ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Towards Efficient Global Wave Propagation in 3D Media Using Scattering Integrals

Third QUEST Workshop - Tatranska Lomnica

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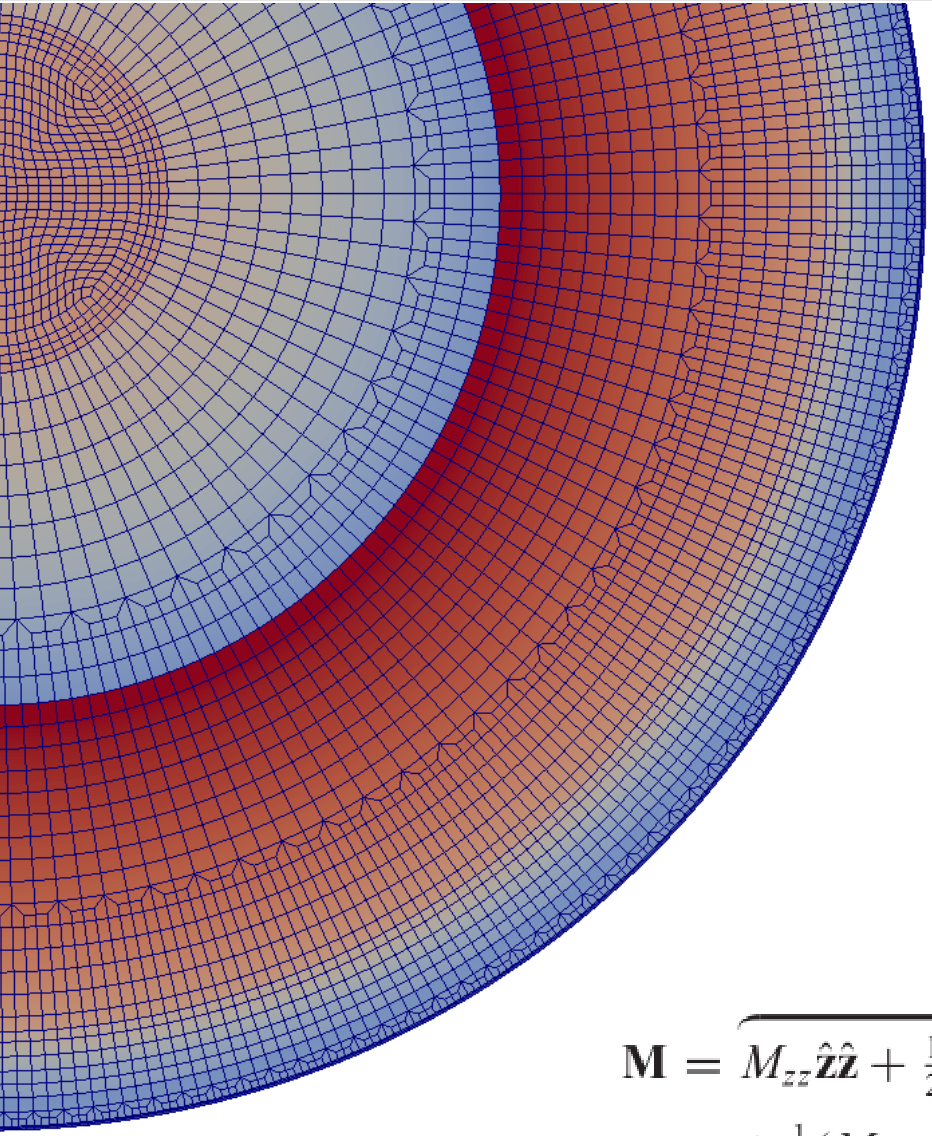
AXISEM



- Spectral Element Method
- 2.5D (axisymmetric) earth models
- Four 2D simulations allow to construct full 3D Wavefields

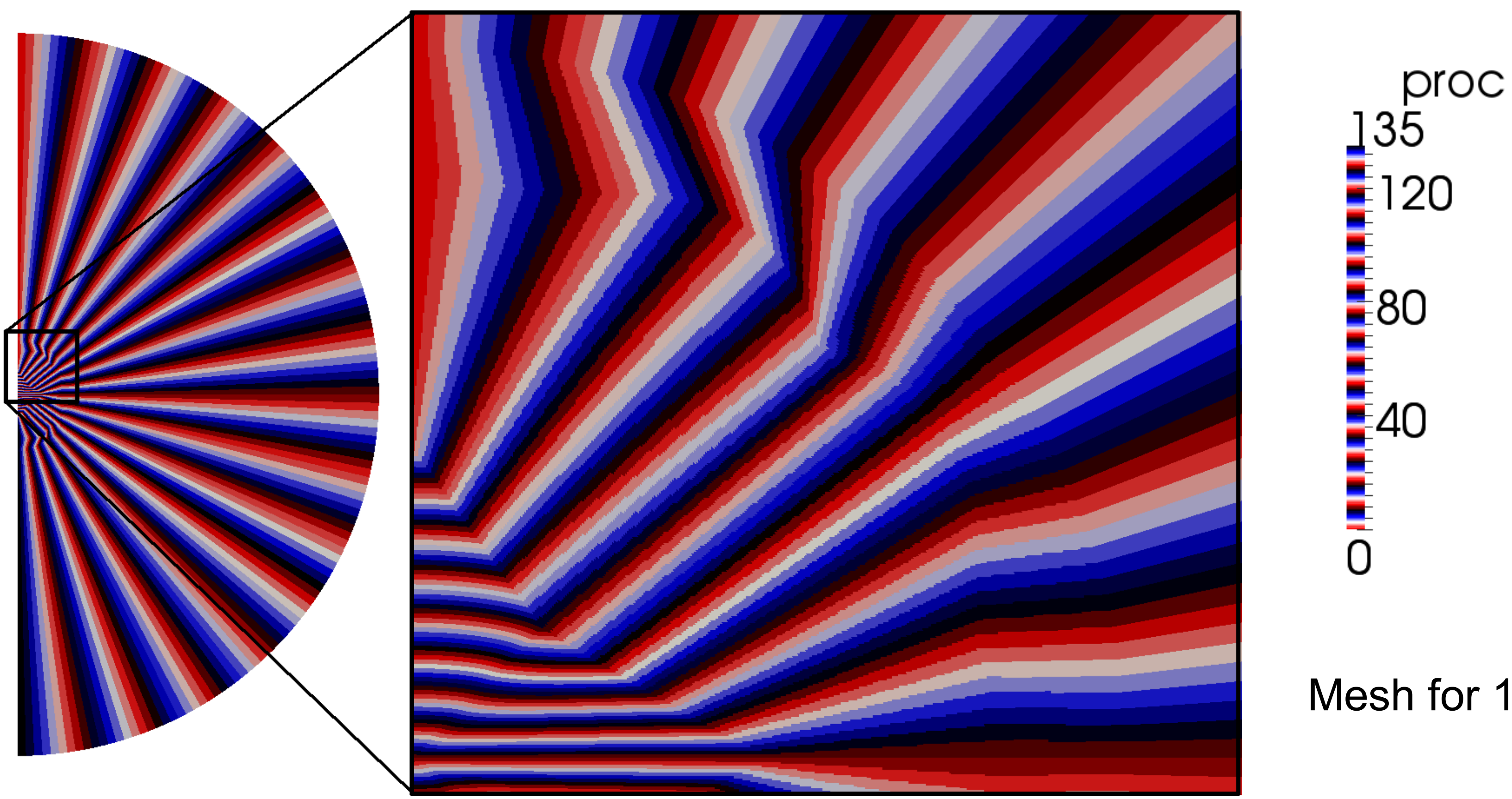
$$\mathbf{M} = \overbrace{M_{zz}\hat{\mathbf{z}}\hat{\mathbf{z}} + \frac{1}{2}(M_{xx} + M_{yy})(\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}})}^{\text{monopole}} + \overbrace{M_{xz}(\hat{\mathbf{x}}\hat{\mathbf{z}} + \hat{\mathbf{z}}\hat{\mathbf{x}}) + M_{yz}(\hat{\mathbf{y}}\hat{\mathbf{z}} + \hat{\mathbf{z}}\hat{\mathbf{y}})}^{\text{dipole}}$$
$$+ \underbrace{\frac{1}{2}(M_{xx} - M_{yy})(\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}}) + M_{xy}(\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}})}_{\text{quadrupole}}.$$

Nissen-Meyer et al (2007 & 2008)

