Geometric Non-Uniqueness in Seismic Tomography

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Let (M, g) be a compact Riemannian manifold with boundary ∂M. We consider the scalar wave equation

$$\rho \partial_t^2 u - \mathrm{div}_g (K \cdot \mathrm{grad}_g u) = -\mathrm{div}_g m$$

subject to boundary conditions

$$(K \cdot \mathrm{grad}_g u, n)_{TM} = 0 \quad \text{on} \quad \partial M$$

and initial conditions

$$u = \partial_t u = 0 \quad \text{for} \quad t = 0$$

For $\xi \in \mathrm{Diff}(M)$ we define

$$\tilde{u} = \xi^* u = u \circ \xi$$

It may be shown that $\tilde{u}$ satisfies

$$\tilde{\rho} \partial_t^2 \tilde{u} - \mathrm{div}_g (\tilde{K} \cdot \mathrm{grad}_g \tilde{u}) = -\mathrm{div}_g \tilde{m}$$

subject to boundary conditions

$$\left(\tilde{K} \cdot \mathrm{grad}_g \tilde{u}, n\right)_{TM} = 0 \quad \text{on} \quad \partial M$$

and initial conditions

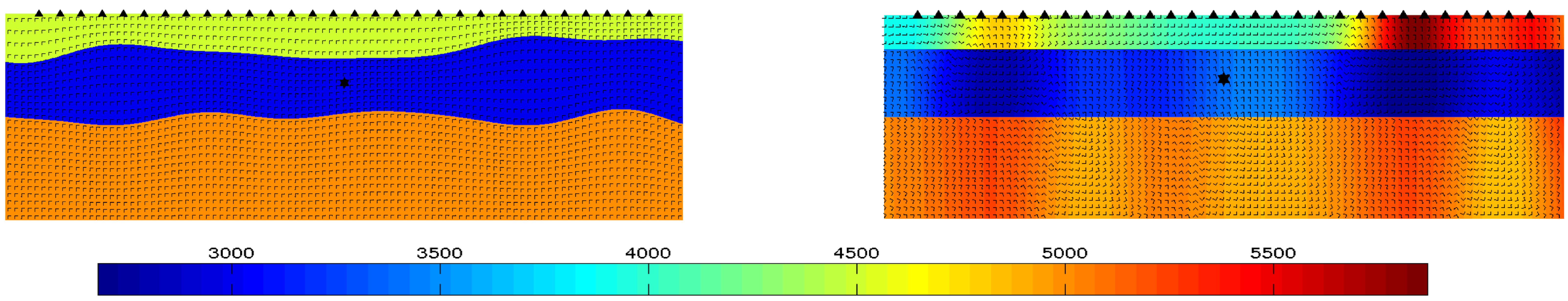
$$\tilde{u} = \partial_t \tilde{u} = 0 \quad \text{for} \quad t = 0$$

where we have introduced

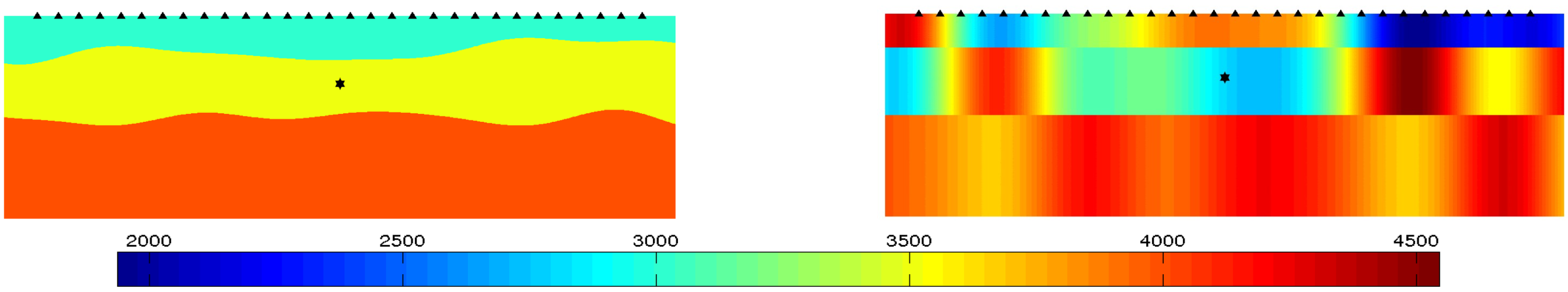
$$\tilde{\rho} = J\xi^* \rho \qquad \tilde{K} = J\xi^* K \cdot C^{-1}$$

