Introduction to frequency domain waveform inversion: theory and applications

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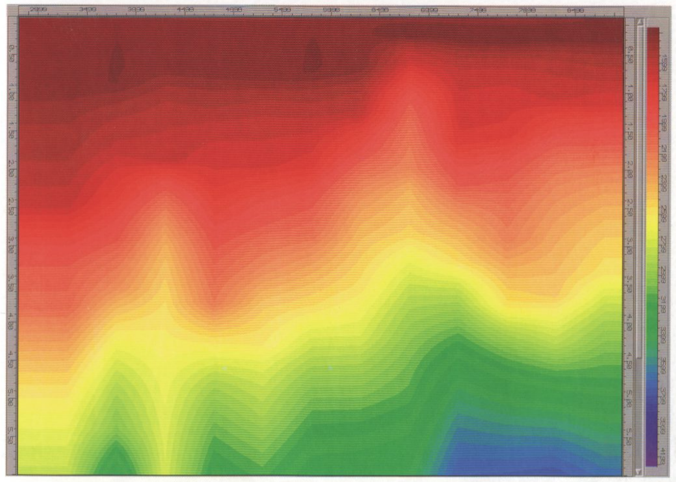
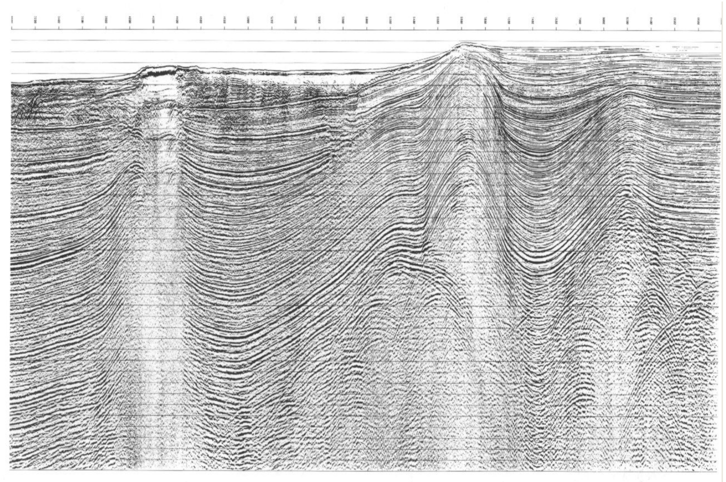
Imaging with Exploration Seismic Data – the Fundamentals

- Common practice in reflection processing:
  - The “model” and the “image” have distinct spectral characteristics
  - Each are derived from distinct aspects of the data



Seismic velocity model



Seismic reflectivity image

Textbook example (Yilmaz 2003)

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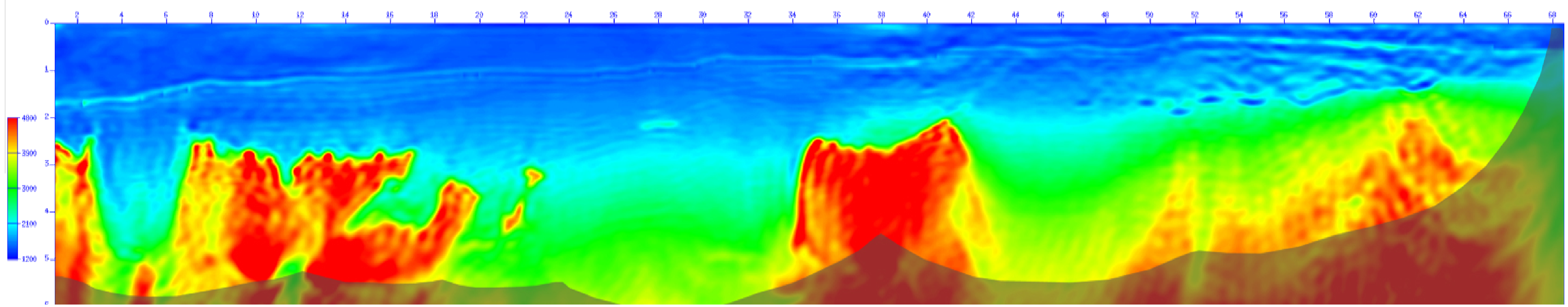
The Basic Idea

- Common practice in reflection processing:
  - The “model” and the “image” have distinct spectral characteristics
  - Each are derived from distinct aspects of the data
- The approach of waveform tomography:
  - No distinction between model and image
  - All available information contributes to spectrum of the result
  - Overlap in methodology (“migration-like”)
  - Significant change in philosophy

Waveform tomography of long offset seismic data



(Pratt and Brenders, EAGE workshop, 2004)

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Outline

- Long offset seismic data
  - An imaging opportunity
- Waveform Tomography
  - Theoretical aspects
  - Making it work
- Nankai Trough Subduction Zone Example
  - Geological implications
  - The Challenge of High Resolution

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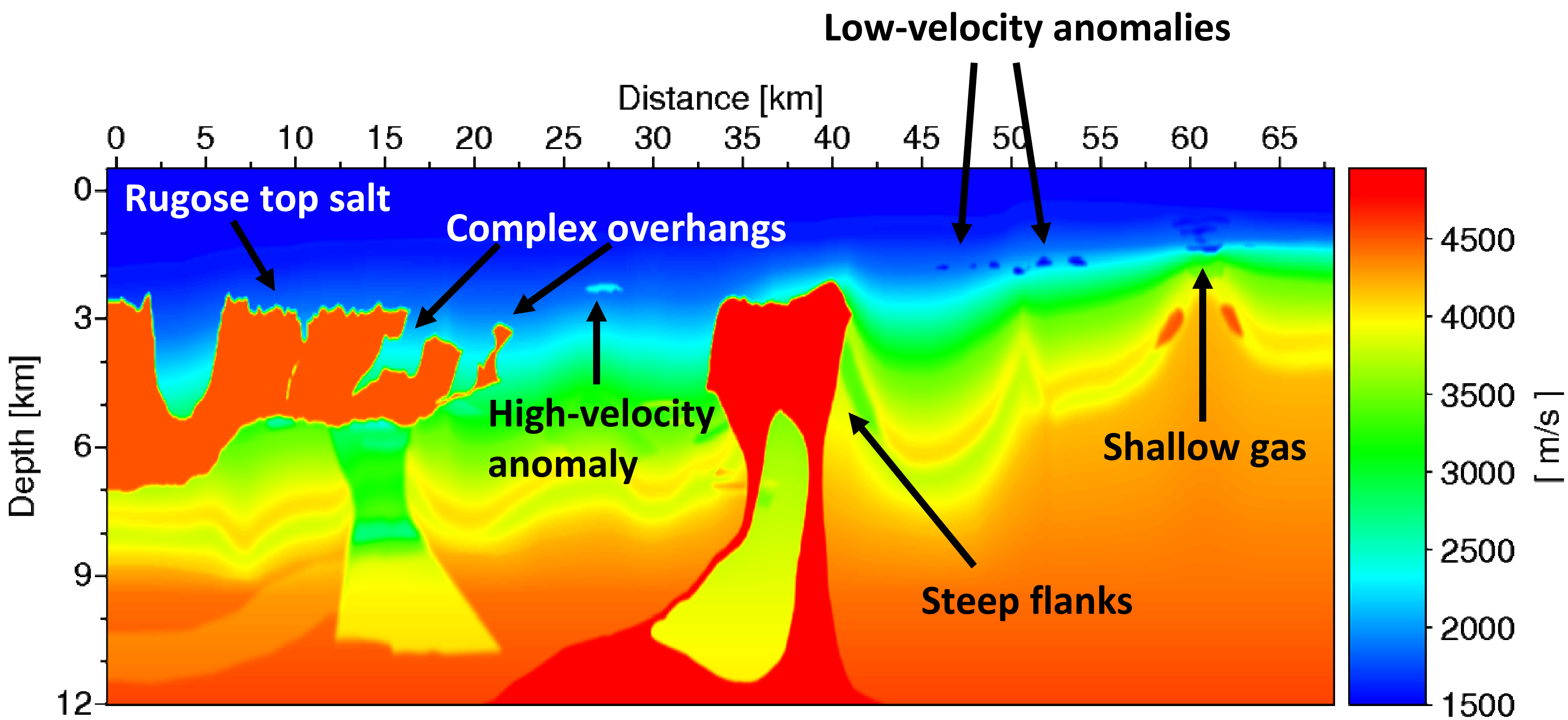
2004 BP Velocity Benchmark: Model



- 67 km wide, 12 km deep
- Built on a 6.25 m x 6.25 m grid
- Range of imaging challenges

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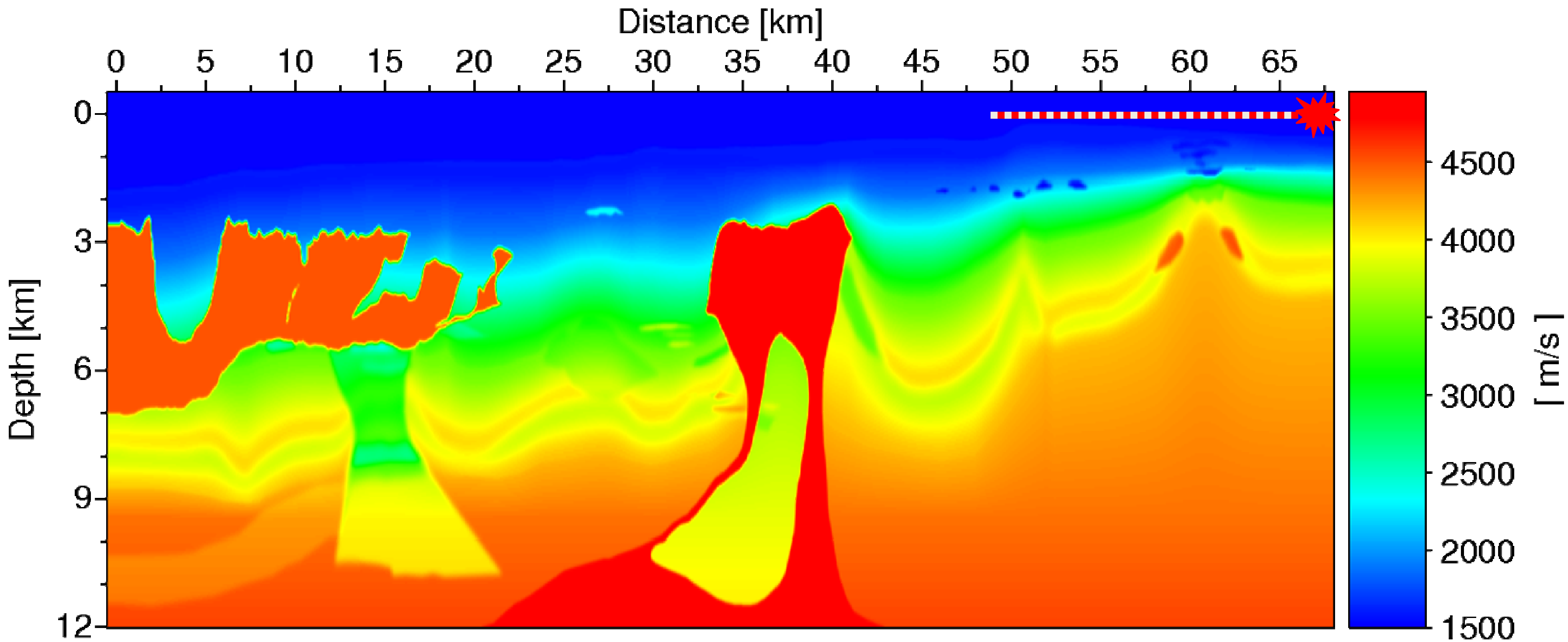
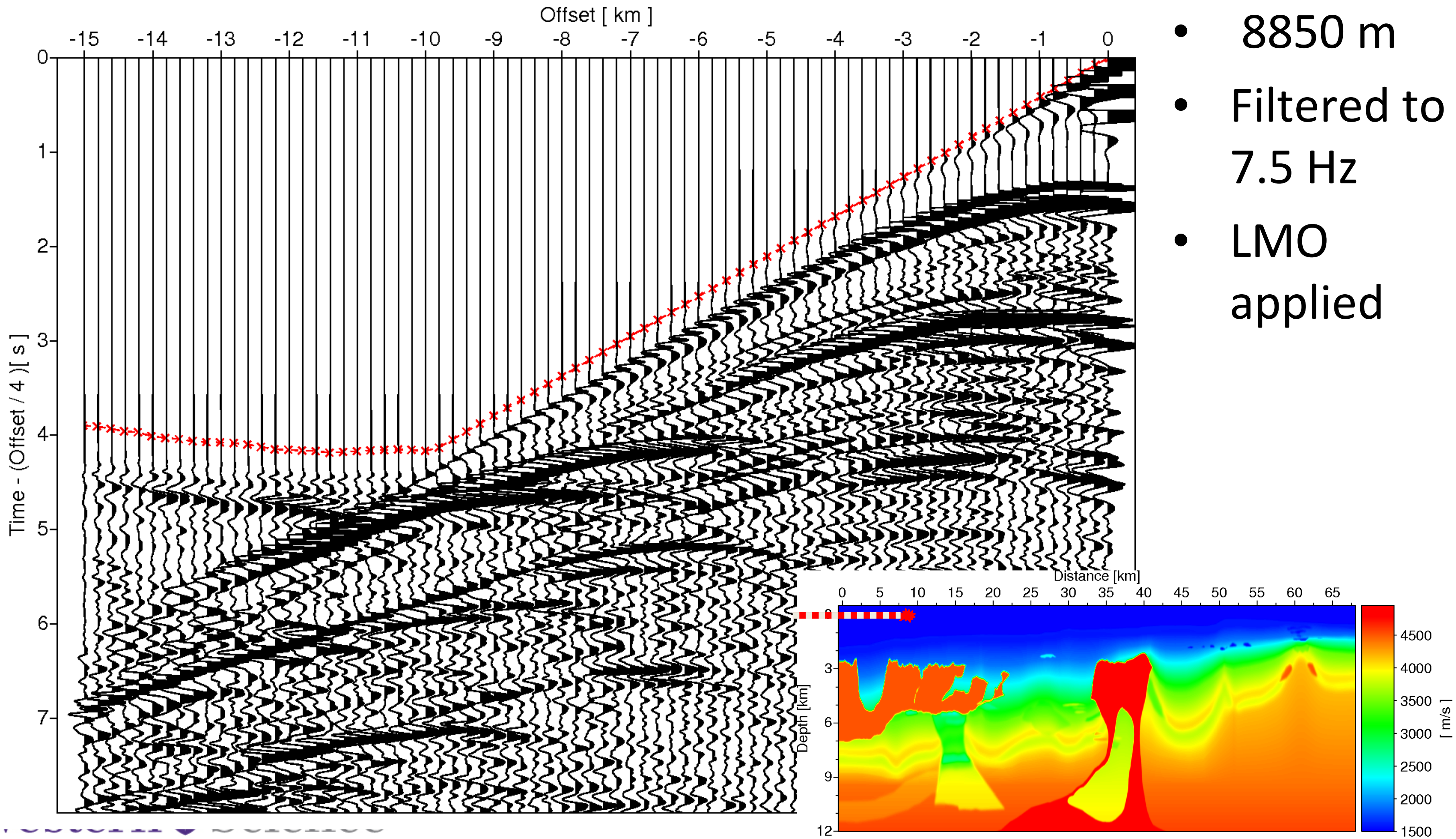
2004 BP Velocity Benchmark: Synthetic Data