AXISEM

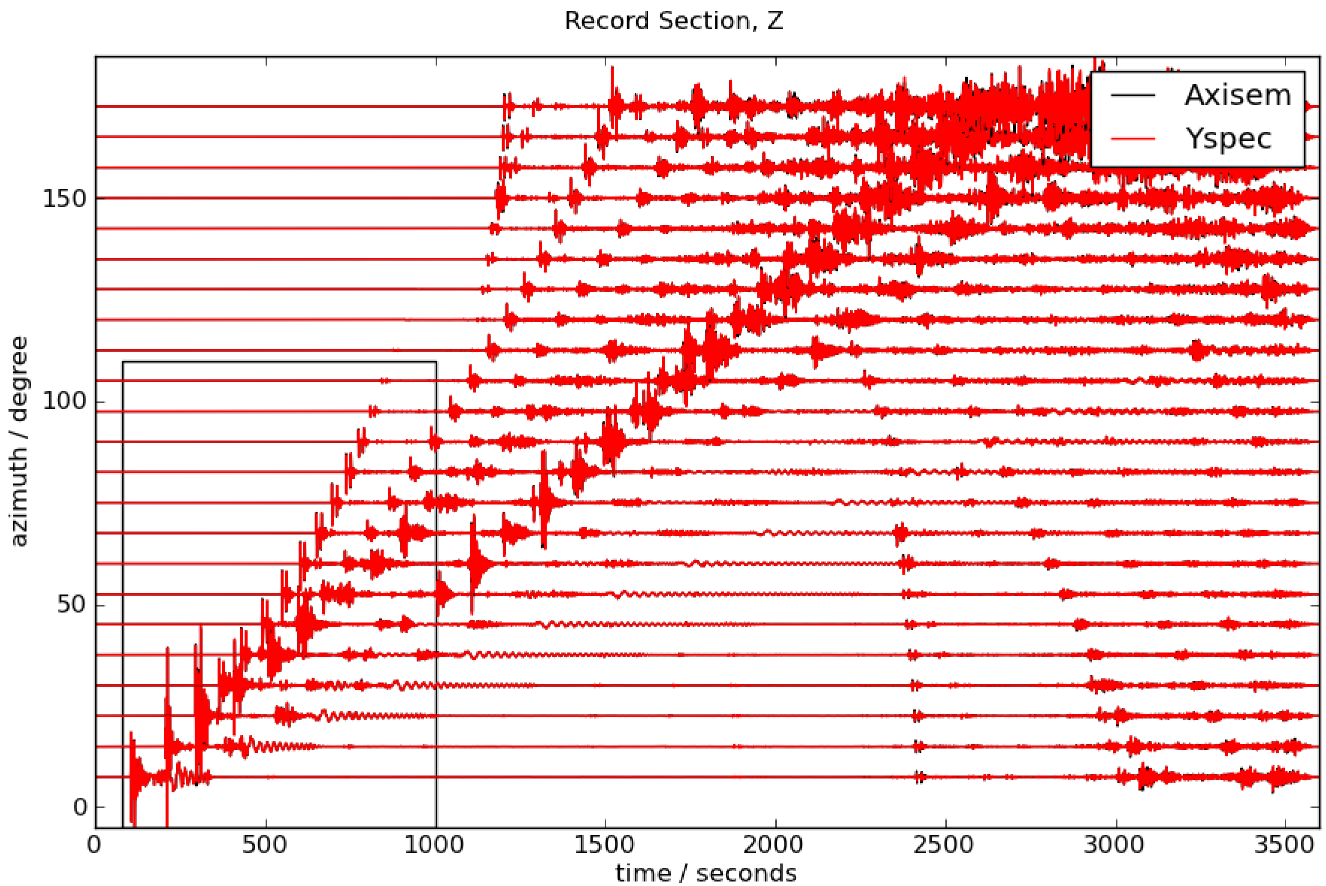
Recent Developments:

- Full Anisotropy (21 independent coefficients – but still 2.5D!)
- Parallelization for more than 16 processes



Mesh for 136 cores @ 2s

High Frequency Benchmark



- Explosive source
- Halfduration 2s, low pass filter at 2s
- PREM (anisotropic)
- AXISEM: 136 cores (~ 8h runtime)

Yspec:

- Al-Attar & Woodhouse (2008)
- Direct radial integration method
  Friederich & Dalkolmo (1995)

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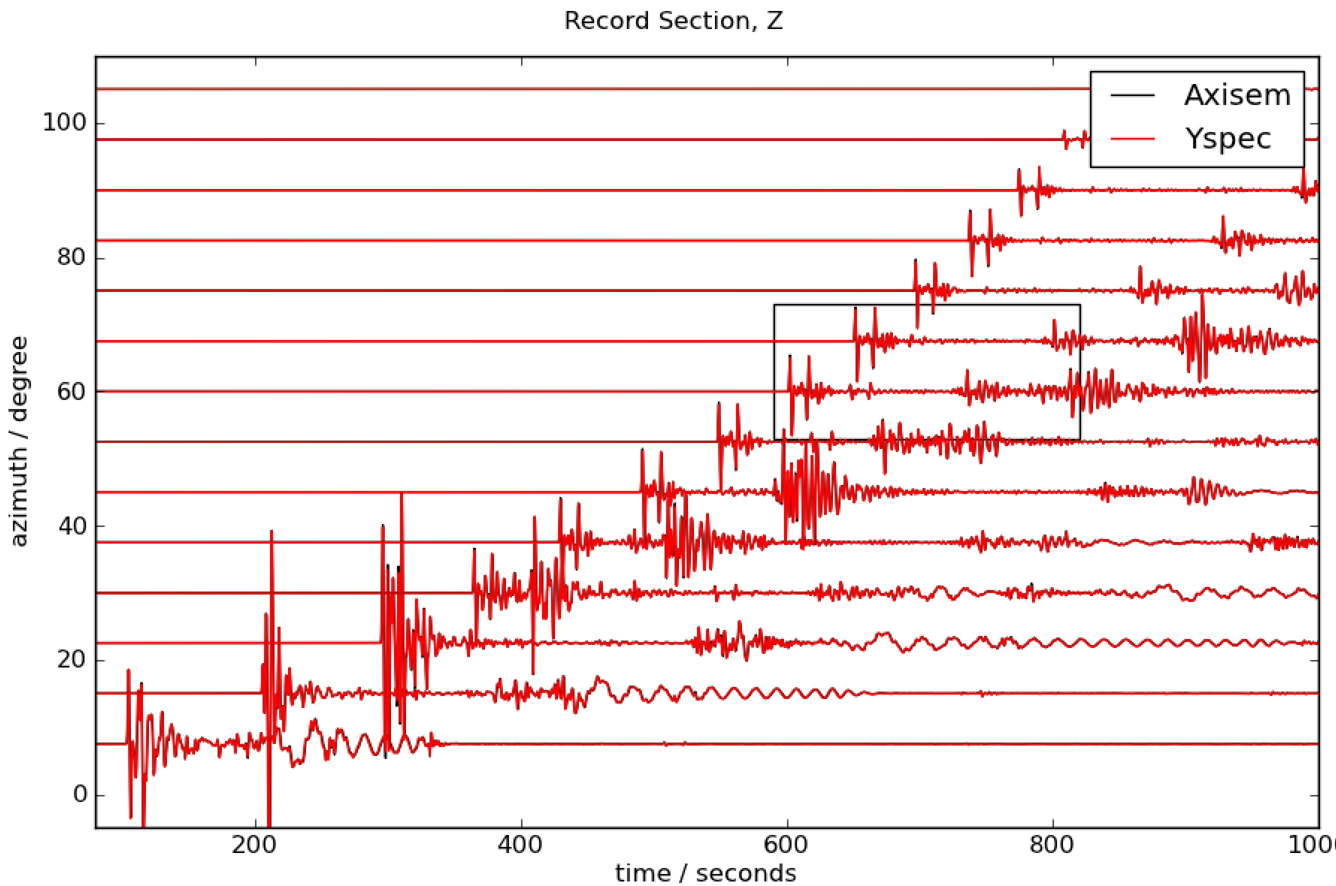

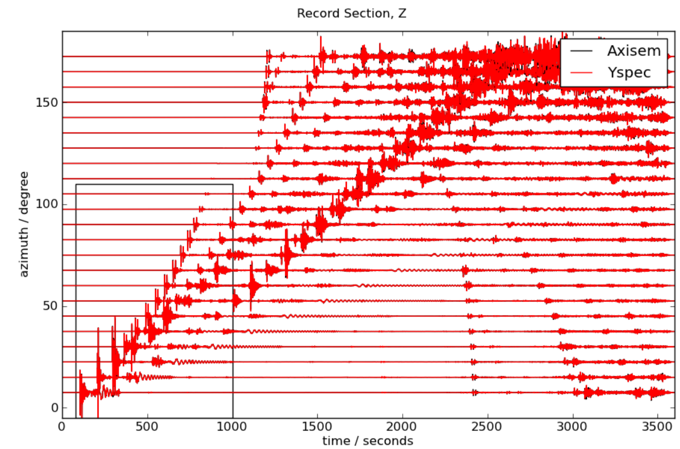

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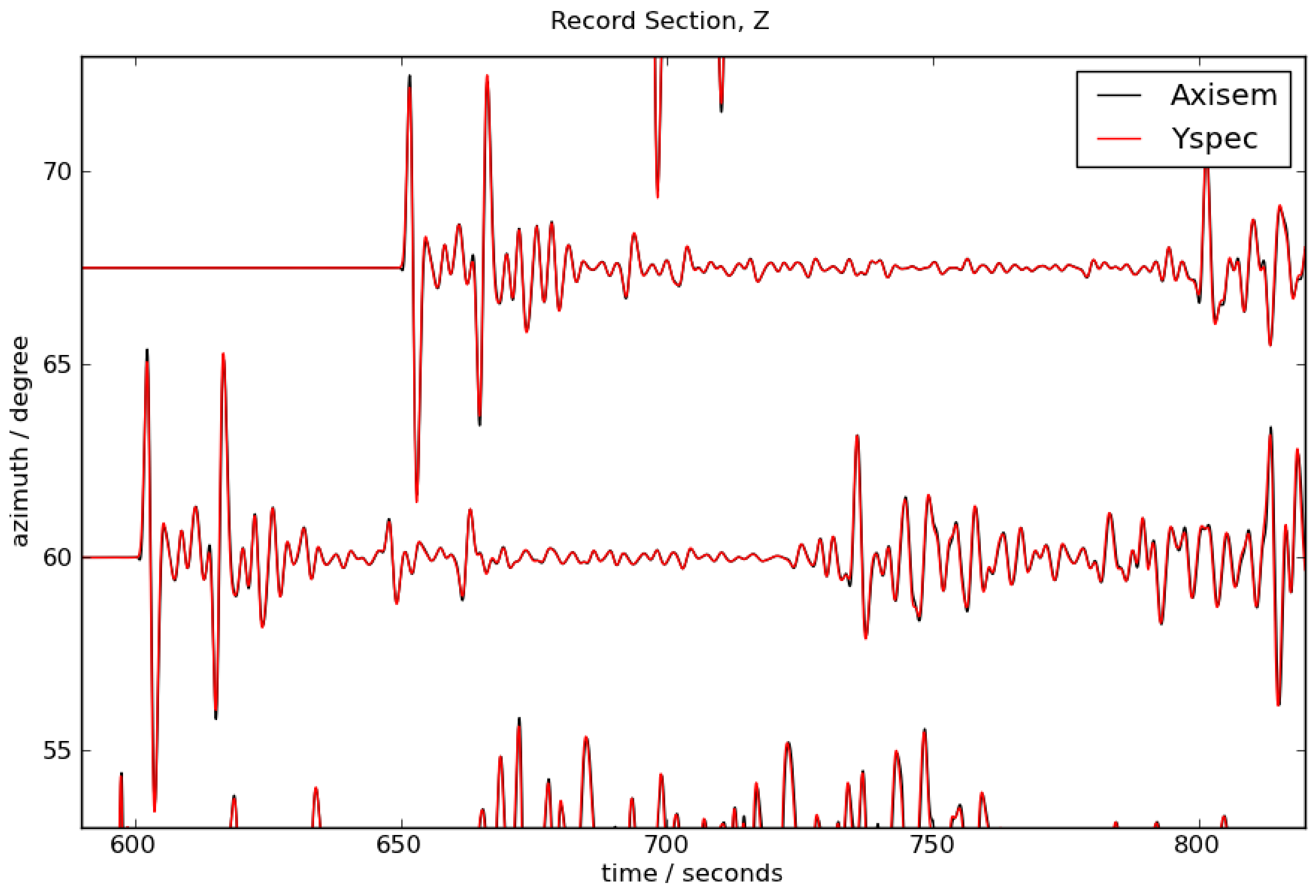

