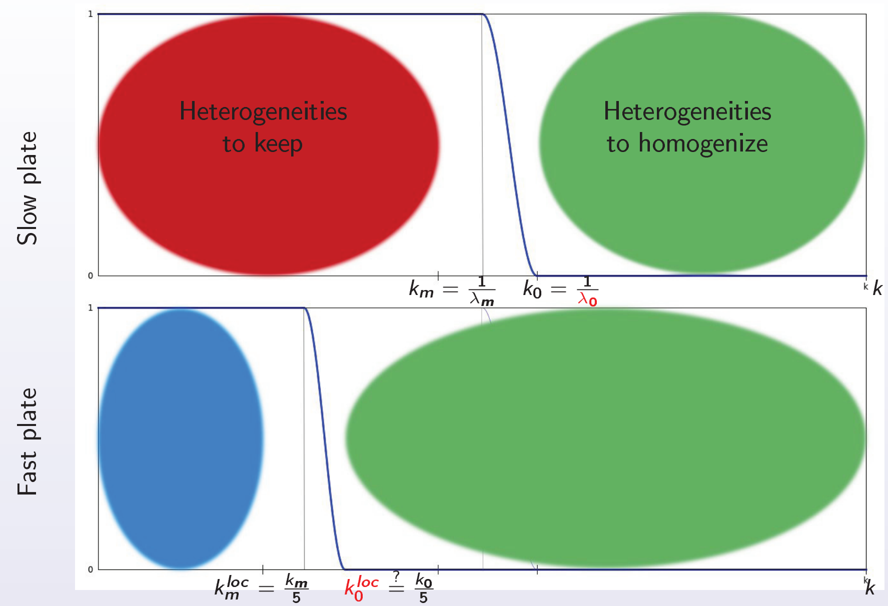
Fast area

$\lambda_m^{loc} \simeq 5\lambda_m \quad \Rightarrow \quad \varepsilon_0^{loc} = \frac{\varepsilon_0}{5}$

Problem : high spatial variability of λ_m

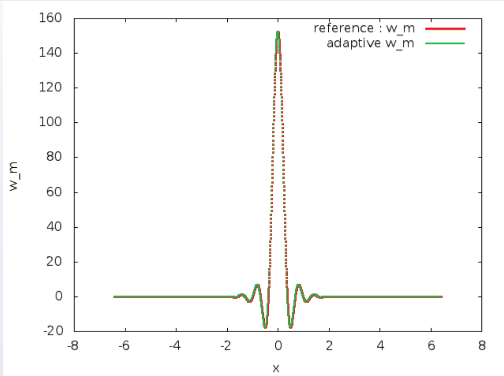


Problem : high spatial variability of λ_m



Solution : a geometrical transformation at step 2

- $\varepsilon = \frac{\lambda}{\lambda_m} < \varepsilon_0 = \frac{\lambda_0}{\lambda_m} << 1$
- Fast space variable : $y = \frac{\xi(x)}{\varepsilon} \Rightarrow$ distortion of the filtering wavelet w_m
- Filter : $F(u) = u * w_m = \int_{\mathbb{R}} u(\mathbf{x}, \frac{\xi(x')}{\varepsilon}) w_m \left(\frac{\xi(x) - \xi(x')}{\varepsilon} \right) |J_\xi| dx'$



Building ξ

Smooth variations for V_m

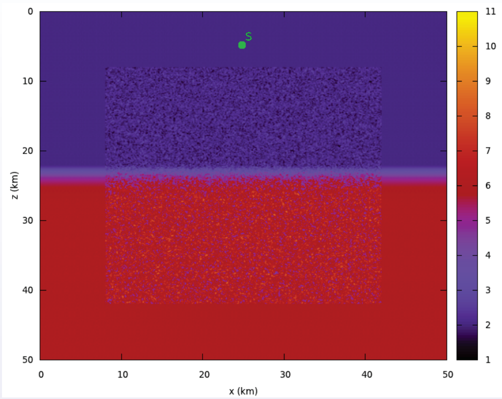
- Gaussian filtering ($\sigma >> \lambda_0$) $\Rightarrow \hat{V}_m$

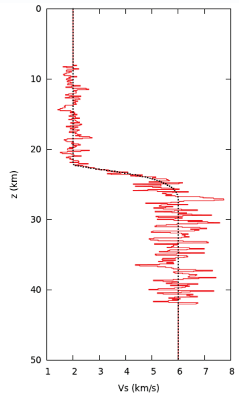
Building the parts of ξ we need

$$\text{Filter : } F(u) = u * w_m = \int_{\mathbb{R}} u(\mathbf{x}, \mathbf{y}') w_m \left(\frac{\xi(x) - \xi(x')}{\varepsilon} \right) |J_\xi| dx'$$

