Heterogeneities and Anisotropy in the Earth

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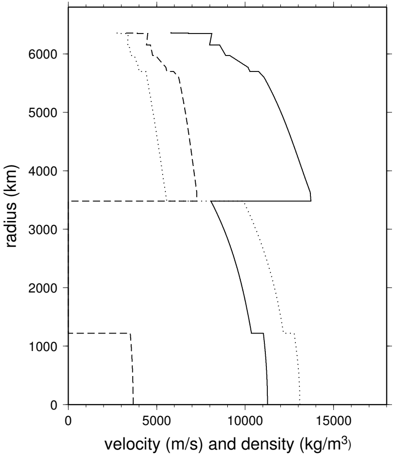
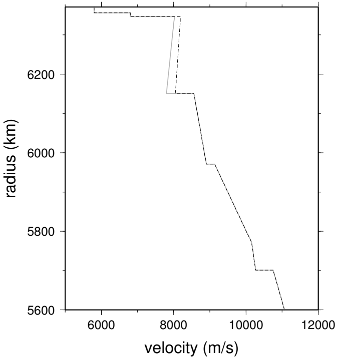
Radial heterogeneities and anisotropy

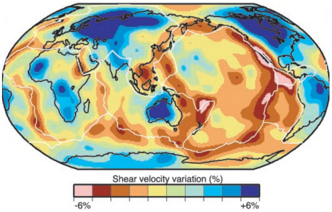


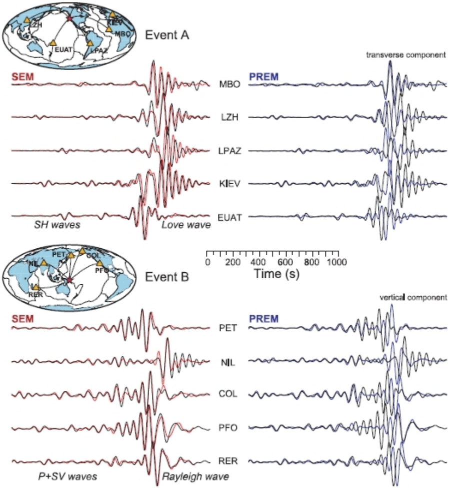


Dziewonski & Anderson (PEPI, 1981)

Lateral variations

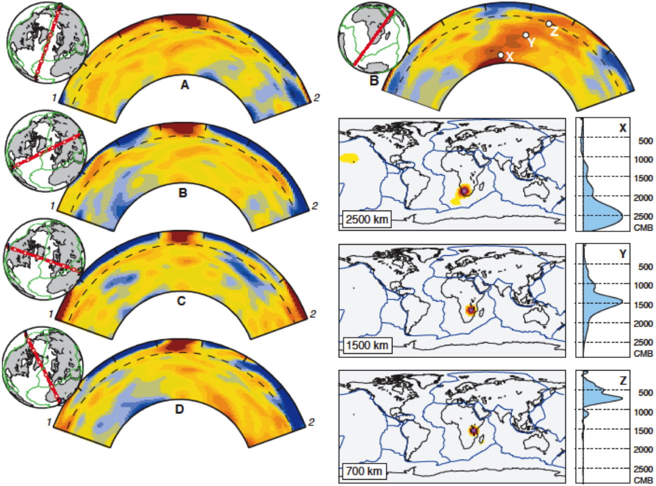
- Woodhouse & Dziewonski, 1984
- Nataf *et al*, 1986
- Li & Romanowicz, 1996
- Ritsema *et al*, 1999
- and many others..





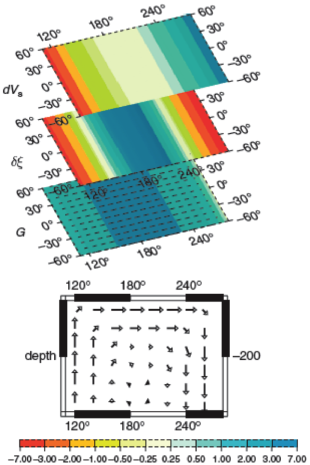
Komatitsch *et al* (Science, 2002)

Geodynamical interpretation

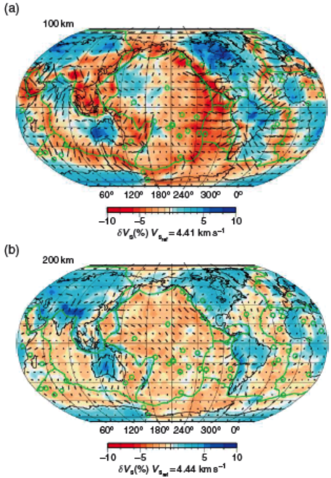


Ritsema *et al* (Science, 1999)