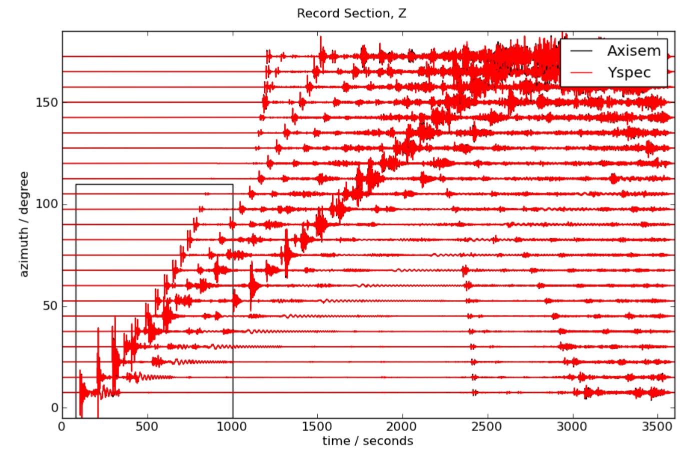

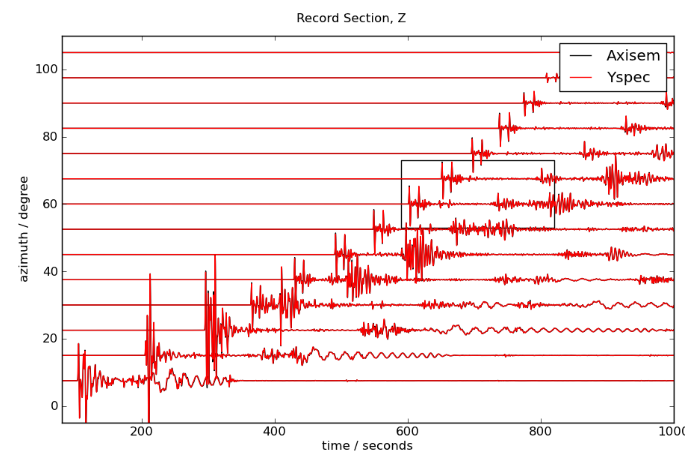

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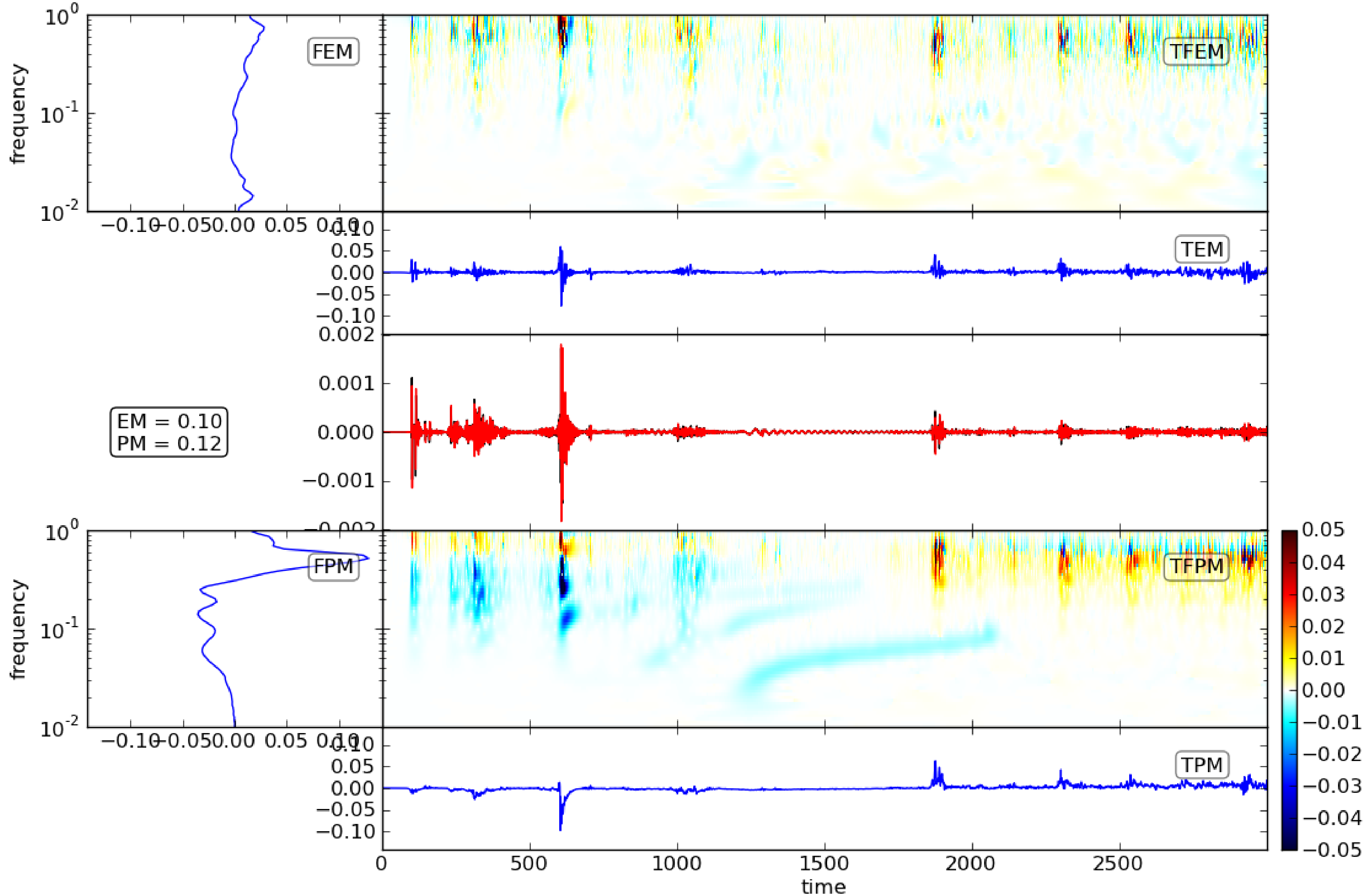
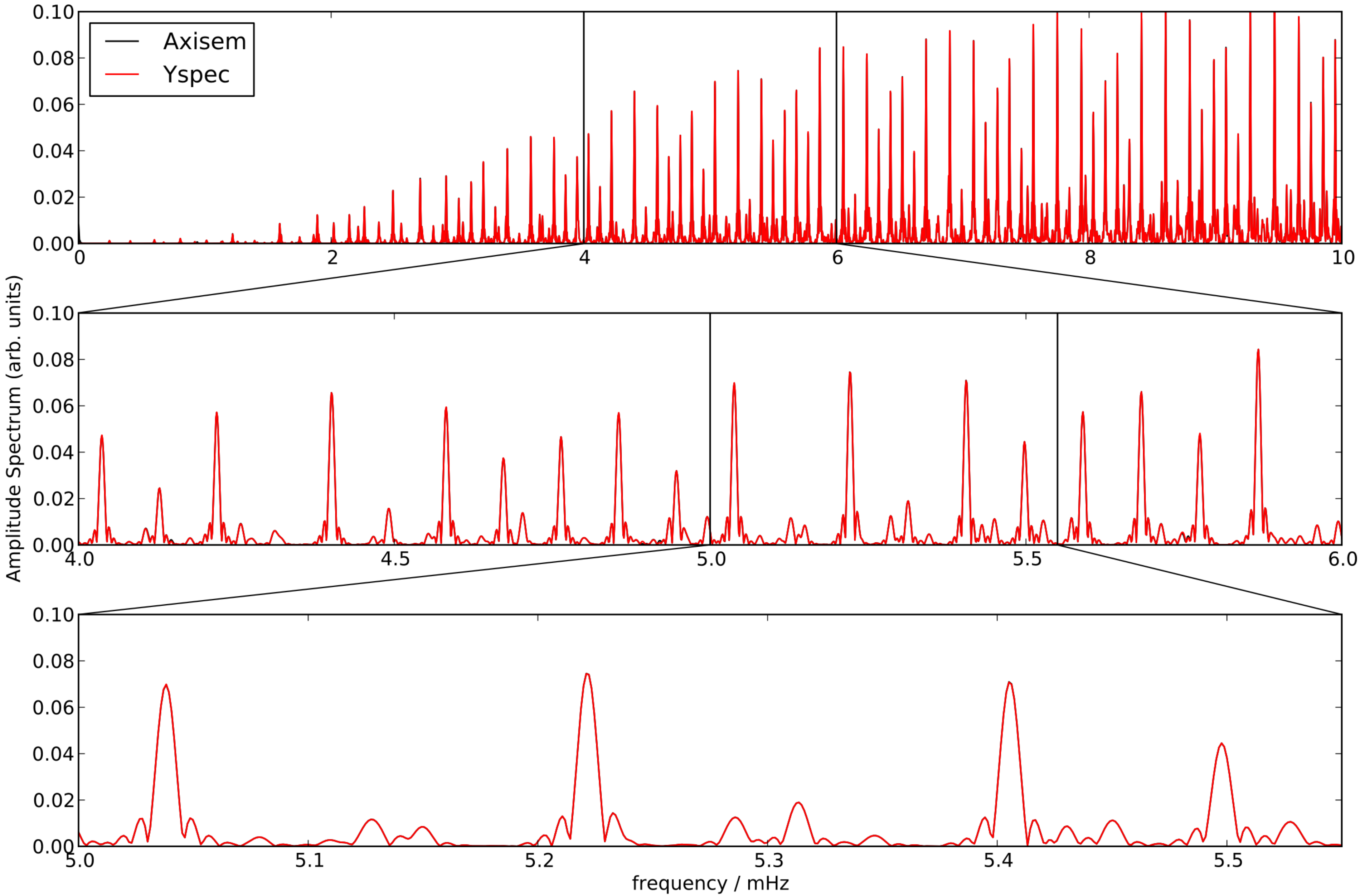