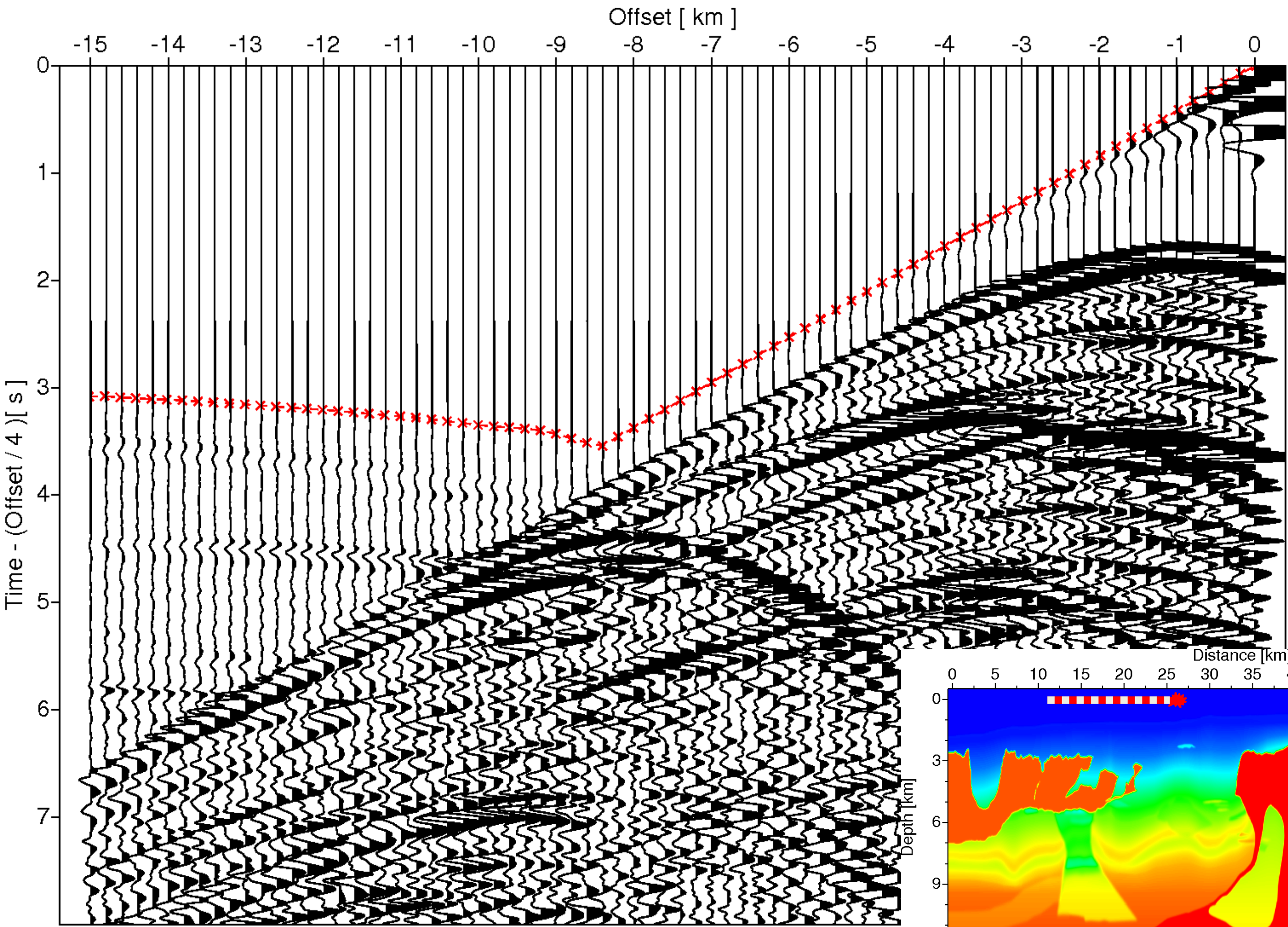
- 2-D finite-difference acoustic wave equation
- Shot spacing: 50 m, 1348 shots total, 1201 receivers per shot
- 15 km offset streamer data provided
- $\Delta t = 6$ ms, $f_{peak} = 27$ Hz

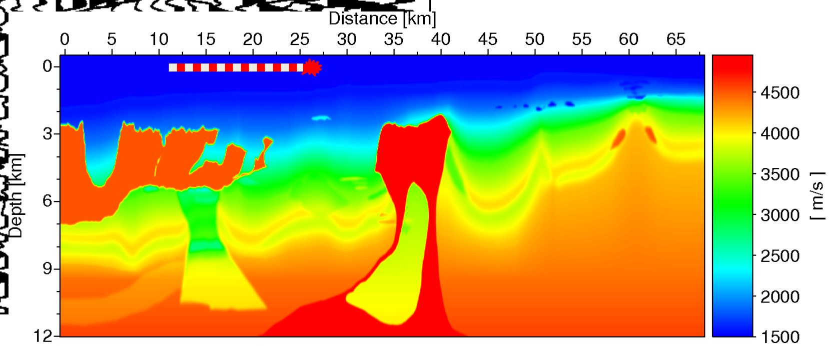
2004 BP Velocity Benchmark: Synthetic Data



- 8850 m
- Filtered to 7.5 Hz
- LMO applied

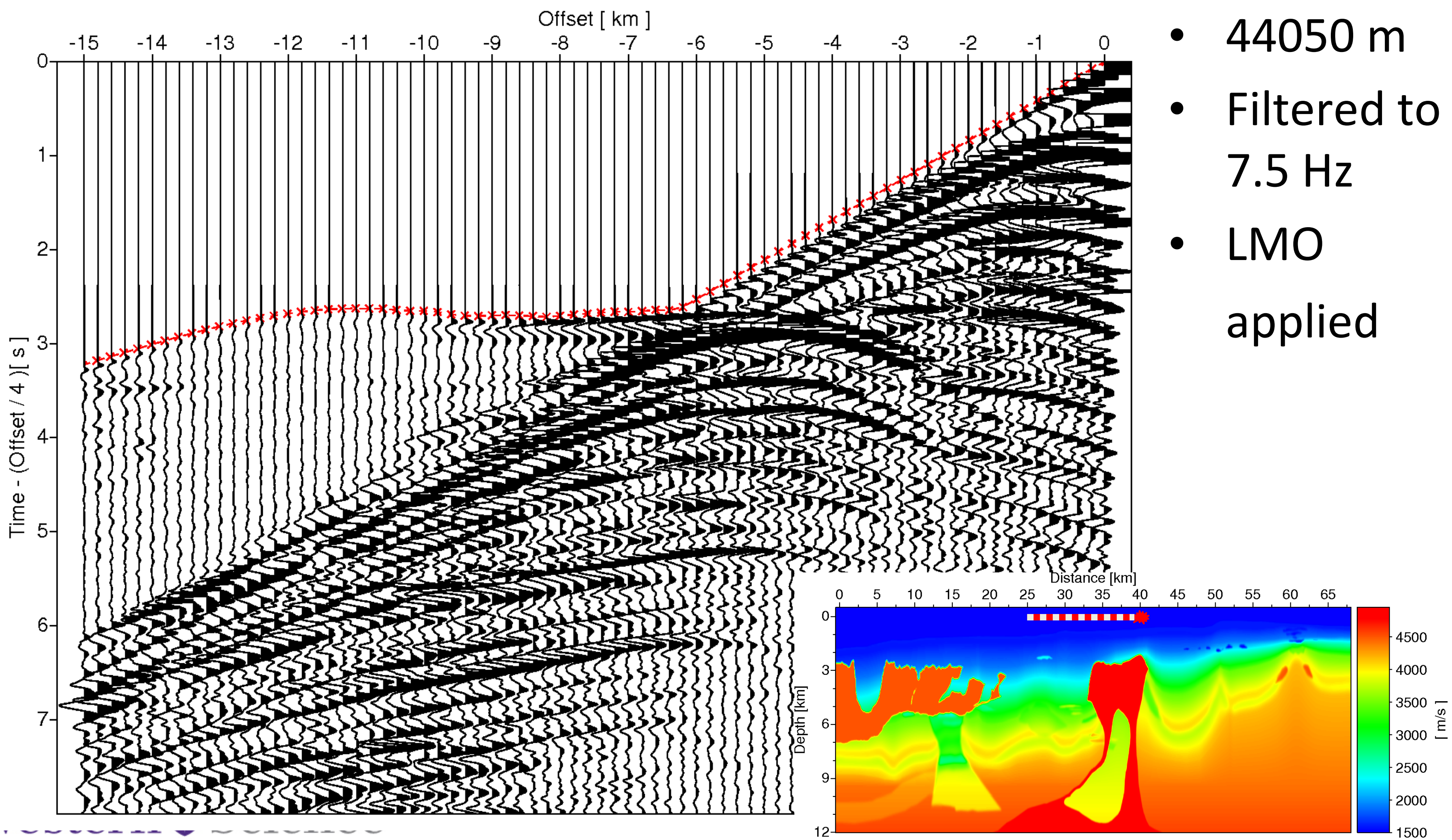
2004 BP Velocity Benchmark: Synthetic Data