Azimuthal anisotropy

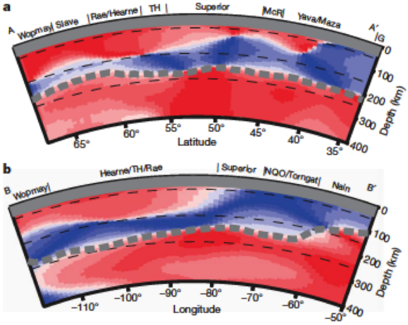


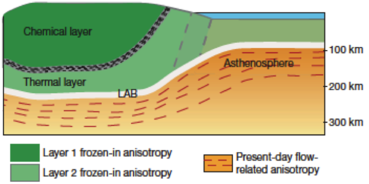
Montagner (EPSL, 2002)



Debayle *et al* (Nature, 2005)

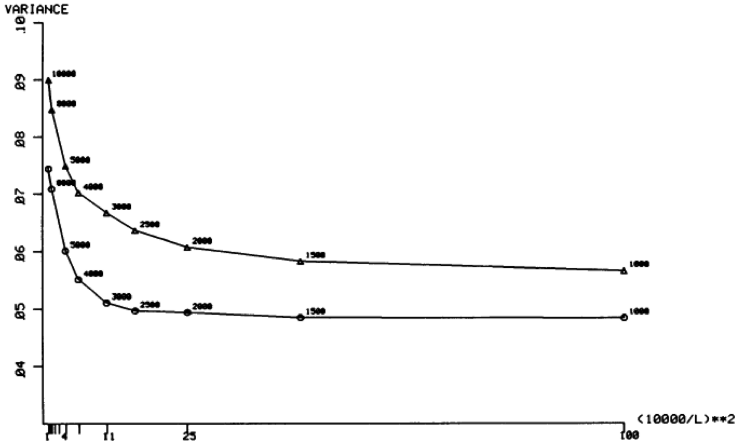
Azimuthal anisotropy





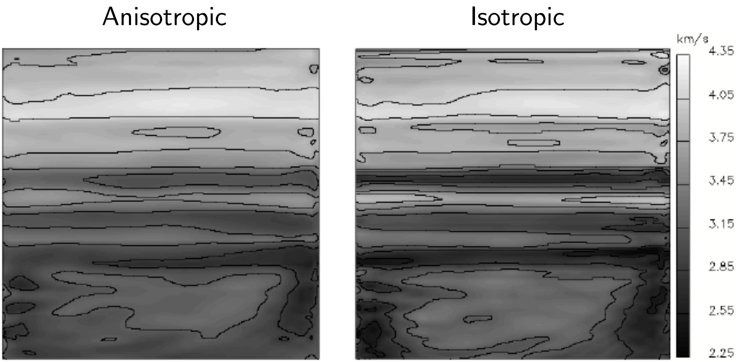
Yuan & Romanowicz (Nature, 2010)

Isotropy vs Anisotropy



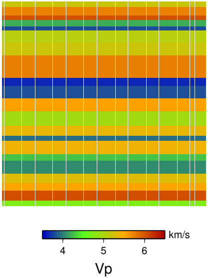
Montagner & Jobert (GJI, 1988)

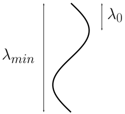
Isotropy vs Anisotropy



Pratt *et al* (63rd EAGE, 2001)

Long-wavelength equivalent medium





$$\varepsilon = \frac{\lambda_0}{\lambda_{min}}$$

$$A = \langle a - f^2 c^{-1} \rangle + \langle c^{-1} \rangle^{-1} \langle f c^{-1} \rangle^2$$
$$C = \langle c^{-1} \rangle^{-1}$$
$$F = \langle c^{-1} \rangle^{-1} \langle f c^{-1} \rangle$$
$$L = \langle l^{-1} \rangle^{-1}$$
$$M = \langle m \rangle$$

Backus (JGR, 1962)

Long-wavelength equivalent medium

$$A = \langle \frac{4\mu(\lambda+\mu)}{\lambda+2\mu} \rangle + \langle \frac{1}{\lambda+2\mu} \rangle^{-1} \langle \frac{\lambda}{\lambda+2\mu} \rangle^2$$
$$C = \langle \frac{1}{\lambda+2\mu} \rangle^{-1}$$
$$F = \langle \frac{1}{\lambda+2\mu} \rangle^{-1} \langle \frac{\lambda}{\lambda+2\mu} \rangle$$
$$L = \langle \frac{1}{\mu} \rangle^{-1}$$
$$M = \langle \mu \rangle$$

Backus (JGR, 1962)

- There are HTI media which are not long-wave equivalent to any layered isotropic medium.
- A HTI medium equivalent to an isotropic layered medium can be reproduced using 3 materials only.
- Some HTI media can be equivalent to isotropic layered media described by 2 materials only.