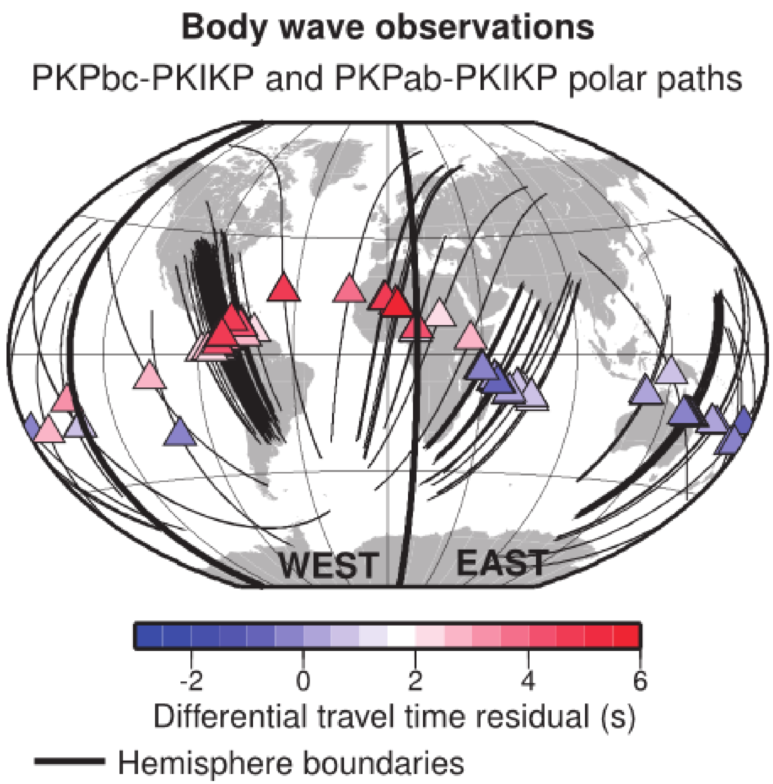
Time-frequency misfit criteria - Kristeková et al (2006 + 2009)
- now also available in ObsPy (www.obspy.org)

Low Frequency Benchmark

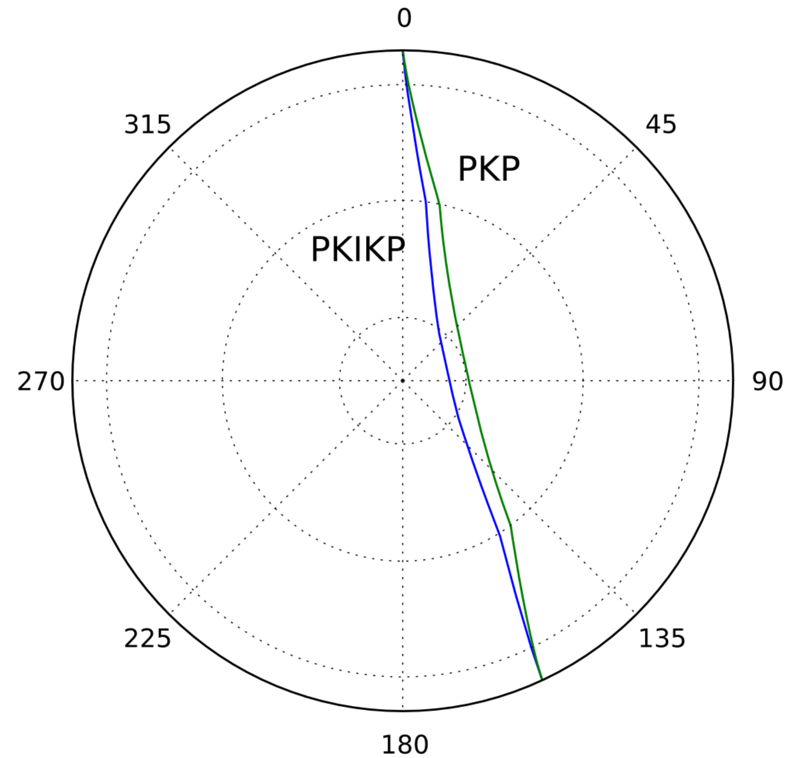


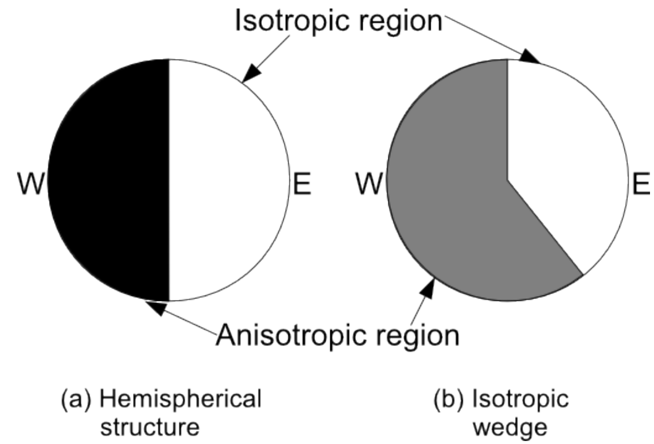
Amplitude spectrum of 48 hours of seismograms: 1.7 million timesteps

2.5D Application



A. Deuss et. al. (2010)





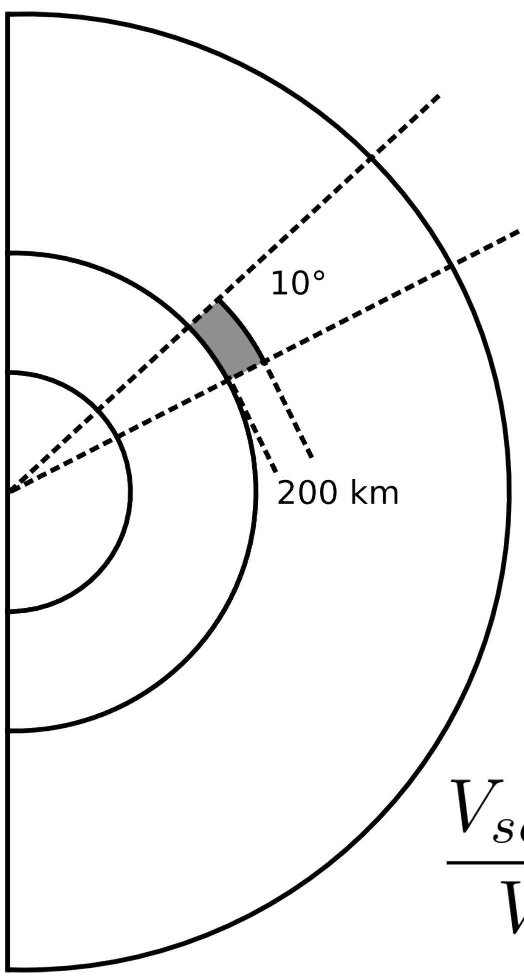
J. Irving et. al. (2009)

- Typically observed at 1-2s period
- Forward Solver used until now: taup

What about 3D structures?

Scattering Integrals: Motivation

Wave Propagation Solver that ...



- … exploits the fact that 3D seismic velocity anomalies are small in amplitude and volume for many applications
- … couples cost and complexity

$$\frac{V_{scatterer}}{V_{earth}} = 0.7 \times 10^{-3}$$

Scattering Integrals: Theory

1D wave equation in frequency domain:

$$-\rho\omega^2 u - \partial_x(\mu \partial_x u) = f$$

Perturbed model:

$$\mu = \mu_0 + \delta\mu$$

Scattering Integrals: Theory

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Scattering Integral Equation: $$u = u_0 + \hat{S}u$$

$$\hat{S}u = \int G_0(x, x')(\partial_{x'}(\delta\mu\partial_{x'}u(x')))dx'$$

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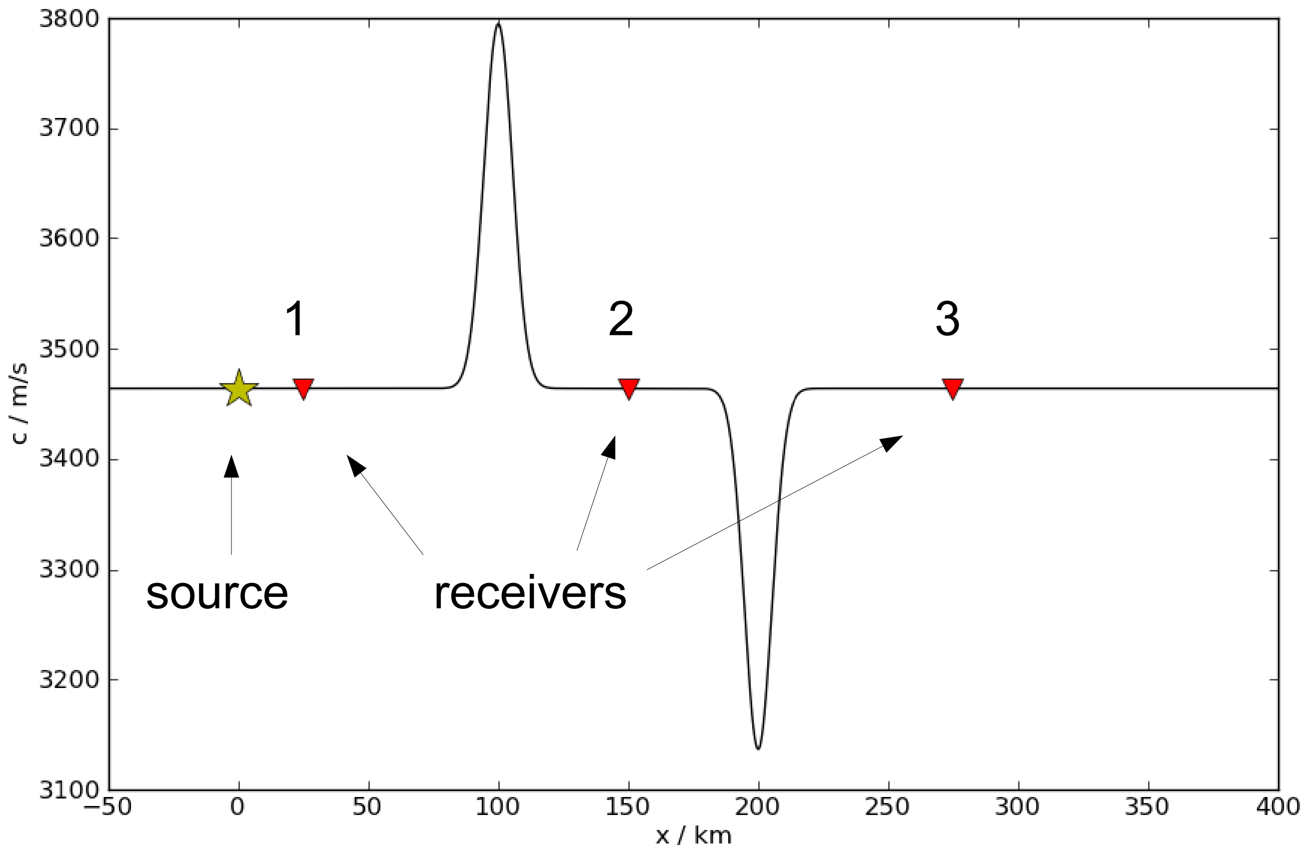
Iterative solution in Neumann Series:

