Numerical implementation of a perfect transparent elastic mirror for the reconstruction of direct and time reversed wavefields

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1 : Initial value

2 : elastic mirror

3 : sources (forward) / sink (backward)

Traveling waves...





Sink



Initial Value



Backward

! Initial value might need to be reseted periodically when attenuation is present !

"Rigid" elastic mirror

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A) Record displacement at all grid nodes on the mirror

B) Force the displacement to match the recorded values

Direct



Time Reversed



Summary for the rigid mirror :

- Perfect accuracy
- Mirror must match the elements border
- Mirror is not transparent
- We need to store

$$N = 3Sh^2$$

Transparent mirror using a set of point sources

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Representation theorem

$$u_{i}(\mathbf{x}) = \int_{V} G_{in}(\mathbf{x}, \mathbf{x}') f_{n}(\mathbf{x}') dV' + \oint_{S} \{G_{in}(\mathbf{x}, \mathbf{x}') n_{j} c_{njkl} \partial'_{k} u_{l}(\mathbf{x}') - u_{n}(\mathbf{x}') n_{j} c_{njkl} \partial'_{k} G_{il}(\mathbf{x}, \mathbf{x}') \} dS'$$

Monopole source



Dipole

source

Direct



Time Reversed



Direct





(Figure form Nissen-Meyer et al 2007)

"Standard method" Express the Dirac delta function in the spectral element basis to get the f coefficients

$$\mathbf{M}_e \ddot{\mathbf{u}} + \mathbf{K}_e \mathbf{u} = \mathbf{f}$$

$$\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_s) S(t)$$

$$u_n = \mathbf{M}_{pq} * \mathbf{G}_{np,q}$$

"Perfect Source" Substitute the displacement with an analytical solution to get the **f** coefficients



Summary for the point source mirror :

- Variable accuracy depending on the number of sources
- Can handle arbitrary geometry (at the expense of adding more sources)
- Mirror is transparent

• We need to store

$$N = 6Sh_s^2$$

Perfect transparent mirror using discrete differences

Forward simulation

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$$\mathbf{M}_e \ddot{\mathbf{u}} + \mathbf{K}_e \mathbf{u} = 0$$

Compute mirror source coefficients by differentiating the windowed wavefield

> w = 0 ouside the mirror w = 1 inside the mirror

 $\mathbf{M}_e(w\ddot{\mathbf{u}}) + \mathbf{K}_e(w\mathbf{u}) = \mathbf{f}_{Mirror}$

Similar to grid injection when used with finite differences (see Robertsson 2000)



Summary for the mirror based on discrete differences :

- Perfect accuracy
- Can handle arbitrary geometry
- Mirror is transparent
- We need to store

$$N = 3\frac{p}{2}Sh^2$$
 General $N = 3Sh^2$ Best

Why transparency matters ...











Local modeling + field extrapolation = synthetic data at receivers





Piecewise inversion ?

- A) Compute green functions from receivers to mirrors in the current model using global simulations
- B) Perform piecewise waveform tomography (Possibly one inversion per core)
- C) Assembly of the updated model
- Iterate...

Conclusion

- We introduced a perfectly transparent elastic mirror allowing to reconstruct the seismic wavefield within a volume enclosed in a surface of arbitrary shape
- Combined with field extrapolation, the mirror can be used to Perform Local waveform tomography using a global dataset
- to be continued...