$$\tilde{m} = J\xi^* m \qquad C = g^{-1} \cdot \xi^* g$$

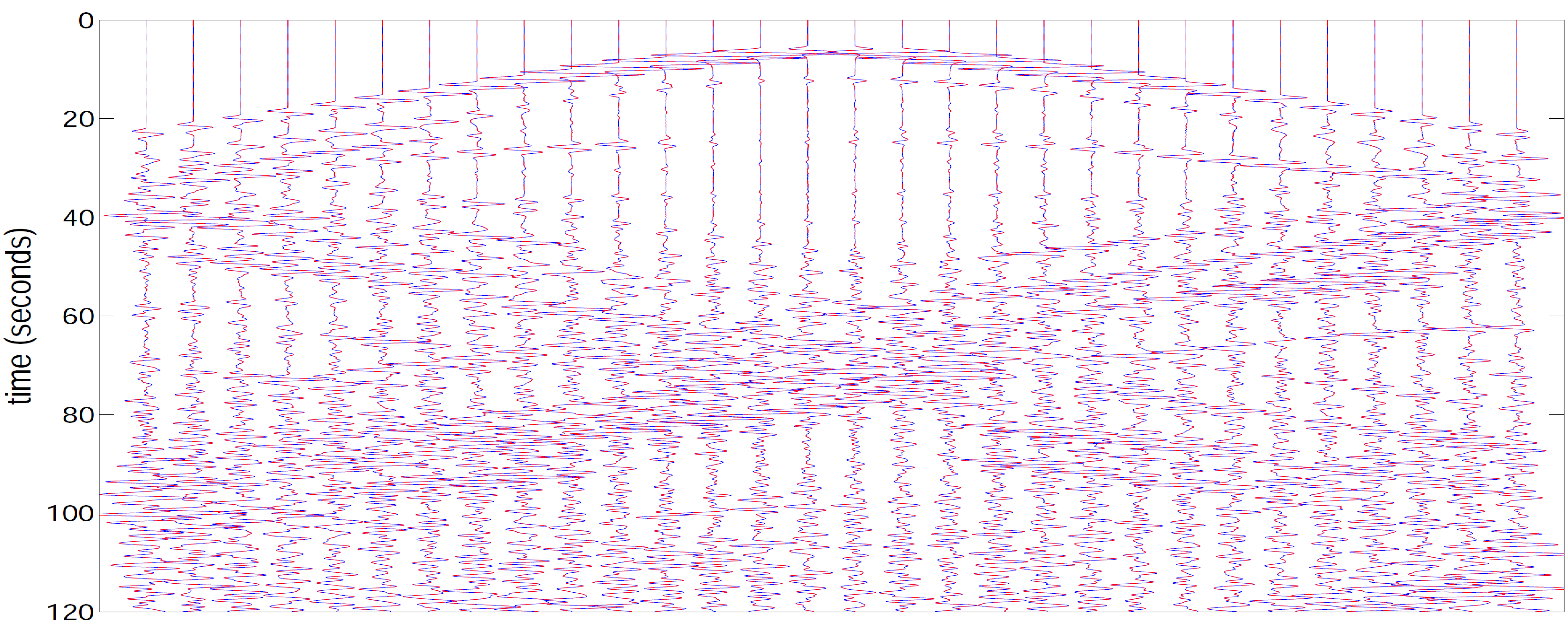
Model 1 Model 2



average wave velocity (m/s)



density (kg/m^3)



Synthetic seismograms for model 1 (blue-solid) and model 2 (red-dashed)