$$u = \sum_n \hat{S}^n u_0$$

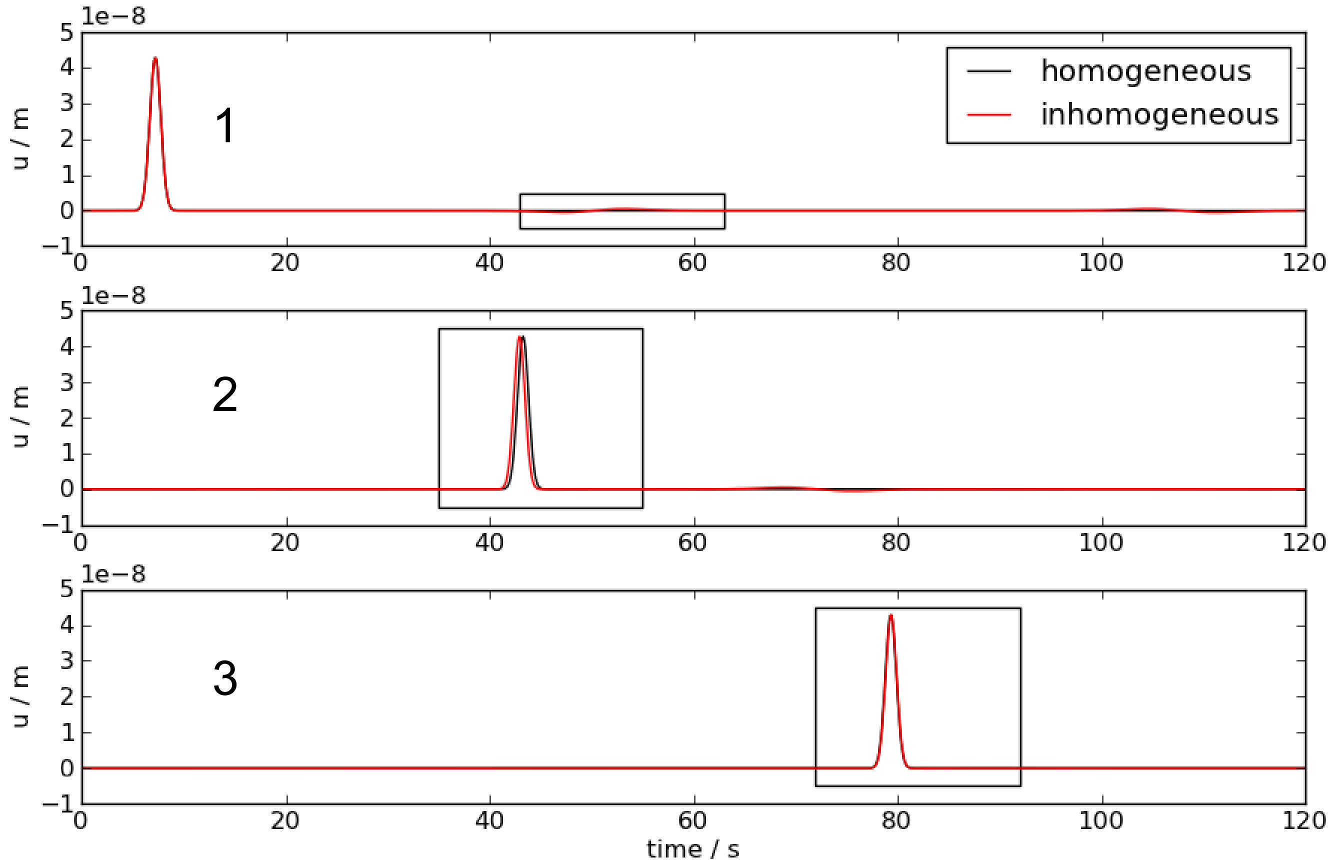
Scattering Integrals: Example I

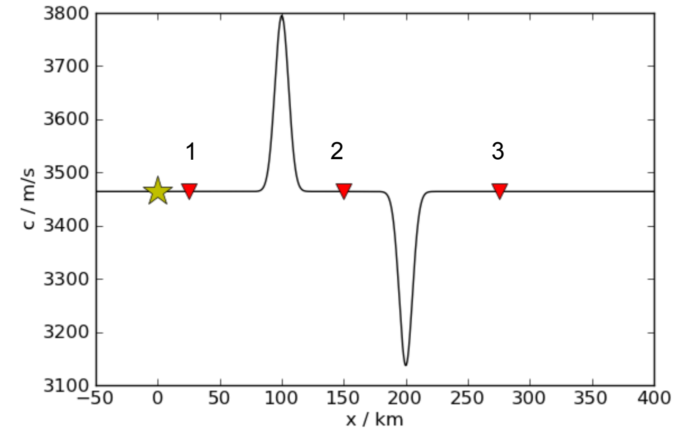


1D velocity model:

- 10% velocity perturbation
- Gaussian Scatterers (width ~ 4 wavelengths at the highest frequencies)