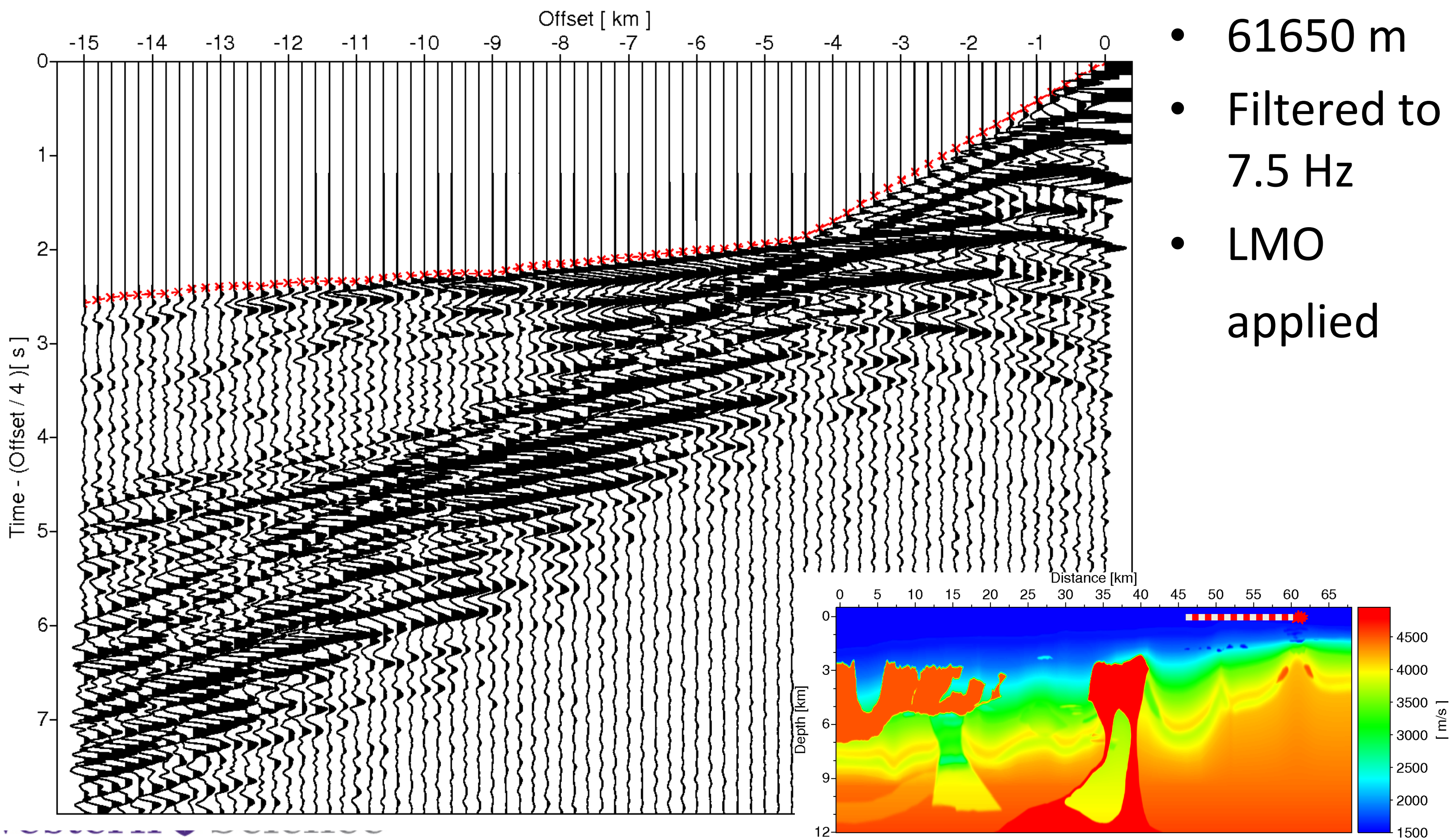
- 26450 m
- Filtered to 7.5 Hz
- LMO applied

2004 BP Velocity Benchmark: Synthetic Data



- 44050 m
- Filtered to 7.5 Hz
- LMO applied

2004 BP Velocity Benchmark: Synthetic Data



- 61650 m
- Filtered to 7.5 Hz
- LMO applied

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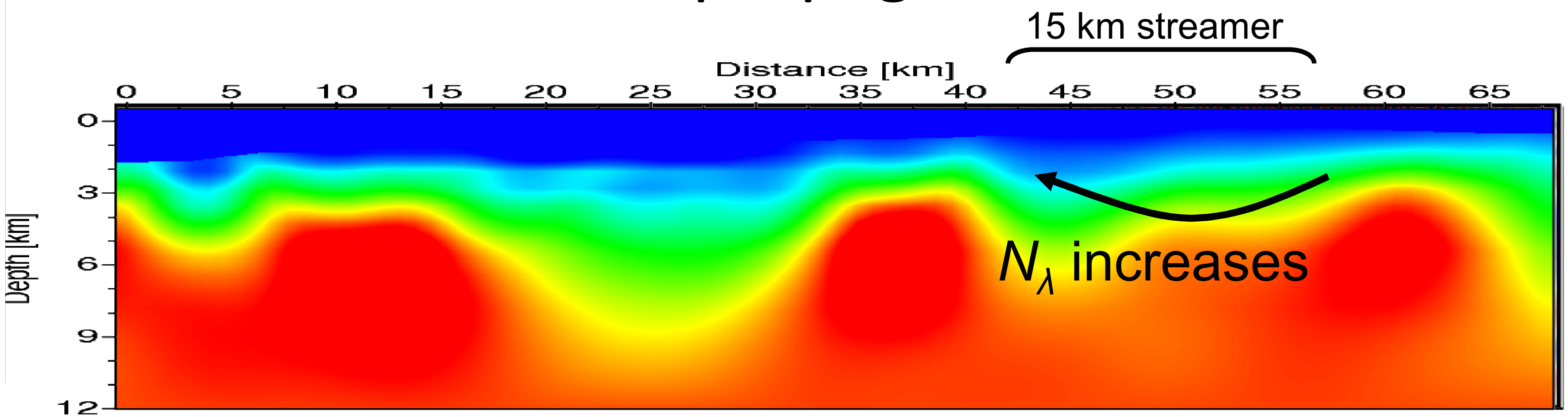
Forward modelling of wide angle propagation



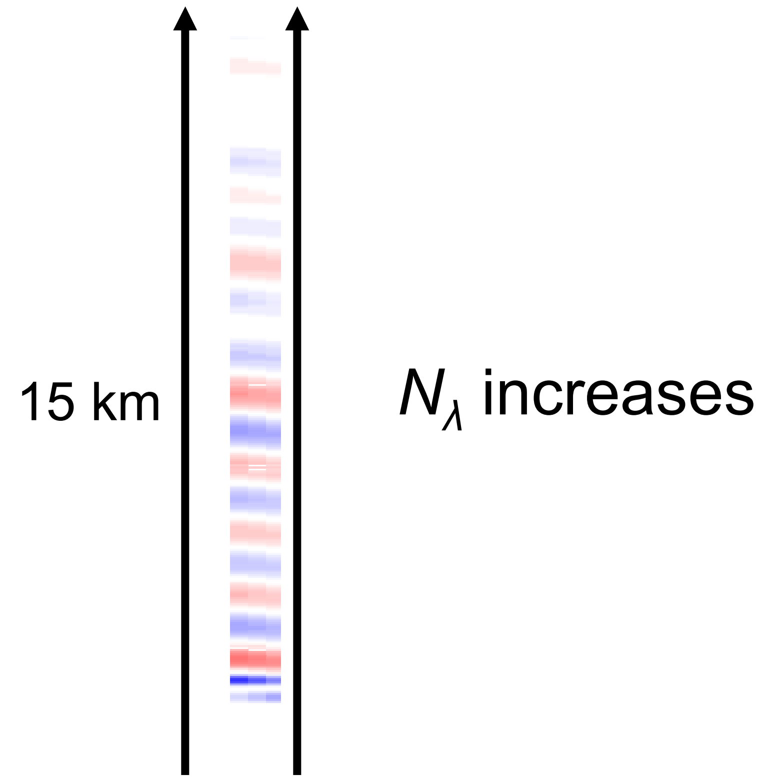
Frequency 3 Hz

Wavelength $\lambda \approx$ 1 km

- 2D Acoustic, isotropic "2-way" wave equation
- Frequency-domain finite differences (implicit)

Animations courtesy Drew Brenders

Forward propagated wavefield at receivers



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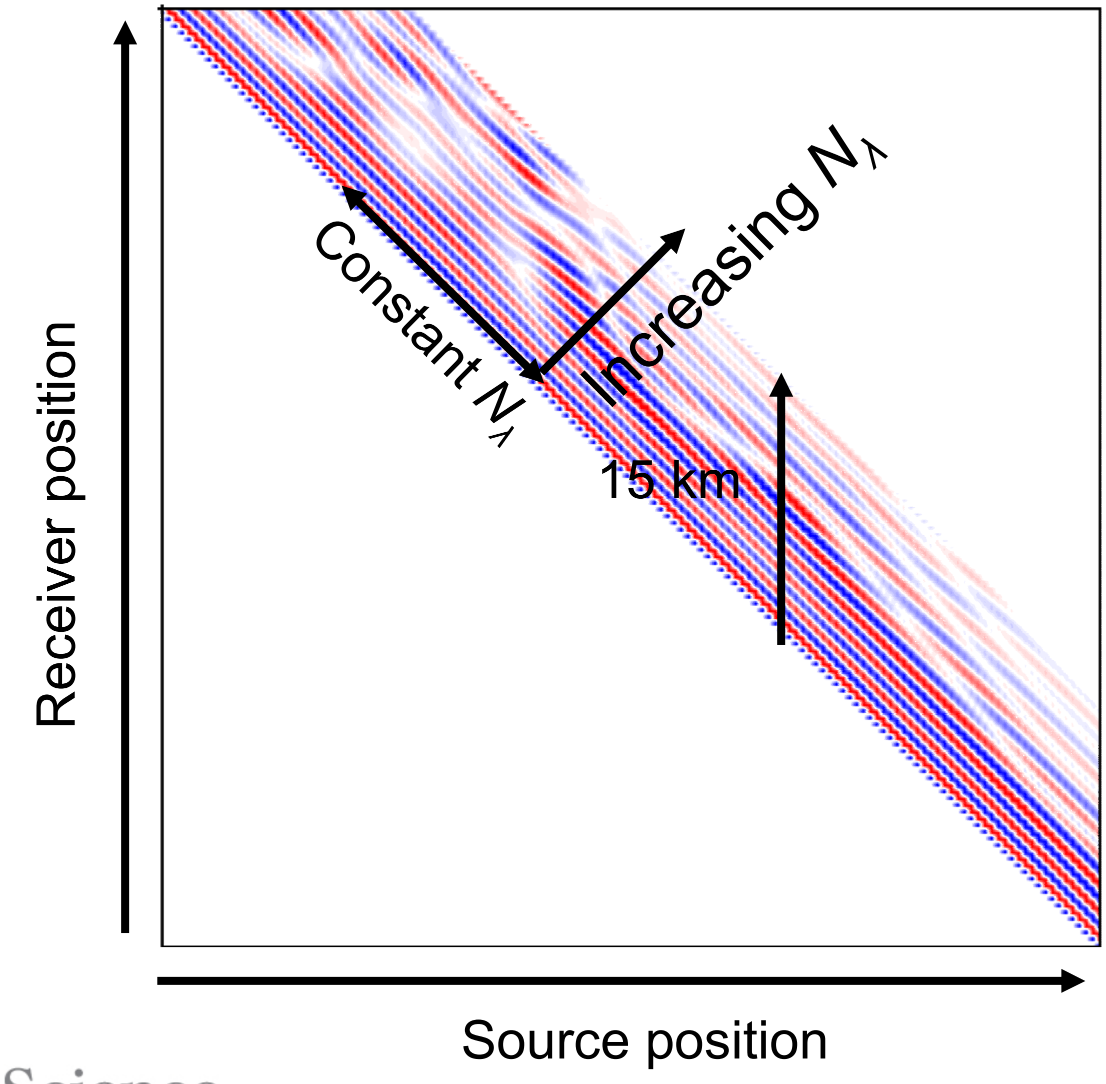
Forward propagated wavefield at receivers: All source locations