- w_m is axisymmetric $\Rightarrow w_m \left(\frac{\xi(x) - \xi(x')}{\varepsilon} \right) = w_m \left(\frac{\|\xi(x) - \xi(x')\|}{\varepsilon} \right)$
- $\hat{V}_m$ arises $\Leftrightarrow w_m$ expands $\Leftrightarrow \xi$ has to contract distances
- smooth variations for $\hat{V}_m$, order 0 approximation :
  $\|\xi(\mathbf{x}) - \xi(\mathbf{x}')\| \sim \frac{\min \hat{V}_m}{\hat{V}_m(\mathbf{x})} \times \|\mathbf{x} - \mathbf{x}'\|$

One academic test : plates





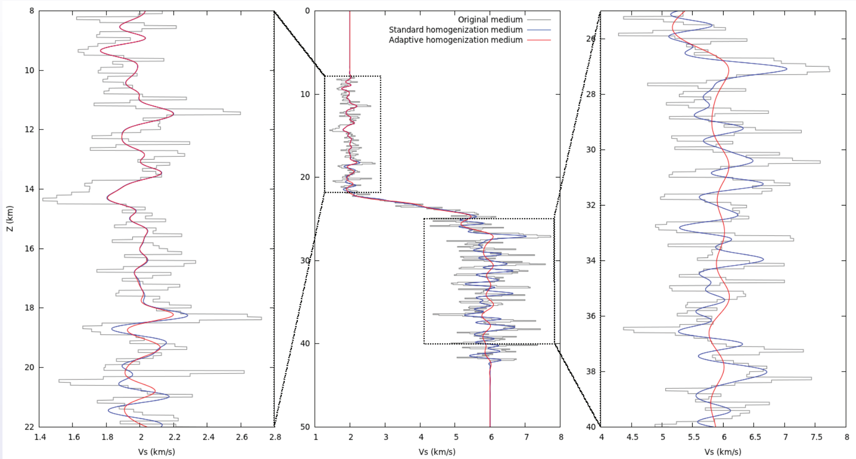
Configuration

- $\lambda_m = 1000\ m$
- $\lambda_0 = 500\ m\,(\varepsilon_0 = 0.5)$
- Slow plate : $\varepsilon_0^{opt} = \varepsilon_0 = 0.5$
- Fast plate : $\varepsilon_0^{opt} = \frac{\varepsilon_0}{3} = 0.166$

Homogenized model along a vertical cut

Slow plate

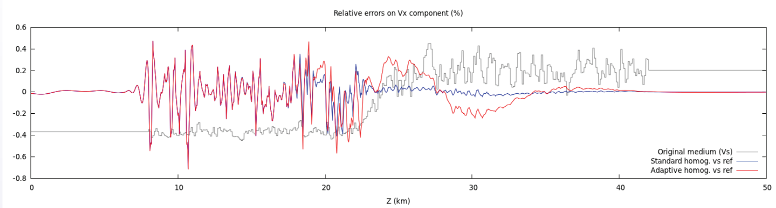
Fast plate



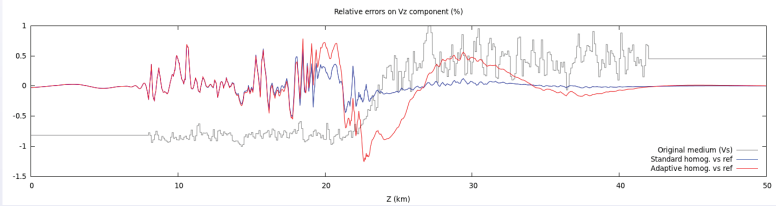
Vertical cut of V_s ($x = 20\ km$)

Waveforms

U_x



U_z



Relative velocity errors (vertical cut) : $\frac{U_{\boldsymbol{ref}} - U^{\mathbf{0},\varepsilon}}{|U^{\mathbf{0},\varepsilon}|} = \varepsilon\chi^1 :\in \left(\frac{U^{\mathbf{0},\varepsilon}}{|U^{\mathbf{0},\varepsilon}|}\right)$

To conclude

A first order adaptive homogenization process has been built

Next steps

- Testing on realistic models,
- Higher order approximation.

Thank you for your attention !