The homogenization technique

1. Scale separation $\mathbf{y} = \frac{\mathbf{x}}{\varepsilon}$:

$$\begin{cases} \mathbf{u}(\mathbf{x};t) \rightarrow \mathbf{u}^{\varepsilon}(\mathbf{x},\mathbf{y};t) \\ \boldsymbol{\sigma}(\mathbf{x};t) \rightarrow \boldsymbol{\sigma}^{\varepsilon}(\mathbf{x},\mathbf{y};t) \end{cases}$$

$$\begin{cases} \rho(\mathbf{x}) \rightarrow \rho^{\varepsilon}(\mathbf{x},\mathbf{y}) \\ \mathbf{C}(\mathbf{x}) \rightarrow \mathbf{C}^{\varepsilon}(\mathbf{x},\mathbf{y}) \end{cases}$$

$$\boldsymbol{\nabla}_x \rightarrow \boldsymbol{\nabla}_x + \frac{1}{\varepsilon}\boldsymbol{\nabla}_y$$

2. Asymptotic expansion in ε:

$$\begin{cases} \mathbf{u}^{\varepsilon}(\mathbf{x},\mathbf{y};t) = \sum_i \varepsilon^i \; \mathbf{u}_i(\mathbf{x},\mathbf{y};t) \\ \boldsymbol{\sigma}^{\varepsilon}(\mathbf{x},\mathbf{y};t) = \sum_i \varepsilon^i \; \boldsymbol{\sigma}_i(\mathbf{x},\mathbf{y};t) \end{cases}$$

3. Homogenization problem = series of equations to be solved for each i:

$$\begin{cases} \rho^{\varepsilon}\partial_{tt}\mathbf{u}_i - \boldsymbol{\nabla}_x \cdot \boldsymbol{\sigma}_i - \boldsymbol{\nabla}_y \cdot \boldsymbol{\sigma}_{i+1} = \mathbf{f}\delta_{i0} \\ \boldsymbol{\sigma}_i = \mathbf{C}^{\varepsilon} : [\boldsymbol{\varepsilon}_x(\mathbf{u}_i) + \boldsymbol{\varepsilon}_y(\mathbf{u}_{i+1})] \end{cases}$$

Capdeville *et al* (GJI, 2010)

The homogenization technique

Solving the homogenization problem yields:

$$\begin{cases} \mathbf{u}_0 = \mathbf{u}_0(\mathbf{x}) \\ \boldsymbol{\sigma}_0 = \boldsymbol{\sigma}_0(\mathbf{x}) \end{cases}$$

$$\mathbf{u}_1 \sim \boldsymbol{\chi}_1(\mathbf{x},\mathbf{y}) : \boldsymbol{\varepsilon}_x(\mathbf{u}_0)$$

where $\boldsymbol{\chi}_1$ is the first order corrector, solution to

$$\boldsymbol{\nabla}_y \cdot \left\{ \mathbf{C}^{\varepsilon} : \left[\mathbf{I} + \frac{1}{2}(\boldsymbol{\nabla}_y \boldsymbol{\chi}_1 + {}^t\boldsymbol{\nabla}_y \boldsymbol{\chi}_1) \right] \right\} = 0$$

Capdeville *et al* (GJI, 2010)

Practical construction of the order 0 problem:

- Solving the elasto-static equation

$$\boldsymbol{\nabla}_x \cdot \left\{ \mathbf{C} : \left[\mathbf{I} + \frac{1}{2}(\boldsymbol{\nabla}_x \boldsymbol{\chi} + {}^t\boldsymbol{\nabla}_x \boldsymbol{\chi}) \right] \right\} = 0$$