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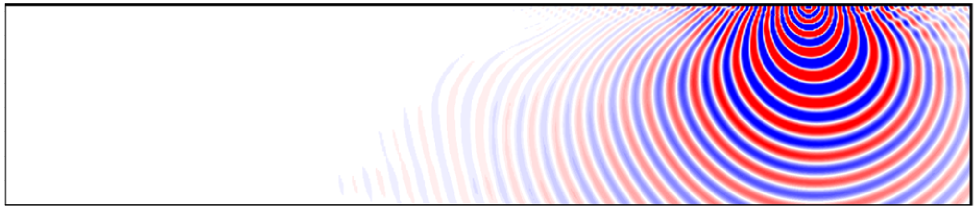
Predicting data perturbations

Frequency 3 Hz

Wavelength $\lambda \approx 1$ km



$$\delta d(\mathbf{r}_s, \mathbf{r}_g) = -\omega^2 \int_V G(\mathbf{r}, \mathbf{r}_s)\, \delta s^2(\mathbf{r})\, G(\mathbf{r}, \mathbf{r}_g)\, d\mathbf{r}$$

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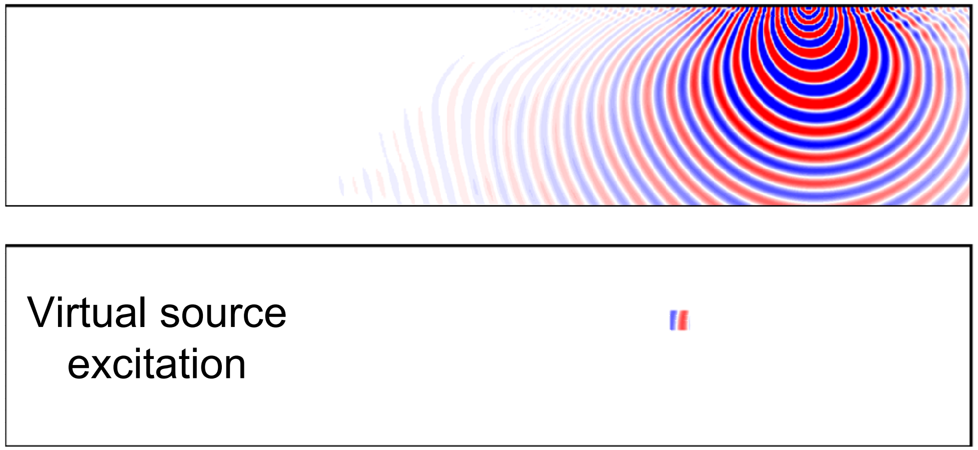
Predicting data perturbations

Frequency 3 Hz

Wavelength $\lambda \approx 1$ km



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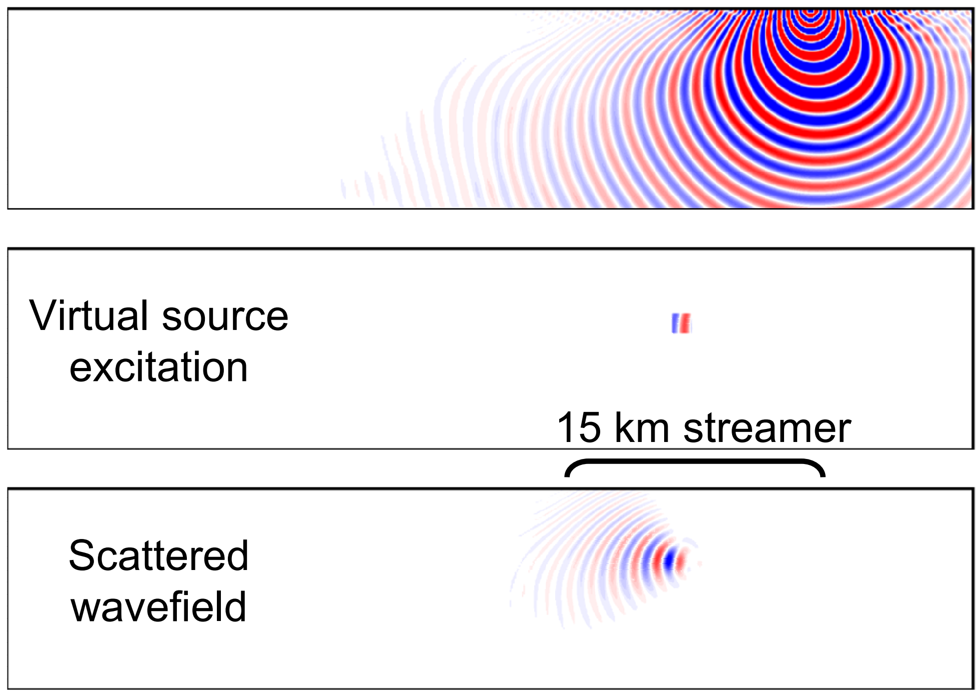
Predicting data perturbations

Frequency 3 Hz

Wavelength λ ≈ 1 km



$$\delta d(\mathbf{r}_s, \mathbf{r}_g) = -\omega^2 \int_V G(\mathbf{r}, \mathbf{r}_s)\, \delta s^2(\mathbf{r})\, G(\mathbf{r}, \mathbf{r}_g)\, d\mathbf{r}$$

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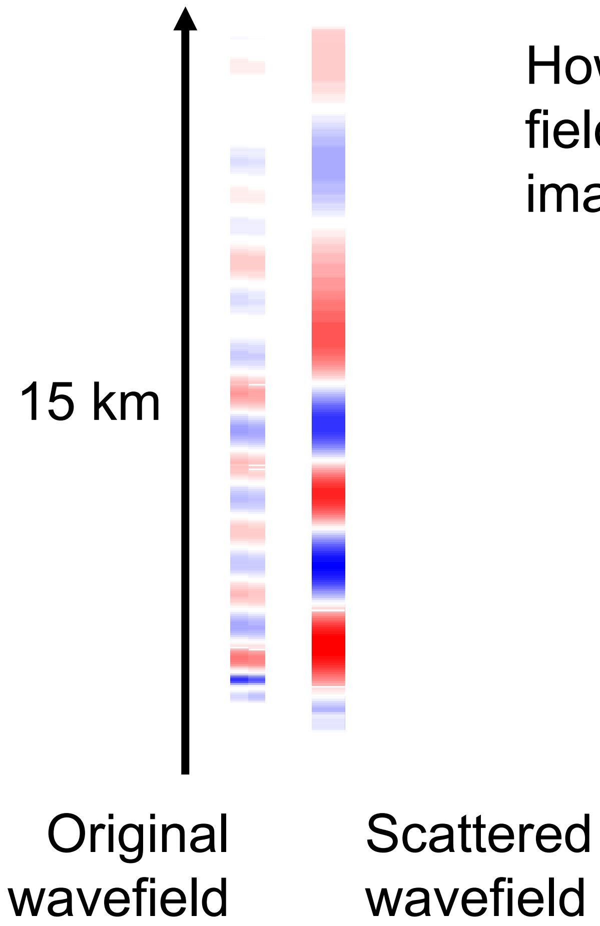
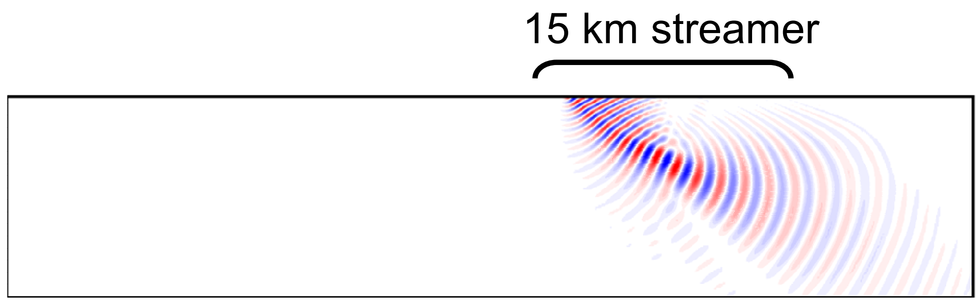
Scattered wavefield at receivers



How can the scattered field be used in imaging/inversion?

The backpropagated wavefield



Frequency 3 Hz

Wavelength $\lambda \approx 1$ km

Time reverse scattered wavefield, and propagate this as if it were a new source

This generates a partial focussing at the perturbation

How can we turn this intuitive idea into an image?

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Adjoint calculation

Born forward equation

$$\delta d(\mathbf{r}_s, \mathbf{r}_g) \approx -\omega^2 \int_V \delta s(\mathbf{r}) \, G(\mathbf{r}, \mathbf{r}_s) G(\mathbf{r}, \mathbf{r}_g) \, d\mathbf{r}$$

Gradient (update image)

$$\delta s(\mathbf{r}) = -\omega^2 \int_D G(\mathbf{r}, \mathbf{r}_s) \delta d(\mathbf{r}_s, \mathbf{r}_g) \, G(\mathbf{r}, \mathbf{r}_g) \, dD$$

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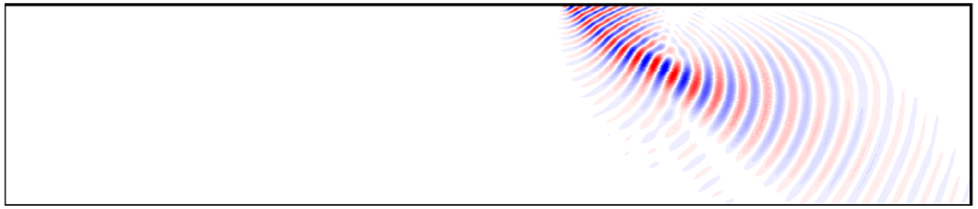
Adjoint calculation: Multiplication of forward and backpropagated wavefields

Frequency 3 Hz

Wavelength $\lambda \approx$ 1 km



$$\delta \widehat{s^2}(\mathbf{r}) = -\omega^2 \int_D G(\mathbf{r}, \mathbf{r}_s)\, \delta d(\mathbf{r}_s, \mathbf{r}_g)\, G(\mathbf{r}, \mathbf{r}_g)\, dD$$

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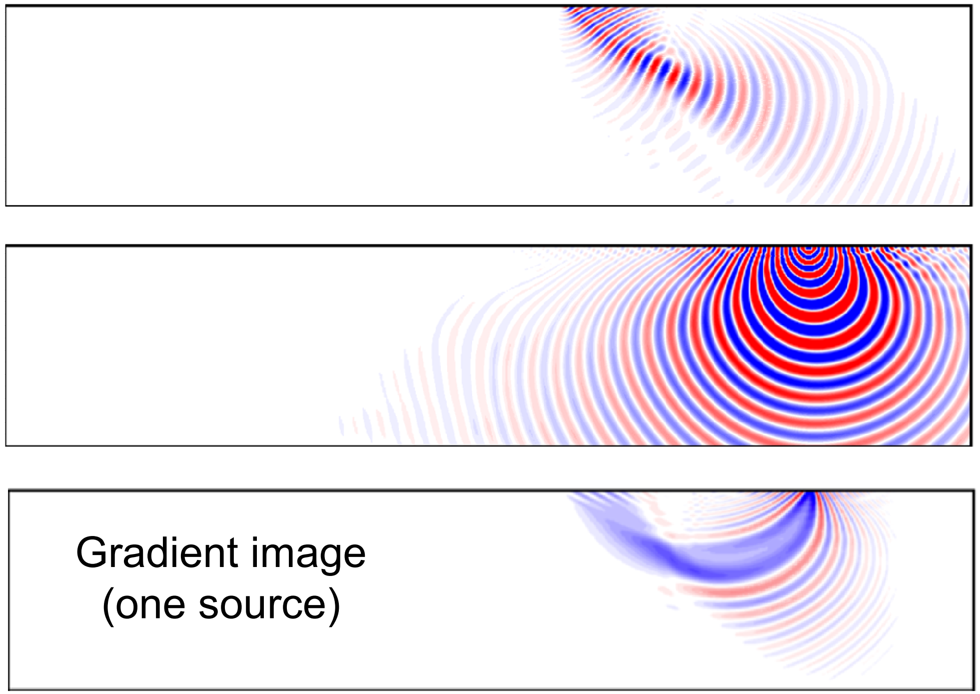
Adjoint calculation: Multiplication of forward and backpropagated wavefields