Scattering Integrals: Example I





SPECFEM1D as reference and for computation of Green's functions

Only small differences between homogeneous and inhomogeneous seismograms

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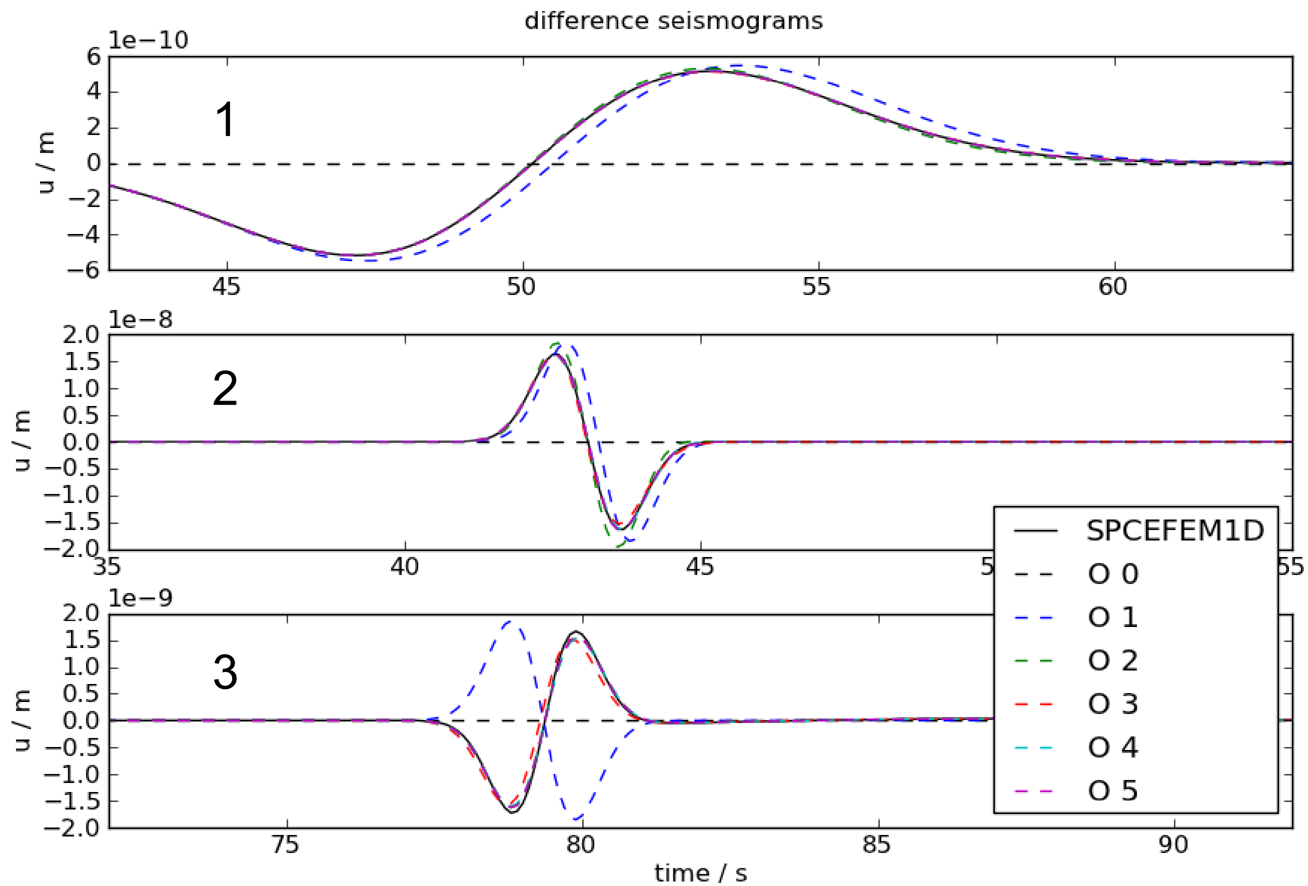
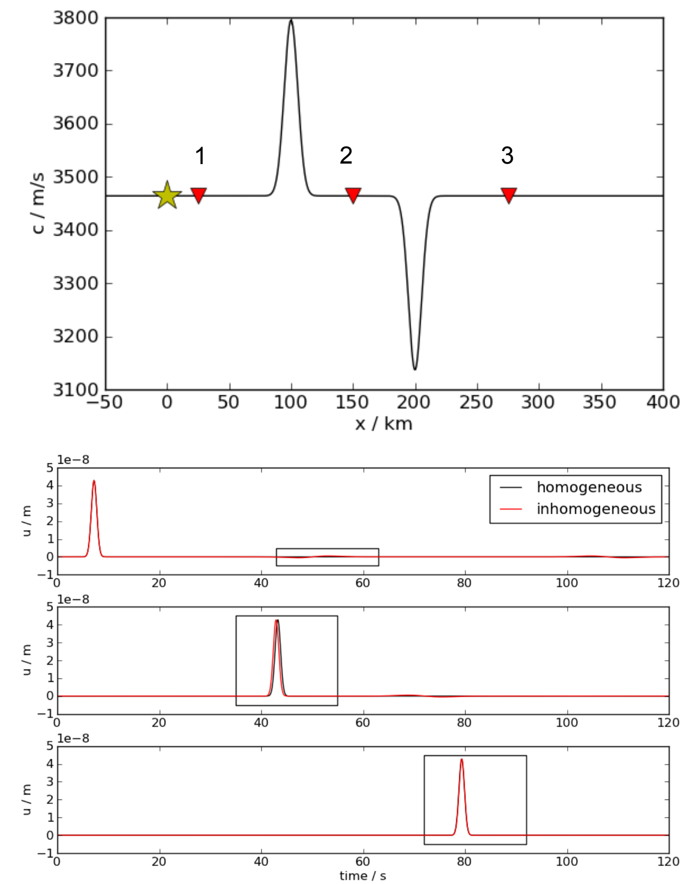
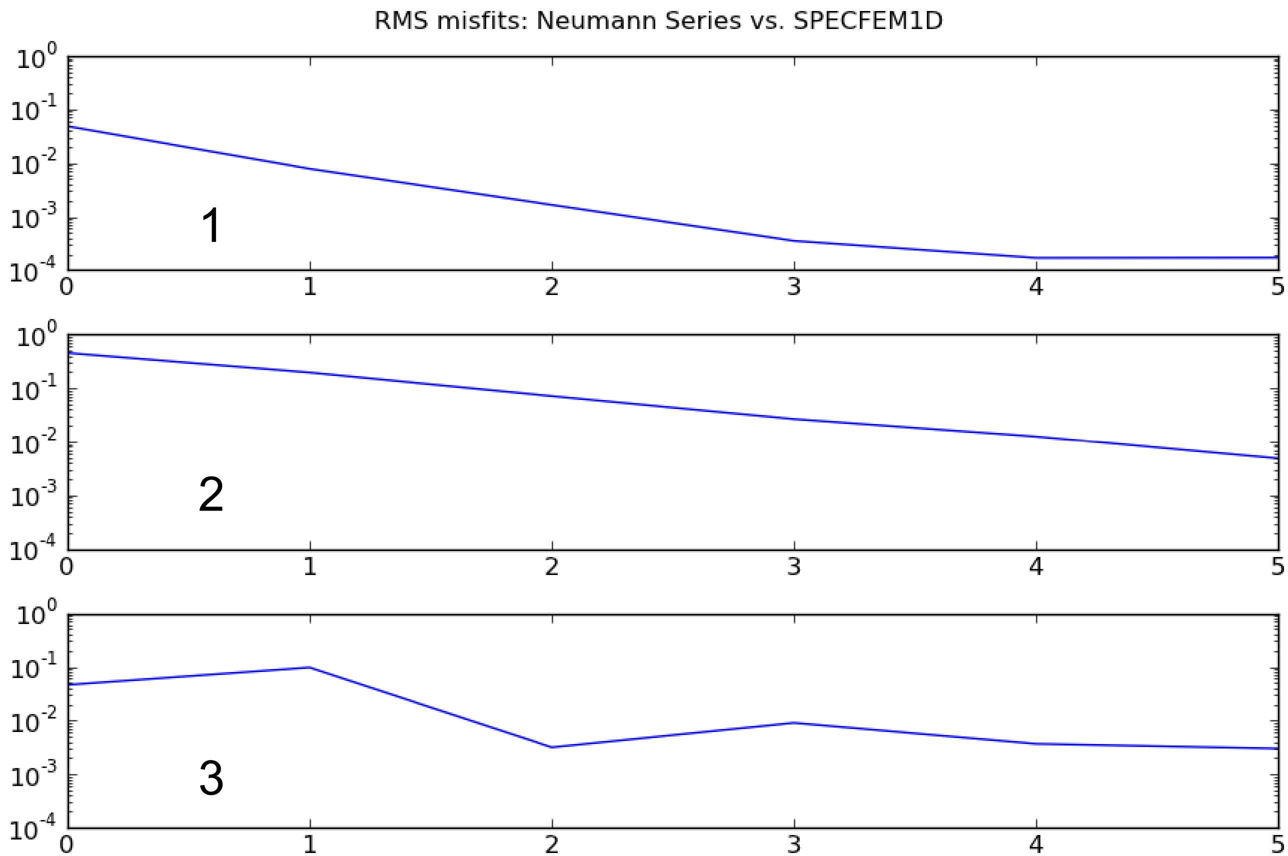


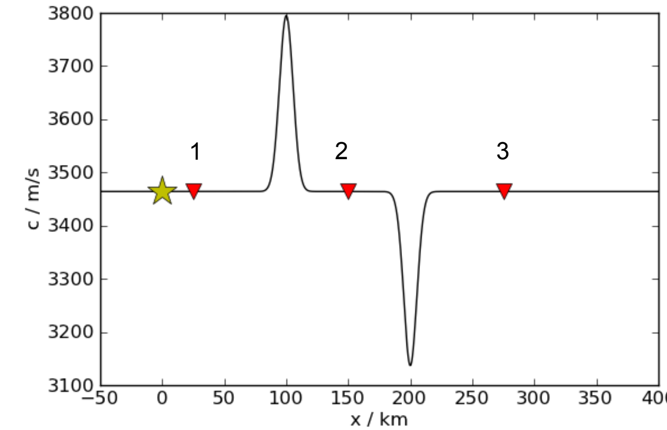


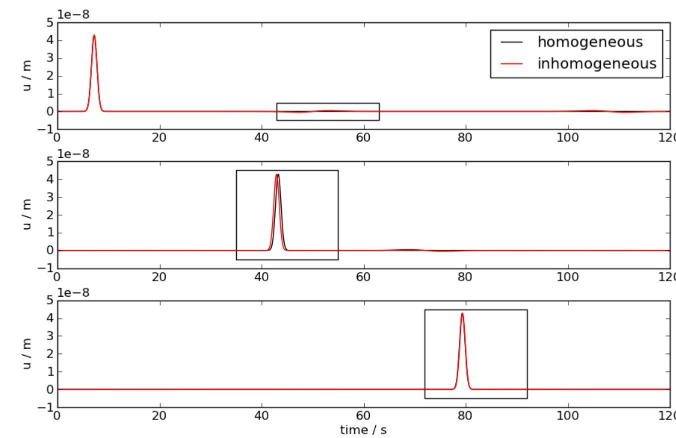
Difference seismograms:

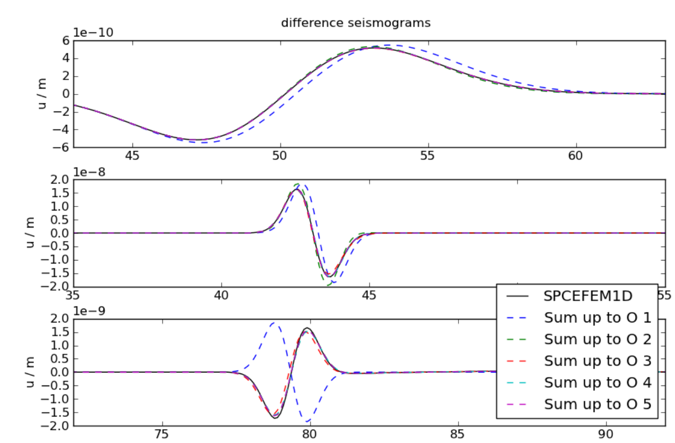
$$\delta u_k = \sum_{n=0}^{k} \hat{S}^n u_0 - u_0$$