- Building the effective medium

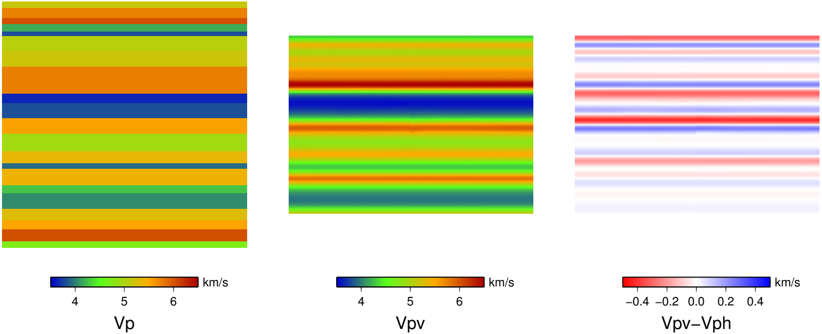
$$\begin{cases} \rho^* = \mathscr{F}^{\varepsilon}(\rho) \\ \mathbf{C}^* = \mathscr{F}^{\varepsilon}(\mathbf{H}) : \mathscr{F}^{\varepsilon}(\mathbf{G})^{-1} \end{cases}$$

avec

$$\mathbf{G} = \frac{1}{2}(\boldsymbol{\nabla}_x \boldsymbol{\chi} + {}^t\boldsymbol{\nabla}_x \boldsymbol{\chi}) + \mathbf{I}$$

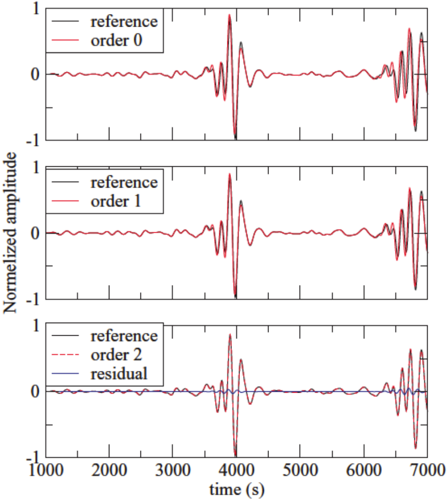
$$\mathbf{H} = \mathbf{C} : \mathbf{G}$$

Application to the layered case



Cupillard & Capdeville (in prep)

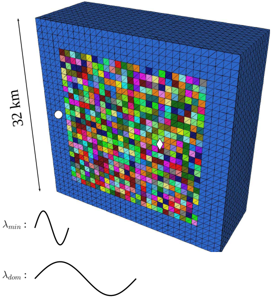
Application to the layered case



Surface waves need higher orders !!

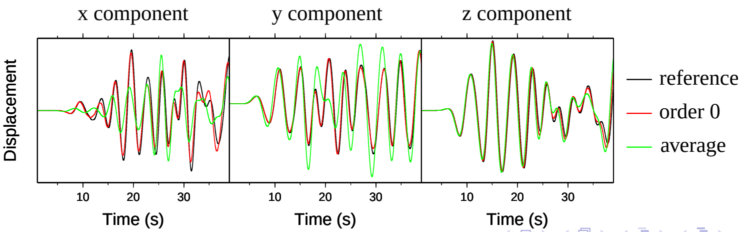
Capdeville & Marigot (GJI, 2007)

Layered in every direction

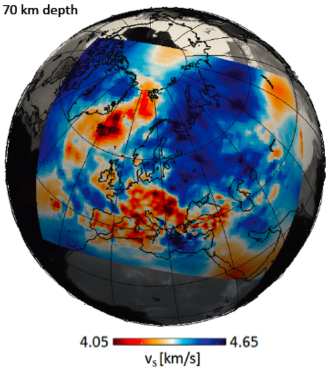


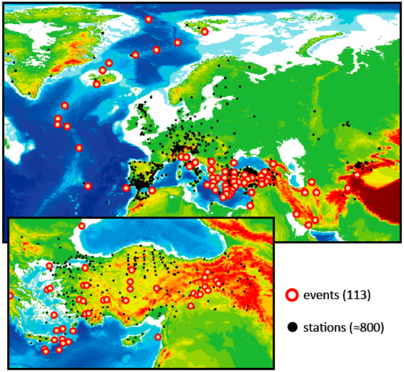
Computing reference traces takes 1 h on 8 CPUs. Computing the order 0 effective medium takes 4 min, and computing traces in this medium takes a couple of seconds.