Frequency 3 Hz

Wavelength $\lambda \approx 1$ km



$$\delta \widehat{s^2}(\mathbf{r}) = -\omega^2 \int_D G(\mathbf{r}, \mathbf{r}_s)\, \delta d(\mathbf{r}_s, \mathbf{r}_g)\, G(\mathbf{r}, \mathbf{r}_g)\, dD$$

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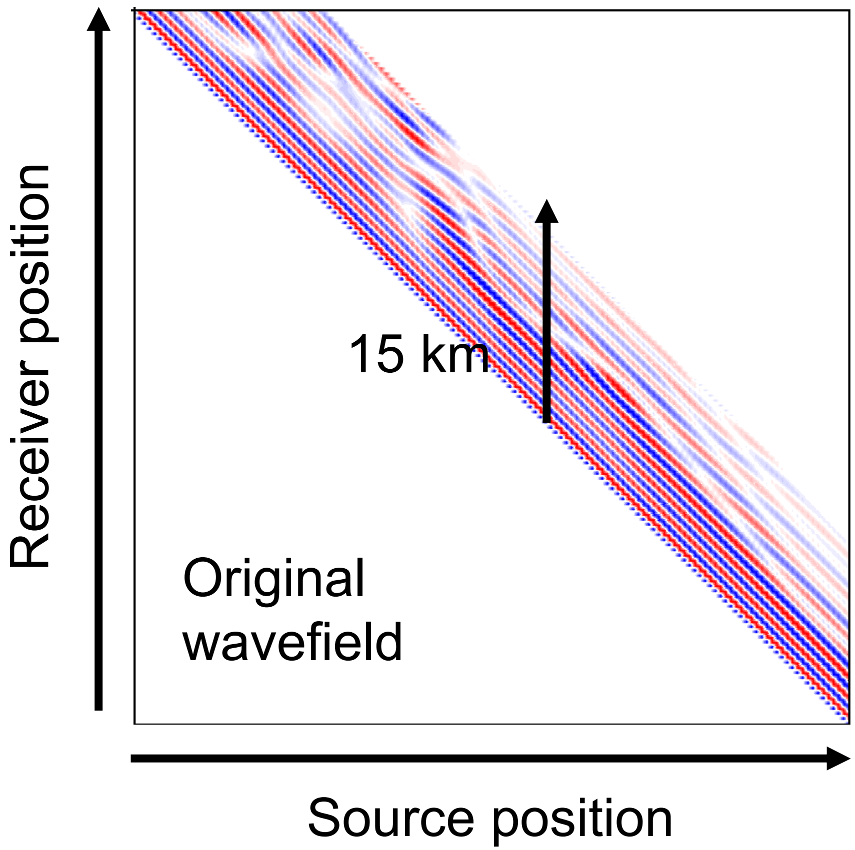
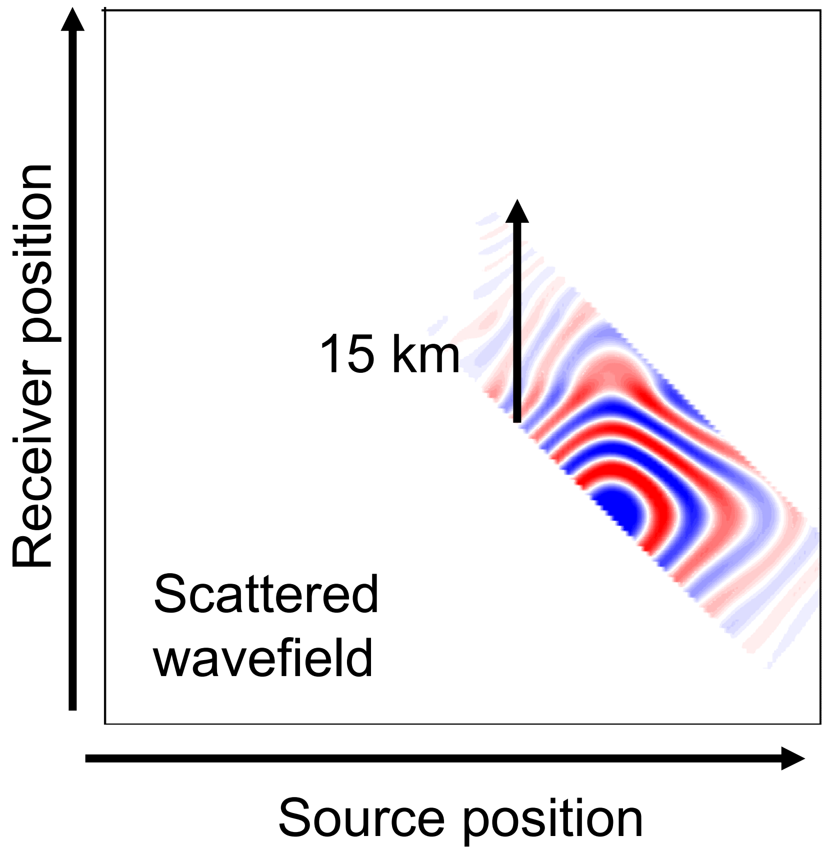
Scattered wavefield at receivers: All source locations





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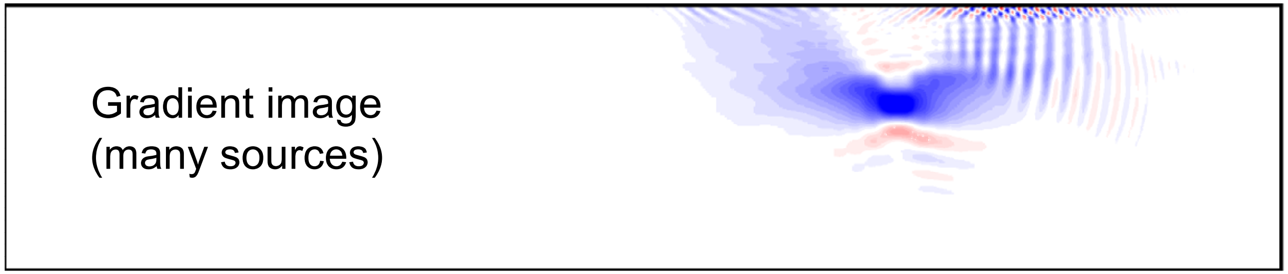
Adjoint calculation, many sources

Frequency 3 Hz

Wavelength $\lambda \approx 1$ km



$$\delta \widehat{s^2}(\mathbf{r}) = -\omega^2 \int_D G(\mathbf{r},\mathbf{r}_s)\,\delta d(\mathbf{r}_s,\mathbf{r}_g)\,G(\mathbf{r},\mathbf{r}_g)\,d\mathrm{D}$$

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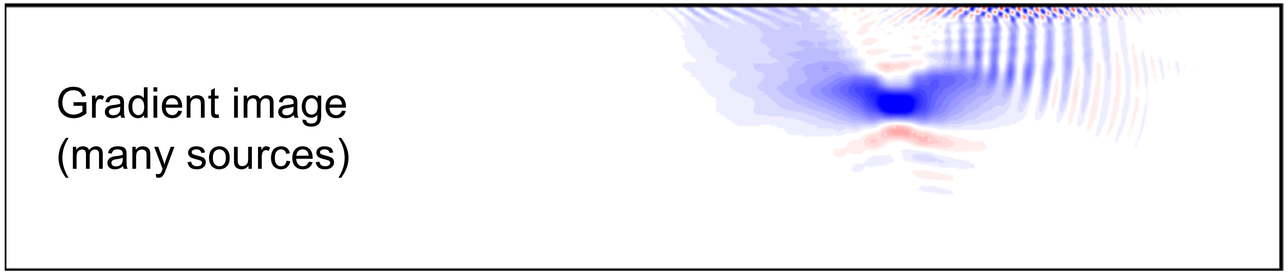
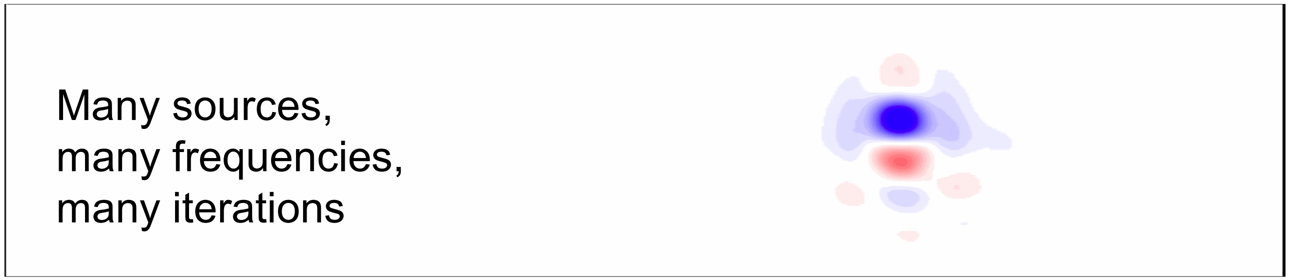
Waveform tomography: An iterative process

Frequency 3 Hz

Wavelength $\lambda \approx 1$ km





$$\delta \widehat{s^2}(\mathbf{r}) = -\omega^2 \int_D G(\mathbf{r}, \mathbf{r}_s)\, \delta d(\mathbf{r}_s, \mathbf{r}_g)\, G(\mathbf{r}, \mathbf{r}_g)\, dD$$

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Practical Aspects

Waveform inversion:

- High resolution / highly non-linear / computationally costly

Seek "robustness" and computational efficiency

- Low frequencies – robust spectral components
- Phase – robust spectral attributes
- Early arrivals – robust temporal wavefield

Leads to consideration of

- Phase-only objective function
- Acoustic formulation

Practical Aspects

FWI – the "Grand vision" of Tarantola et al:

- Models would predict the full wavefield
- Models would respect full elastic wave propagation physics
- Models would reveal full information on all elastic parameters (including insight into posteriori distributions)

The "phase we are going through"

- Bootstrap from kinematic starting models (velocity analysis, traveltimes)
- Acoustic formulation – phase-only
- Acoustic formulation – phase & amplitude
- Elastic formulation – amplitude and phase

Practical Aspects

- Bootstrap from kinematic starting models (velocity analysis, traveltimes)

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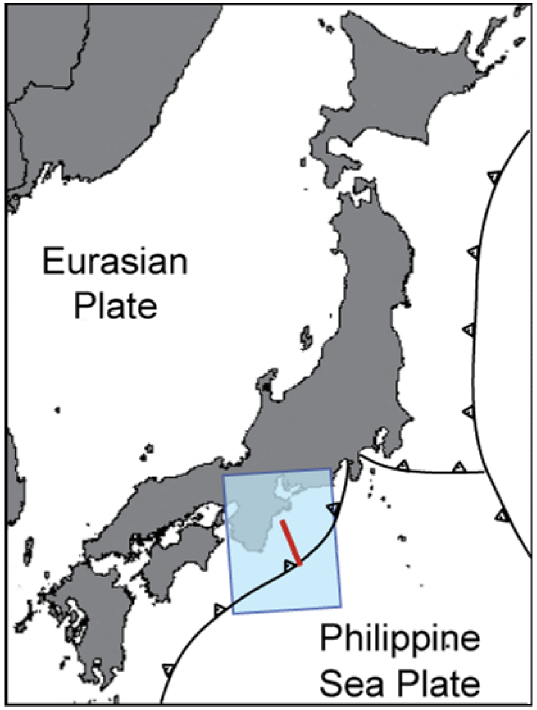
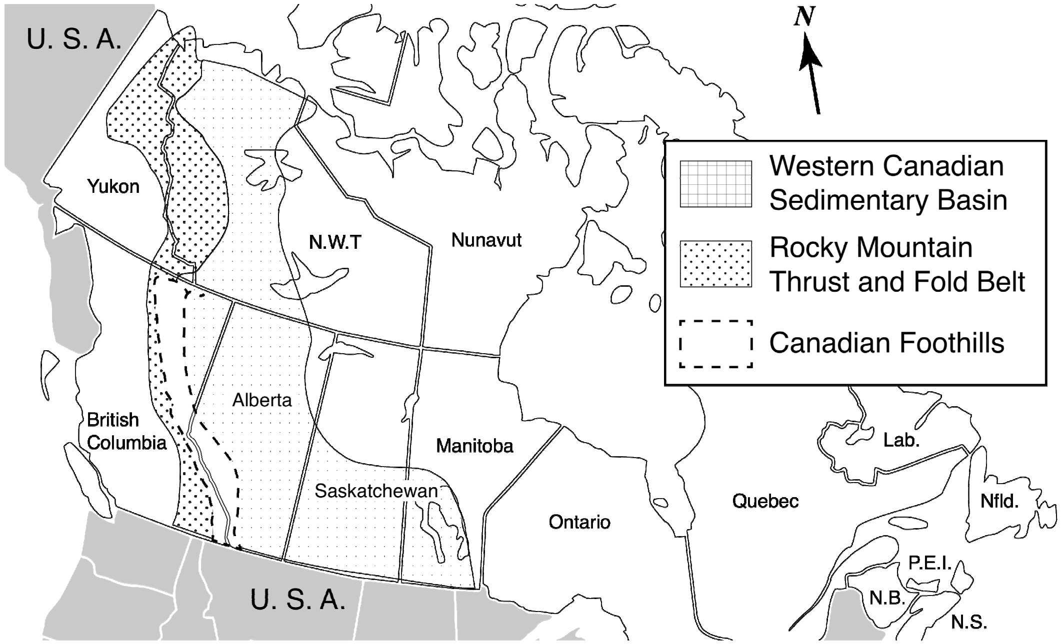
- Acoustic formulation – phase & amplitude