Scattering Integrals: Example I



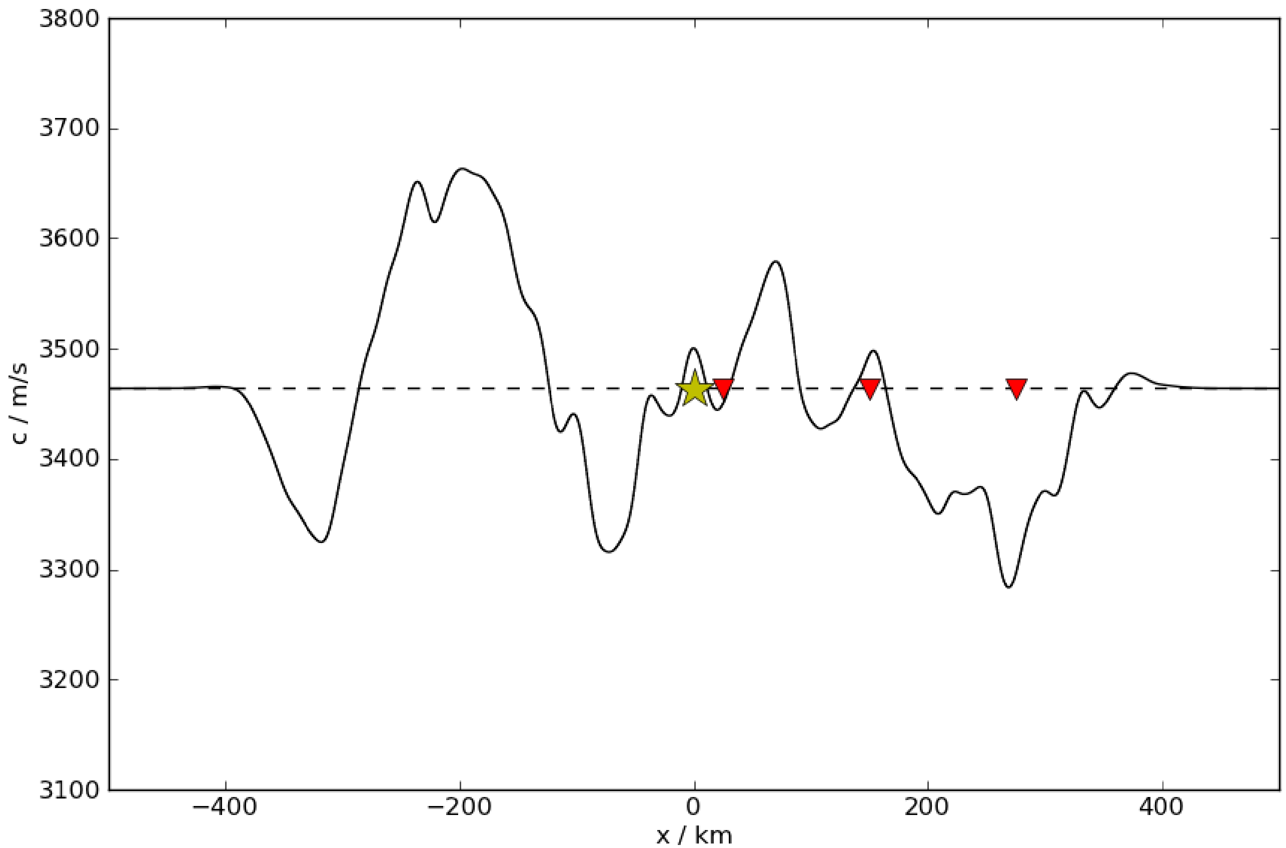






→ Neumann series converges!

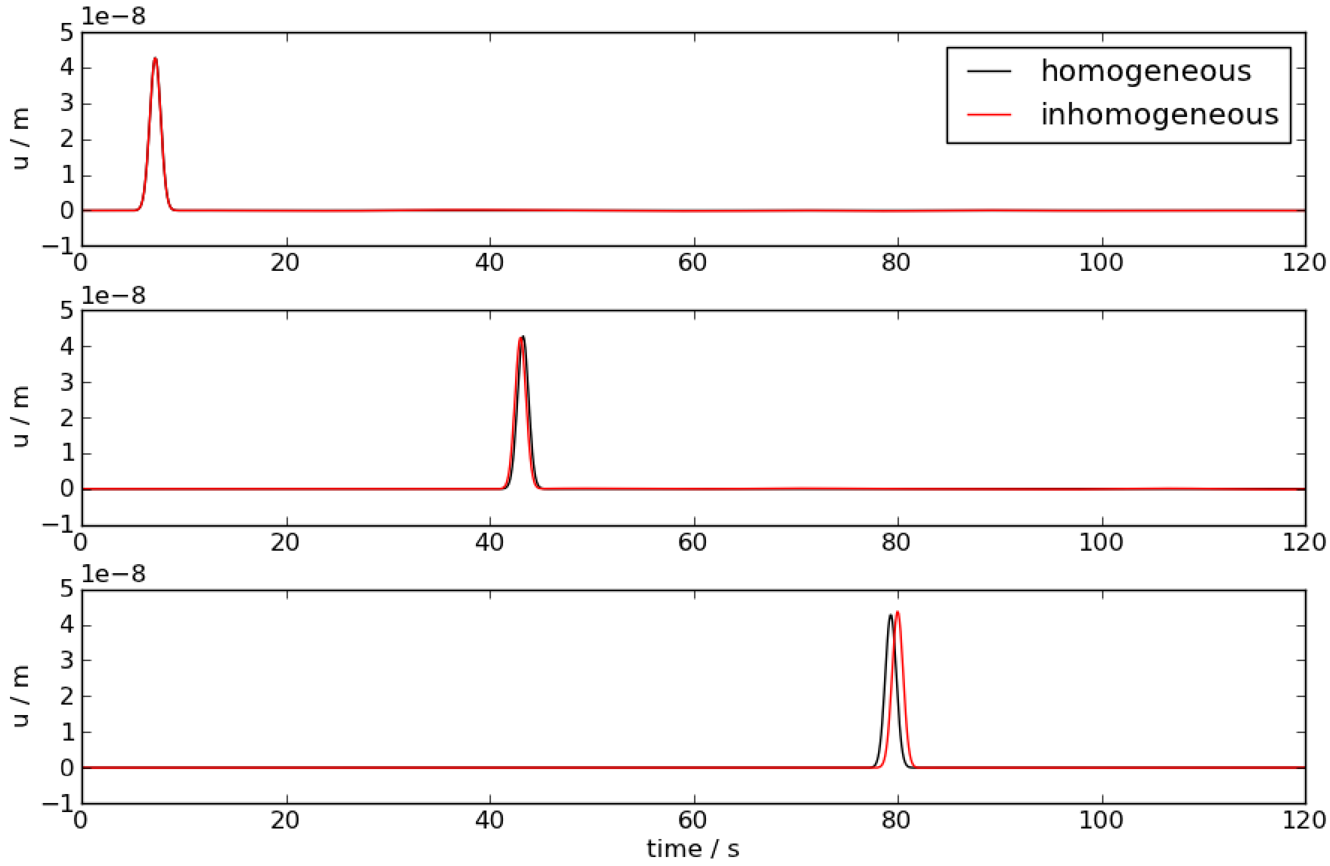
Scattering Integrals: Example II

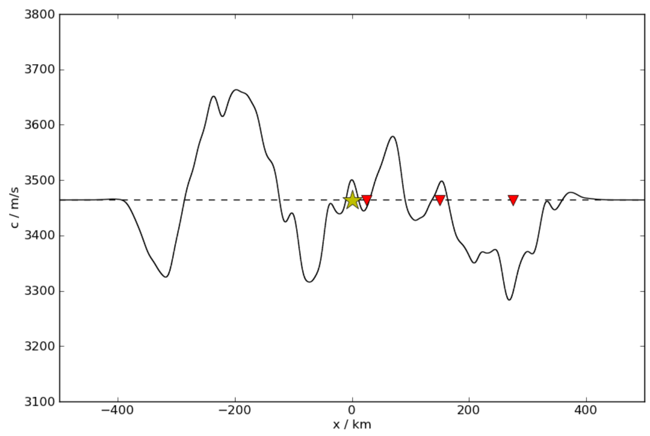


1D velocity model:

- Gaussian medium with correlation length 100km
- 5% velocity perturbation

Scattering Integrals: Example II





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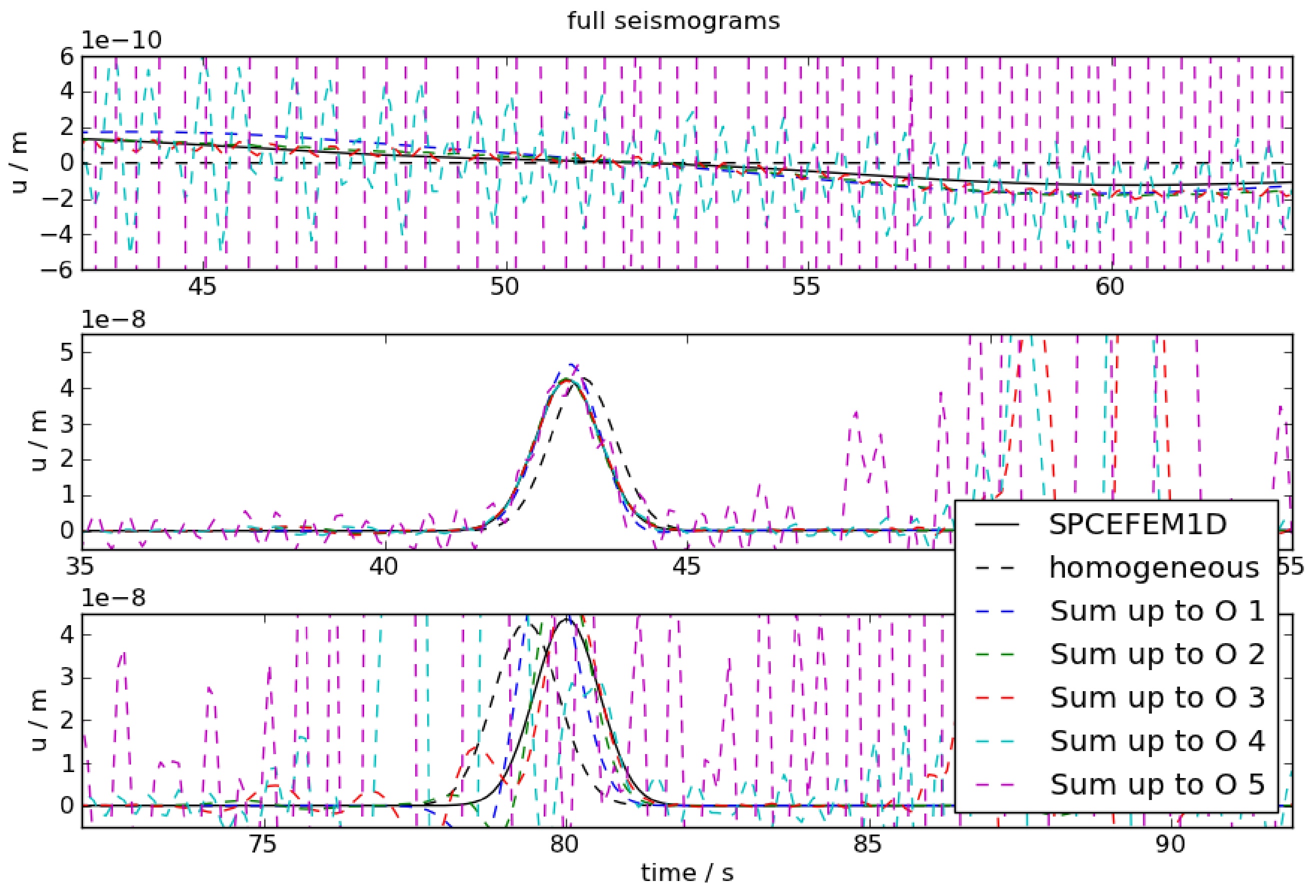