(Cupillard & Capdeville, in prep)



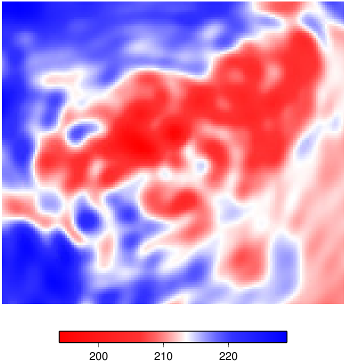
A realistic application



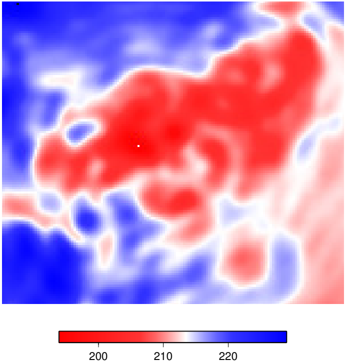


Fichtner *et al* (in prep)

A realistic application

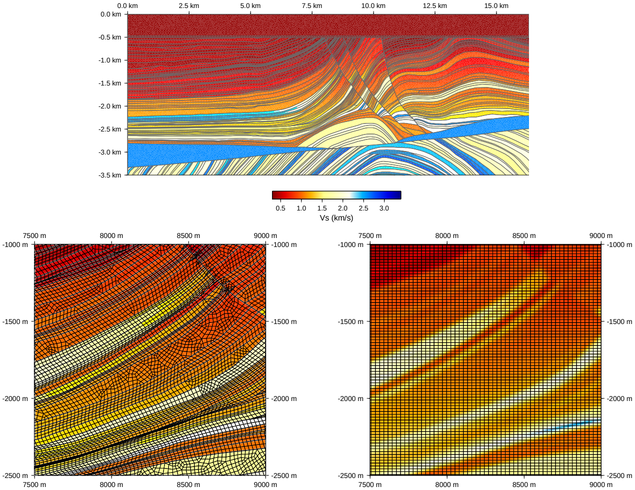


initial C11 (x10^9) at 150km depth



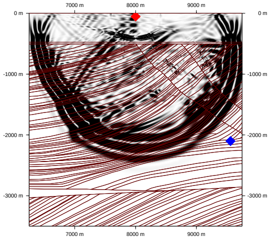
smoothed C11 (x10^9) at 150km depth

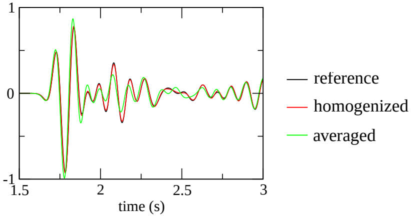
A 2D example: Marmousi2

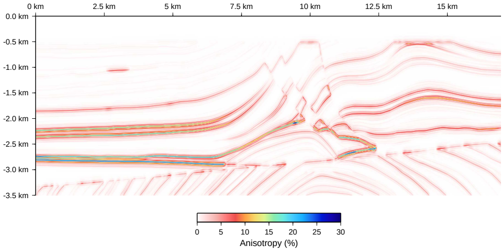


Capdeville *et al* (GJI, 2010)

A 2D example: Marmousi2



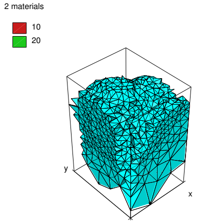


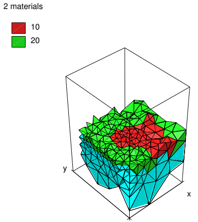


Capdeville *et al* (GJI, 2010)

3D Implementation

1. Resolution of the elastostatic problem using tetrahedral finite elements generated by TetGen. Isoparametric, superparametric or subparametric elements are possible.
2. Parallel implementation using MPI and METIS. The periodic boundary condition is replaced by a Dirichlet condition, so each subdomain have to be surrounded by a trash layer.





3. Inversion of the stiffness matrix using PARDISO.
4. Low-pass filtering in the space domain using OpenMP.

Conclusions

- Anisotropic media can be long-wavelength equivalent to finer isotropic media
- The homogenization technique is able to compute such equivalent media
  - The effective medium is what the waves see
  - Its parameterization is the coarsest (with respect to the minimum wavelength you want to propagate)
  - Accurate seismic wave computation with no meshing effort and no extra CPU cost due to tiny elements
  - Independant of the wave solver
- Different parameterizations (isotropic vs anistropic) yield different interpretations
  - Resolution test (Backus & Gilbert, 1968; Fichtner & Trampert, 2011)
  - Dehomogenization

Perspectives

- Dehomogenization (model space exploration):
  - Generate random models (with random elastic properties, discontinuities and parametrization)
  - Homogenize them
  - Compare them to the model obtained from an adjoint-based inversion. The key-point here is to find to good criteria for the comparison.
- Building a good starting model
  - Multi-scale inversion in Europe
  - Smoothing fine near-surface structures for seismic exploration