- Elastic formulation – amplitude and phase

Data sets

Marine	Land
Nankai subduction zone Crustal scale imaging	**Canadian Foothills** Hydrocarbon exploration

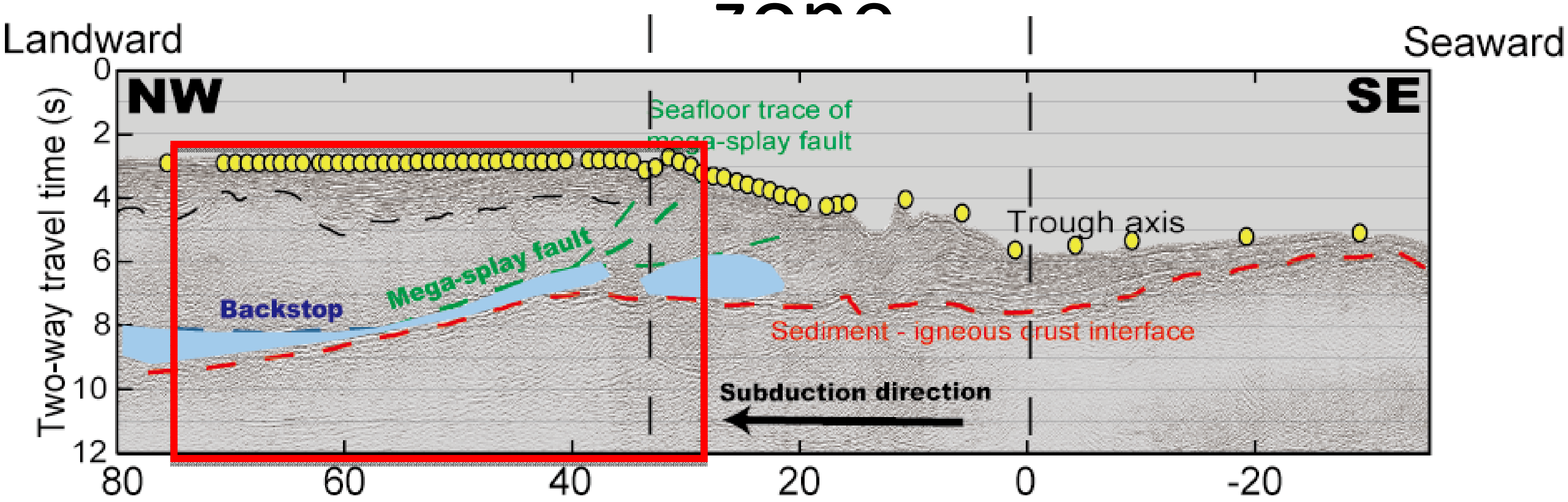




Kamei and Pratt, 2010, SEG
Kamei et al., 2011, EPSL, under review

Brenders et al., 2009, 2010, SEG
Brenders, 2011, PhD Thesis

Marine data set: Nankai subduction zone



(Courtesy of T. Tsuji)

# of OBSs	54 (2 km water depth)
OBS interval	1 km
# of shots	285
Shot interval	200 m (10 m towed depth)
Max offset	60 km
Min available freq	2 Hz

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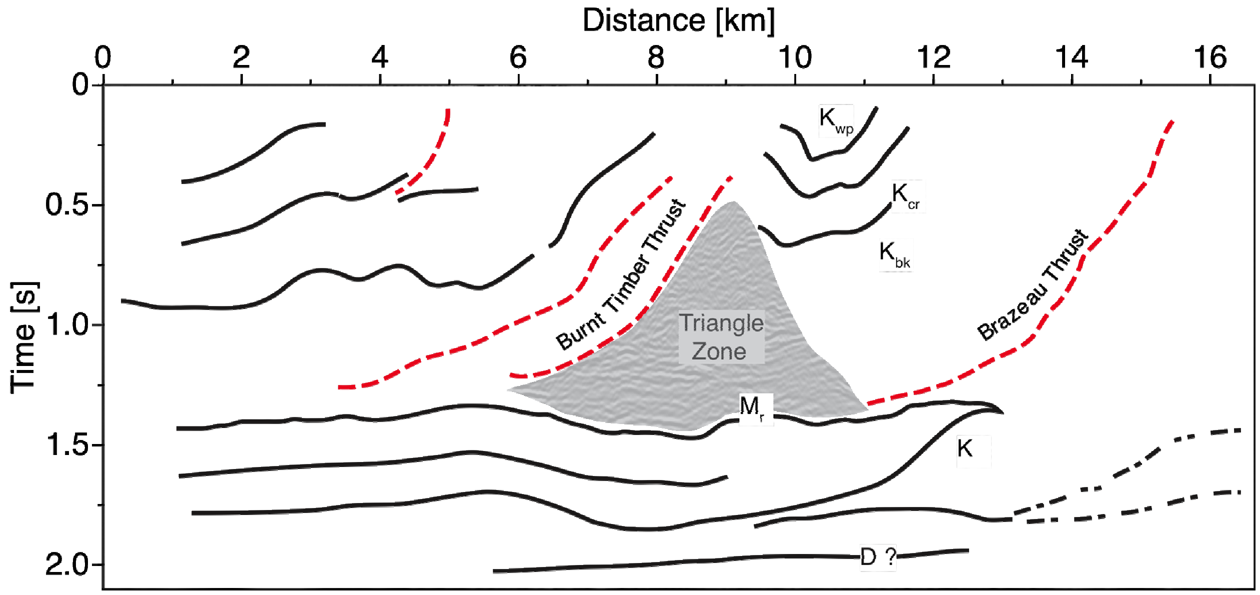
Land data set: Canadian foothills



# of channels	666 (live)
Receiver Interval	25 m
# of shots	168
Shot interval	100 m
Max offset	16.6 km
Min available freq	4 Hz

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Waveform inversion strategy

- Acoustic forward modeling and P-wave imaging
- Data preprocessing
  - Bandpass filter
  - Top and bottom mute
  - Amplitude scaling
  - Choice of frequency and damping schedule
  - Extraction of Laplace-Fourier components from the data

Waveform inversion strategy

- Minimize L_2 norm: $$2E = \sum_{\omega} \delta \mathbf{d}^T \delta \mathbf{d},$$

  by conjugate gradient method

- Multiscale approach I: Data preconditioning
  - Selection of offset range(s)
  - Sequential inversion of a set of frequencies
  - Laplace-Fourier domain (Time damping)

- Multiscale approach II: Image preconditioning
  - Wavenumber filtering and/or smoothing

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Waveform inversion strategy

- Minimize L_2 norm:

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Objective functions

$$2E = \sum_{\omega} \delta\mathbf{d}^T \delta\mathbf{d},$$

The way in which we define the data residuals is critical

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Discussion on objective functions....

Conventional residual

$$\delta d_j = u_j - d_j$$

predicted observed

Phase-only residual

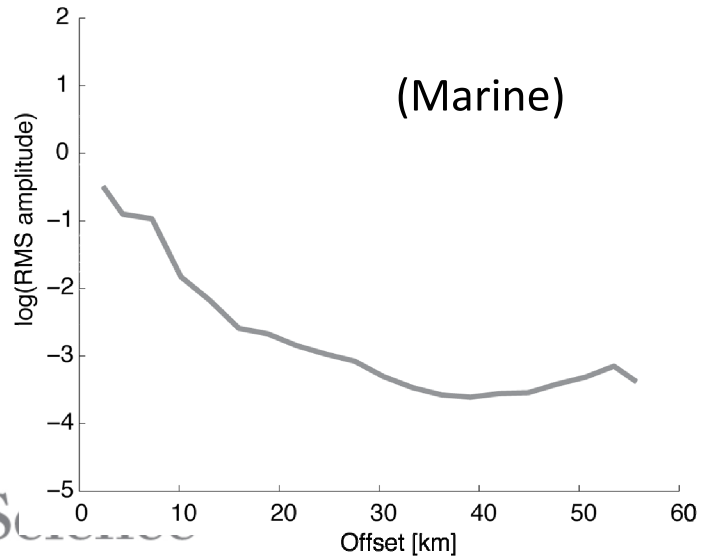
$$\delta d_j = \Im\left[\ln\left(\frac{u_j}{d_j}\right)\right]$$

$$= \arg(u_j) - \arg(d_j)$$

Discussion on objective functions....

	Conventional residual	Phase-only residual
	$\delta d_j = u_j - d_j$ (predicted: u_j; observed: d_j)	$\delta d_j = \Im\left[\ln\left(\frac{u_j}{d_j}\right)\right] = \arg(u_j) - \arg(d_j)$
Dynamic range	$10^{-5} < \lvert\delta d_j\rvert \approx \lvert d_j\rvert < 10$	$-\pi < \lvert\delta d_j\rvert < \pi$

(Marine)



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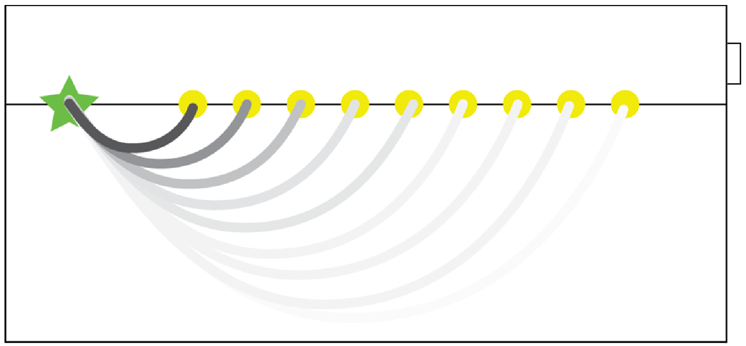
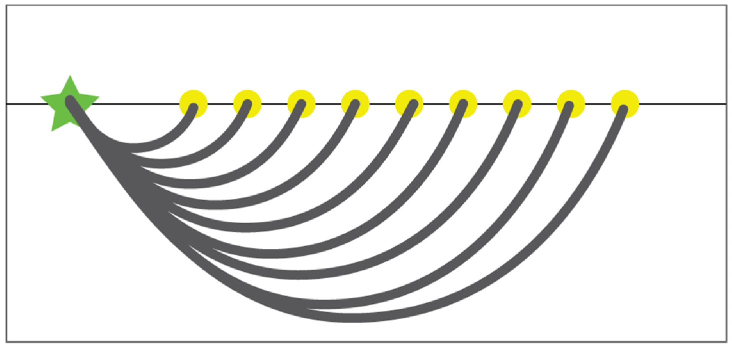
Discussion on objective functions....