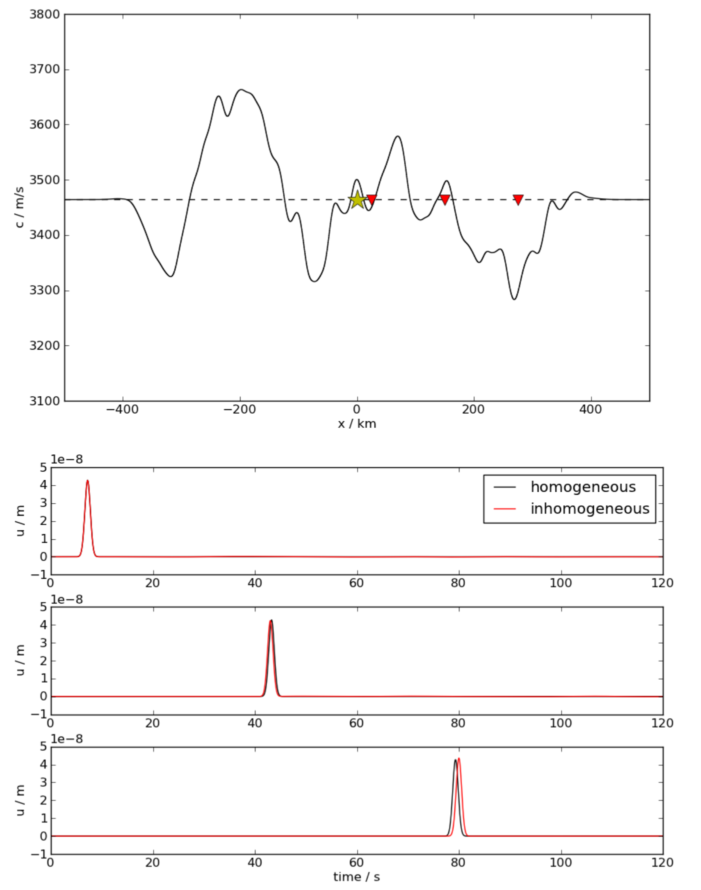

Neumann Series diverges!

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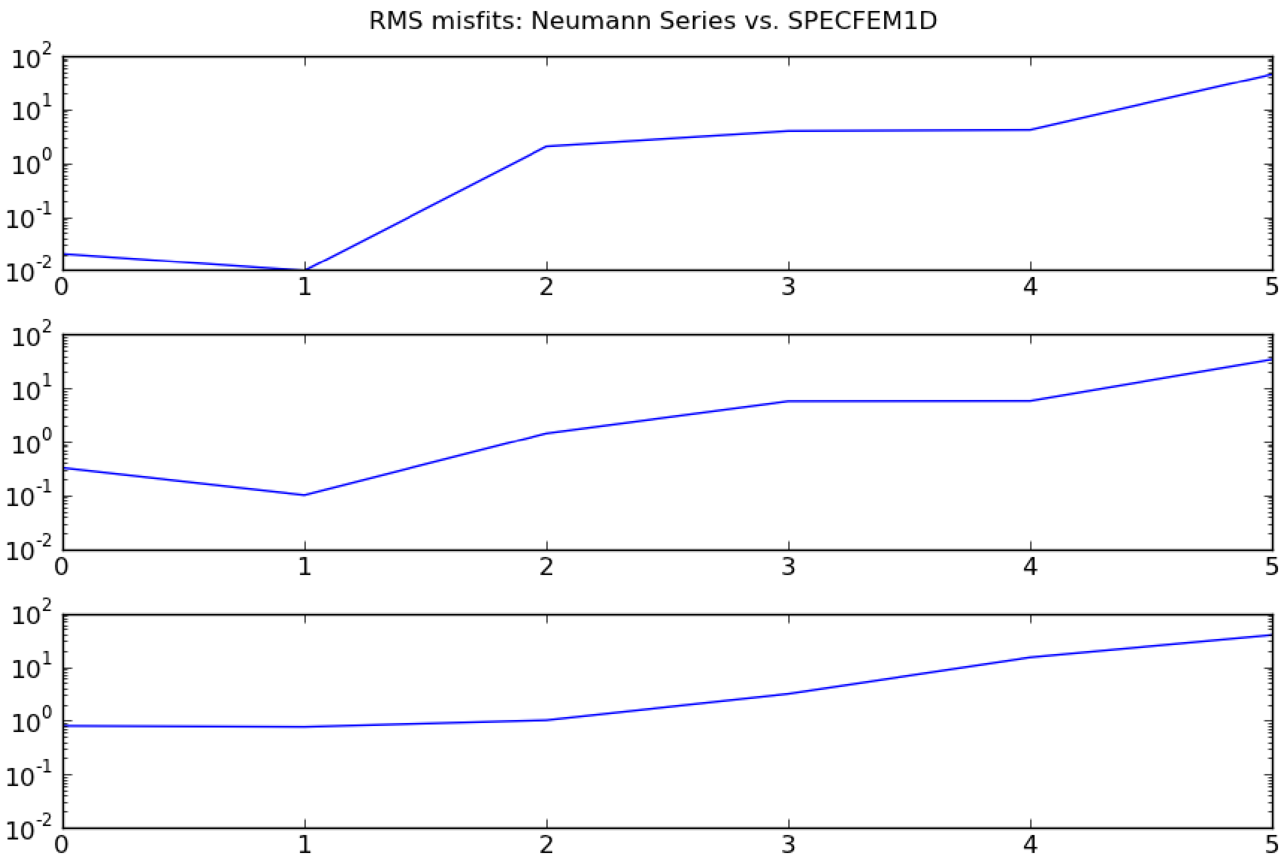

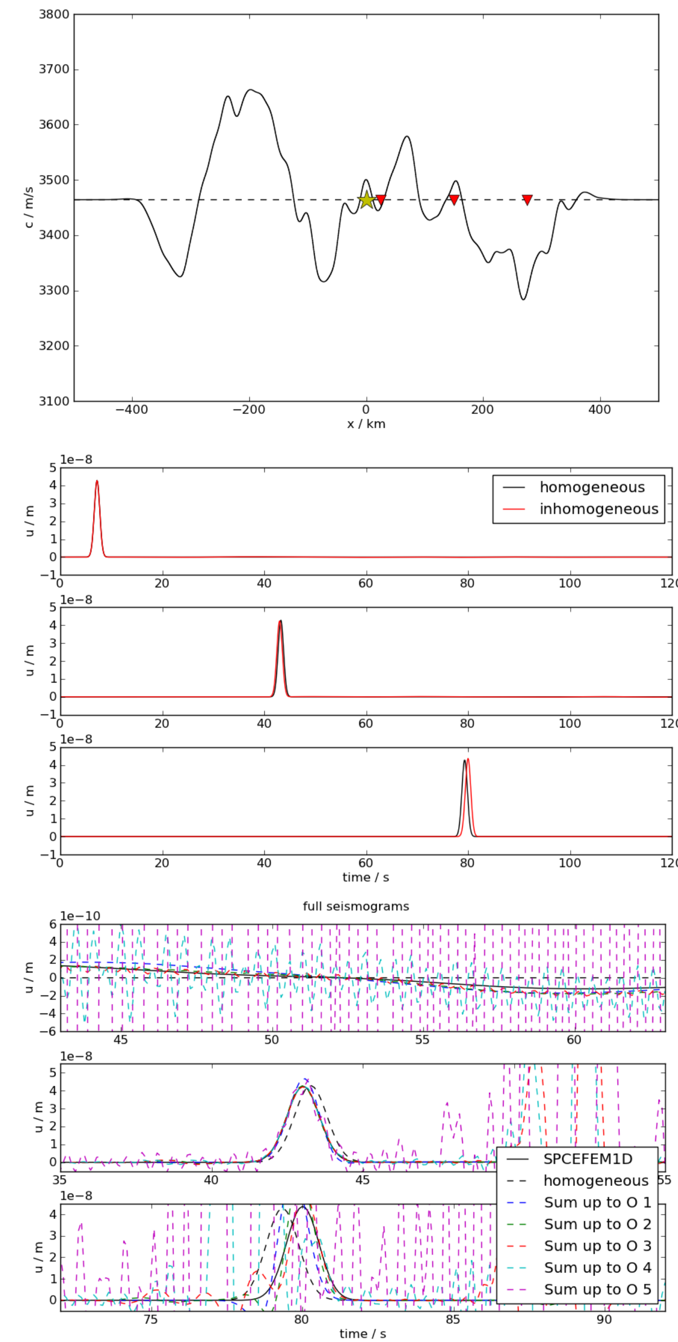

Neumann Series diverges!

→ this does not mean, that there exists no solution to the Integral Equation

→ need for a smarter way to solve it

Iterative Dissipative Method (IDM)

For Maxwell Equations: Singer (1995) and Pankratov & Avdeyev (1995)

Energy Relation:

$$\int \left(|\sqrt{-\mathrm{Im}(\mu)}\partial_x u|^2 \right) dx = \int \mathrm{Im}\left(F \partial_x u^* \right) dx$$

Period average dissipated Energy

Period average Energy introduced by the source

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Modified Scattering Operator:

$$||\hat{K}\eta|| = ||\eta||$$

Corresponding Integral Equation:

$$\chi = \chi_0 + \hat{K} R \chi \qquad (R < 1)$$

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Unconditionally convergent Neumann Series:

$$\chi = \sum_n (\hat{K}R)^n \chi_0$$

→ this defines a well posed method to solve the wave equation in inhomogeneous dissipative media

Conclusions

- Successful benchmarks comparing AXISEM and Yspec for high and low frequencies
- Neumann Series for the 1D Scattering Integral Equation diverges even for small perturbations
- Basic concept of Iterative Dissipative Method for solution of the wave equation