	Conventional residual	Phase-only residual
	$\delta d_j = u_j - d_j$ (predicted: u_j; observed: d_j)	$\delta d_j = \Im\left[\ln\left(\frac{u_j}{d_j}\right)\right] = \arg(u_j) - \arg(d_j)$
Dynamic range	$10^{-5} < \lvert\delta d_j\rvert \approx \lvert d_j\rvert < 10$	$-\pi < \lvert\delta d_j\rvert < \pi$
Effective offset	narrow	wide (entire)





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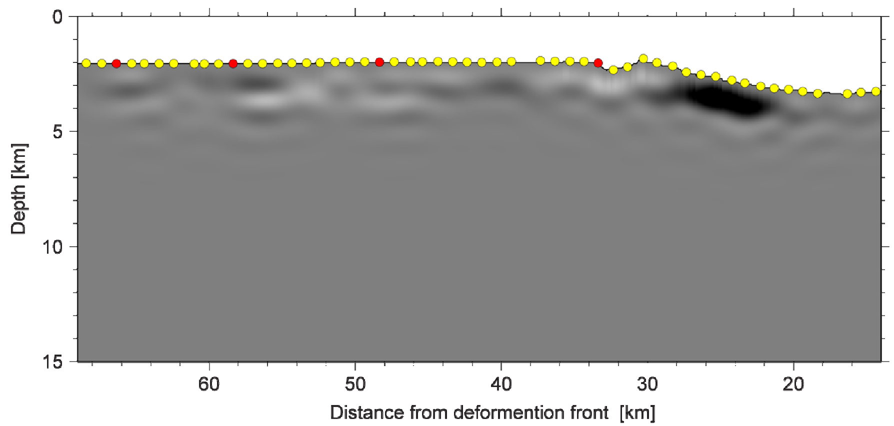
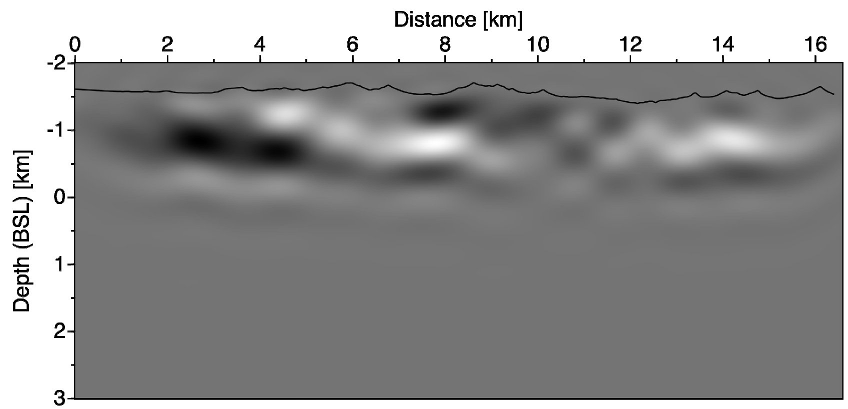
Gradient: Conventional objective function

$$\delta d_j = u_j - d_j$$

Marine	**Land**

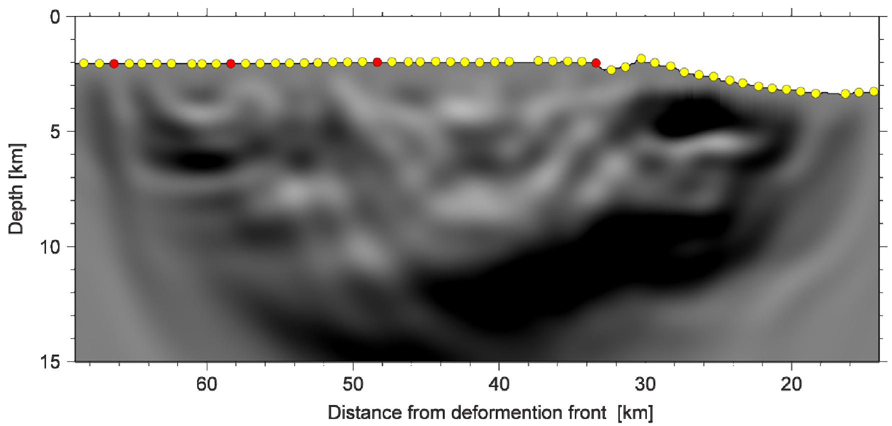




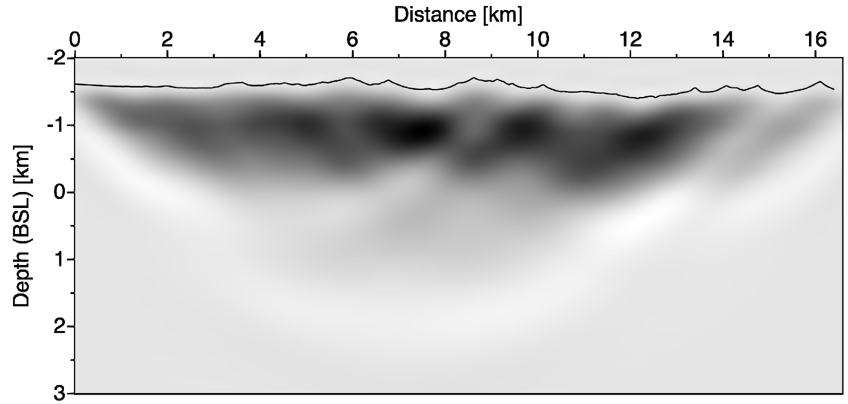
Gradient: Phase-only objective function

$$\delta d_j = \Im\left[\ln\left(\frac{u_j}{d_j}\right)\right]$$

Marine



Land



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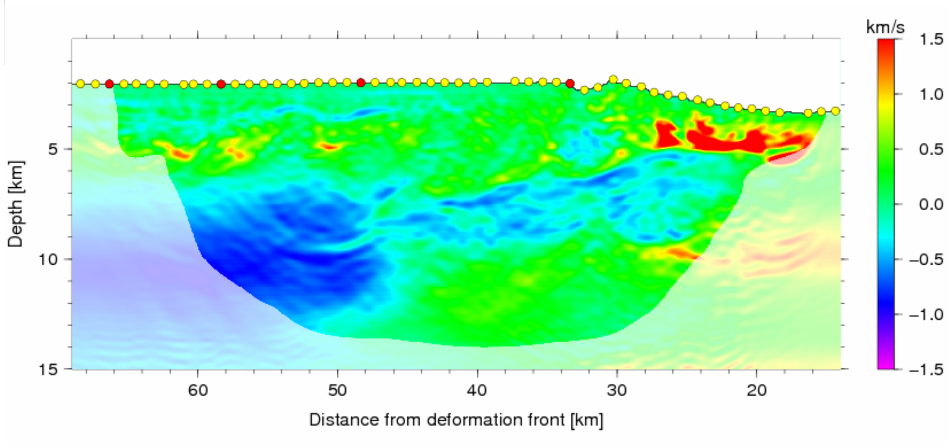
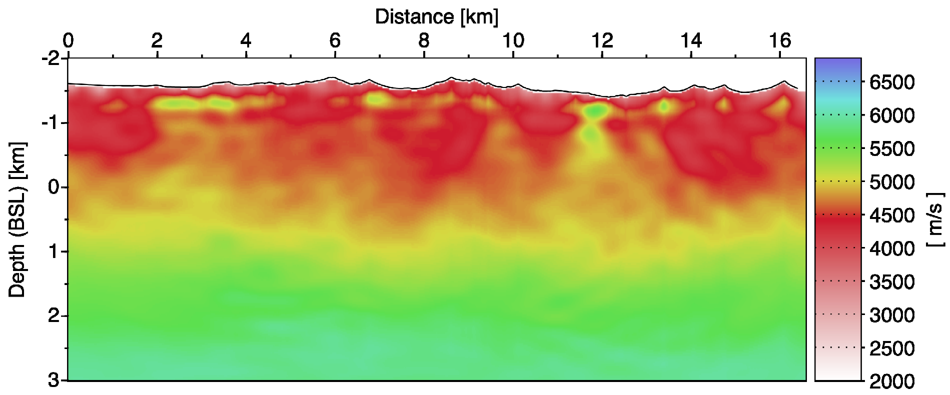
Results: Conventional objective function

$$\delta d_j = u_j - d_j$$

Marine (2.25 – 8.5 Hz)



Land (4.0 – 12.0 Hz)



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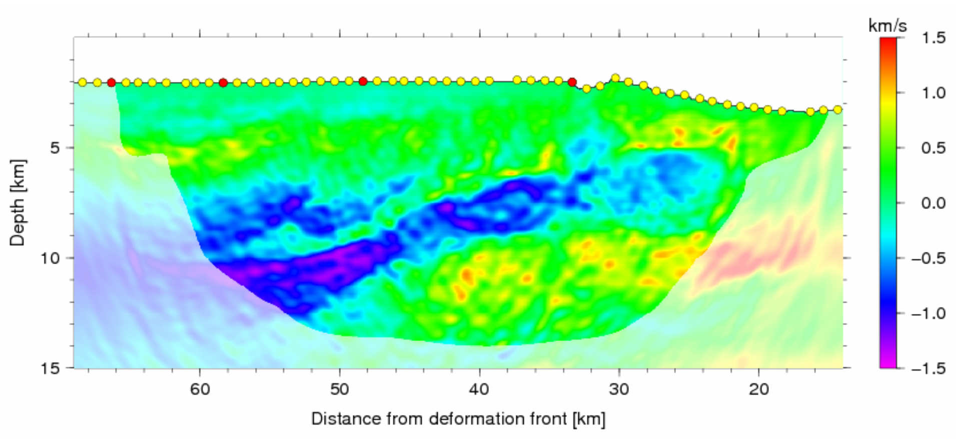
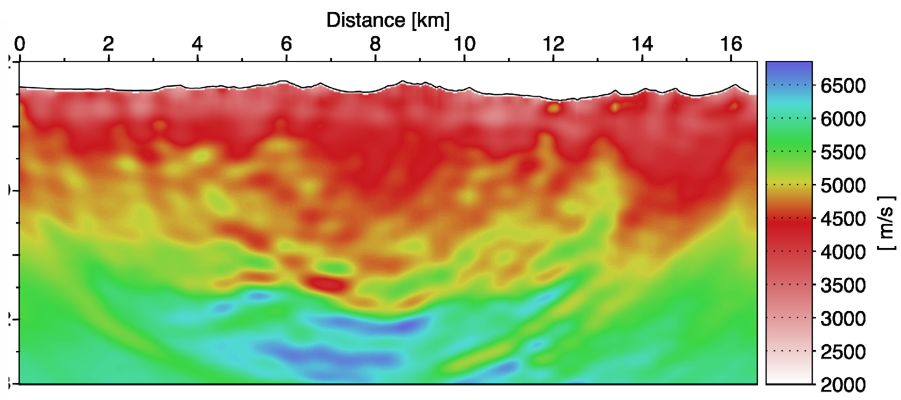
Results: Phase-only

$$\delta d_j = \Im\left[\ln\left(\frac{u_j}{d_j}\right)\right]$$

Marine (2.25 – 8.5 Hz)



Land (4.0 – 12.0 Hz)



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“Phase-only” – are we giving up on amplitudes?

The “phase” information is much richer than simply arrival time information

Complex wavefields with overlapping events have complex phase behaviour

Relative amplitude of events in the data govern the phase behaviour

Laplace-Fourier approach

Time damping (Sirgue and Pratt, 2004)	$\leftrightarrow$	**Laplace-Fourier approach** (Shin and Cha, 2009)
$u(t) \rightarrow u(t)\exp(-t/\tau)$	$\tau = 1/s$	$u(t) \rightarrow u(t)\exp(-st)$

$$\int u(t)\exp(-t/\tau)\exp(-i\omega t)dt = \int u(t)\exp\left[-i(\omega + i/\tau)t\right]dt = u(\omega + i/\tau)$$

$$u(\omega) \rightarrow u(\omega + i/\tau)$$

$$u(\omega) \rightarrow u(\omega + is)$$

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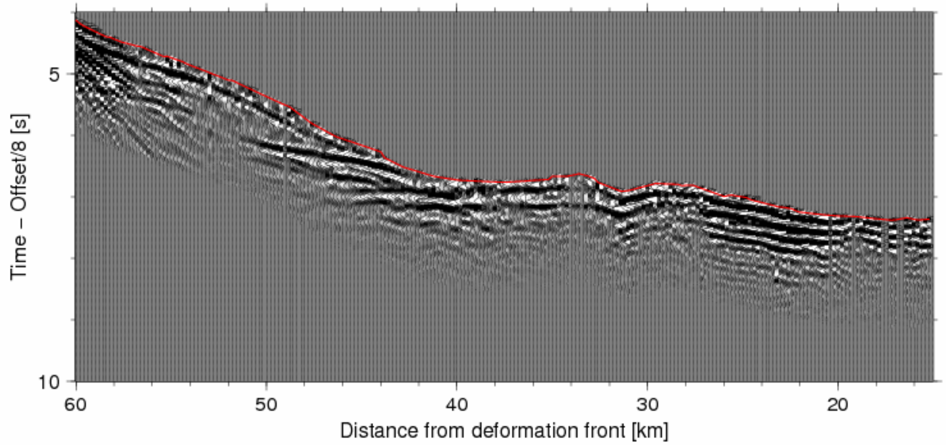
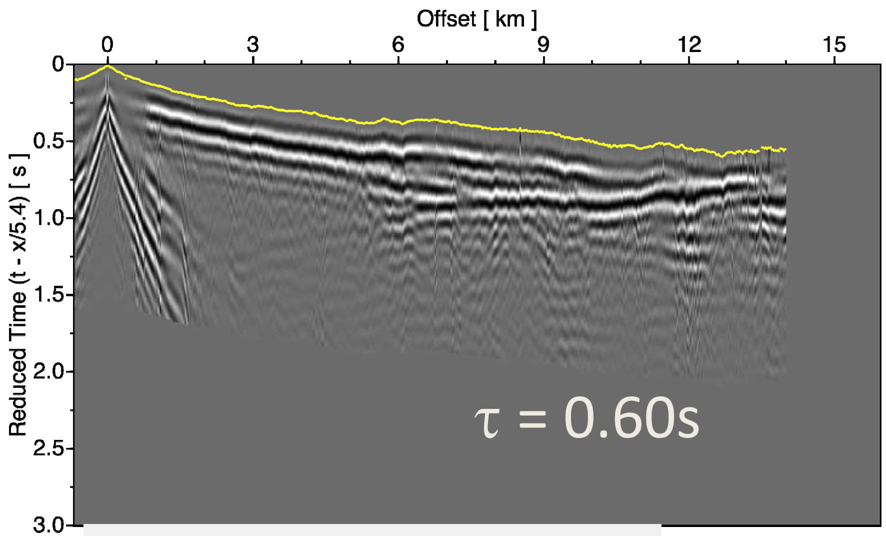
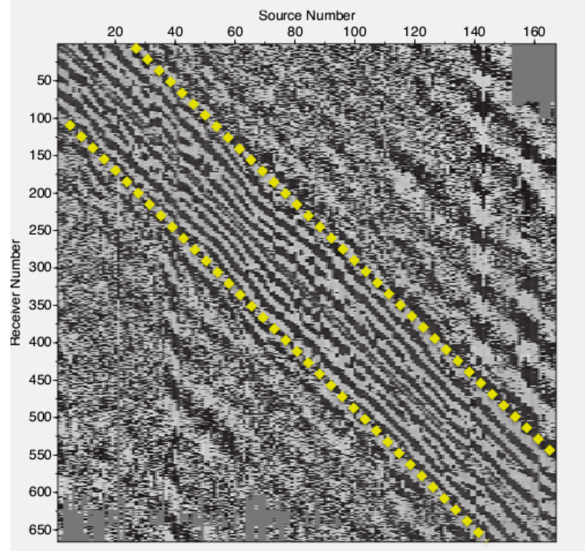
Examples of time damping; damping 1

Marine



Land





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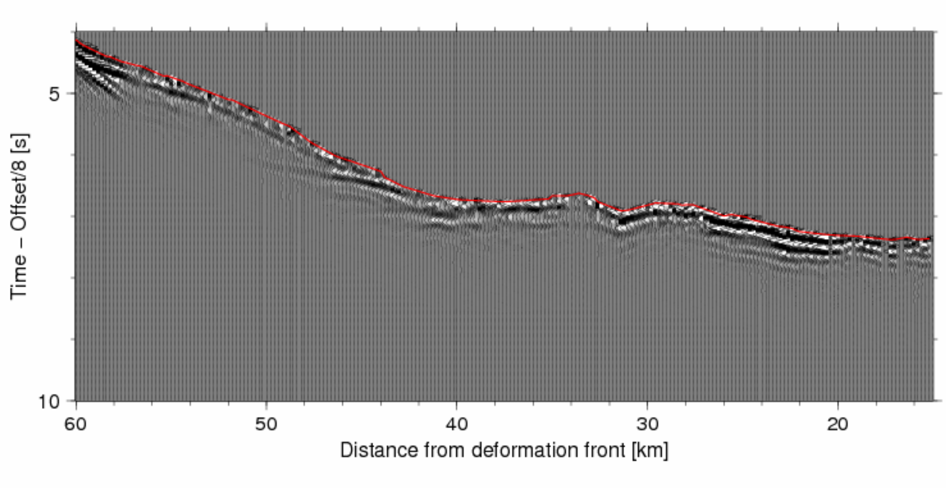
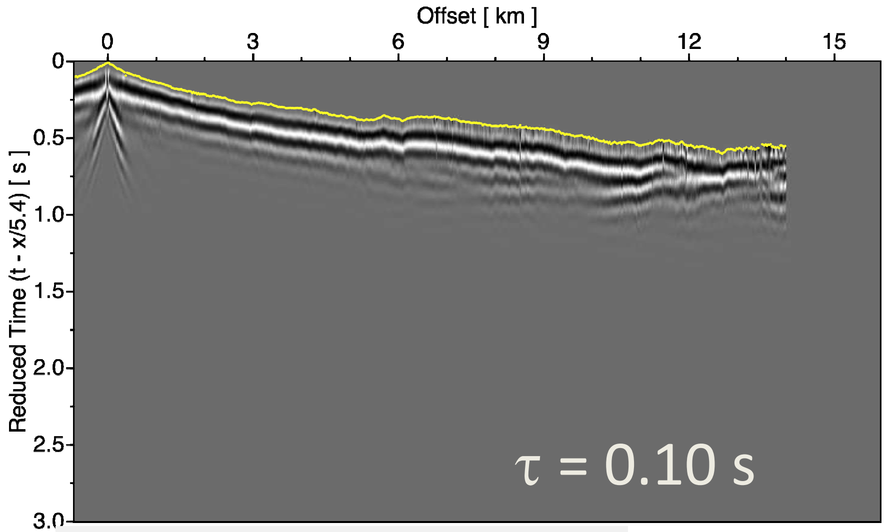
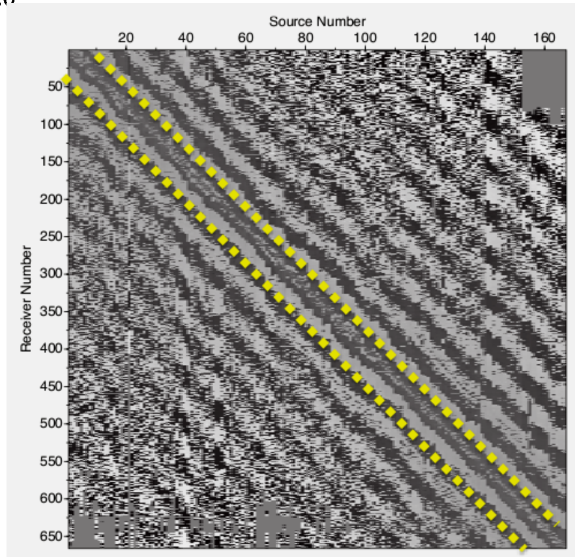
Examples of time damping; damping 2

Marine



Land





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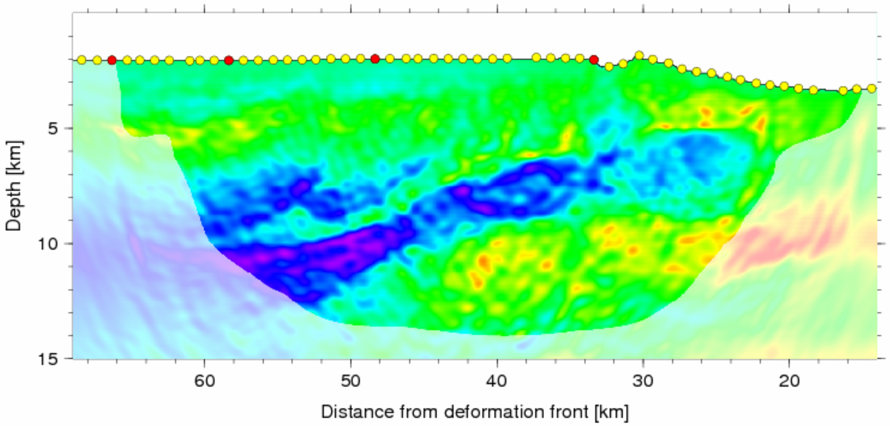
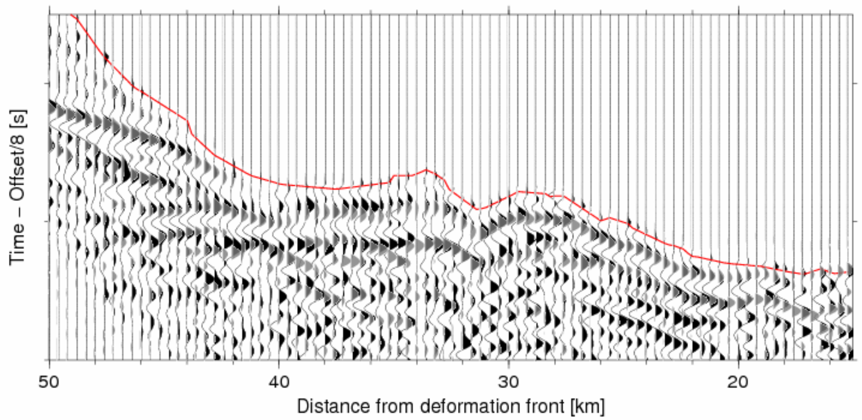
Models and waveform fit

Marine





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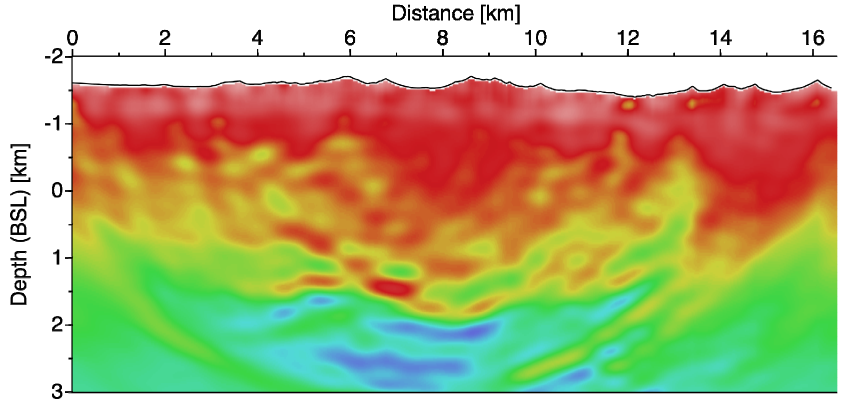

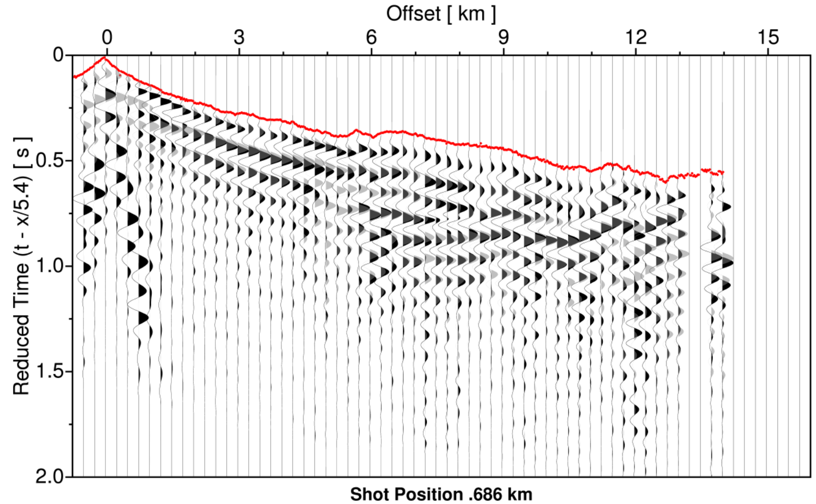

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Outline

- Long offset seismic data
  - An imaging opportunity
- Waveform Tomography
  - Theoretical aspects
  - Making it work
- Nankai Trough Subduction Zone Example
  - Geological implications
  - The Challenge of High Resolution

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