Numerical implementation of a perfect transparent elastic mirror for the reconstruction of direct and time reversed wavefields

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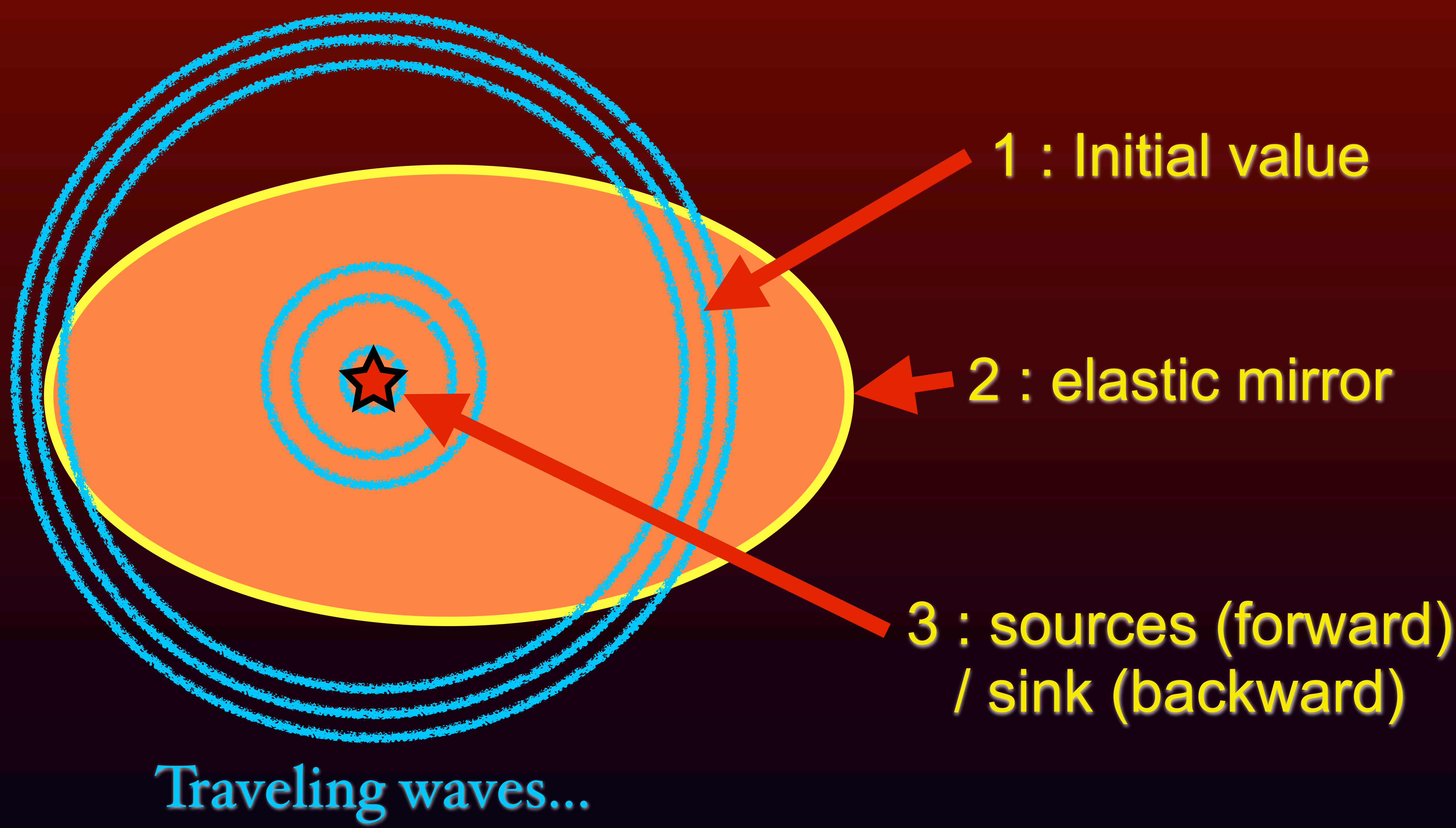
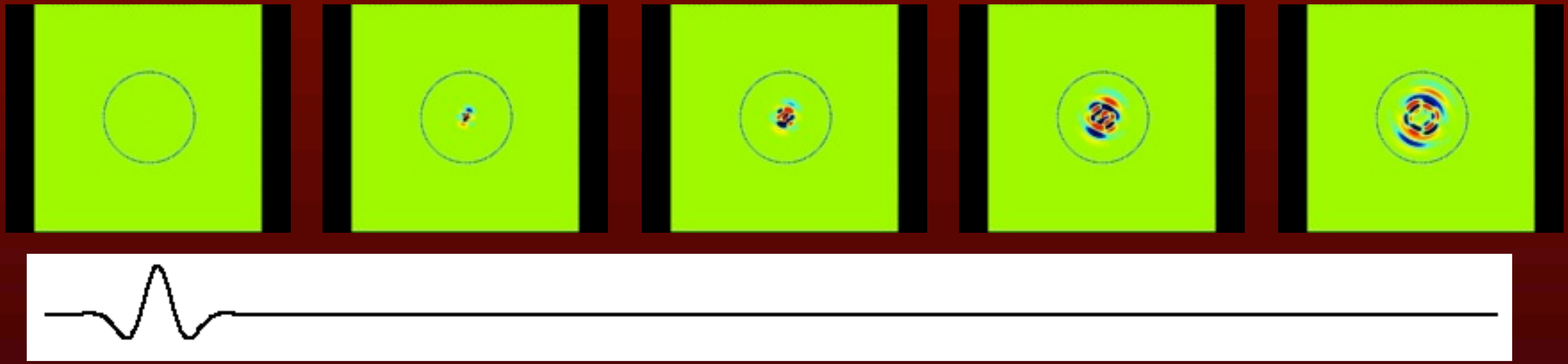
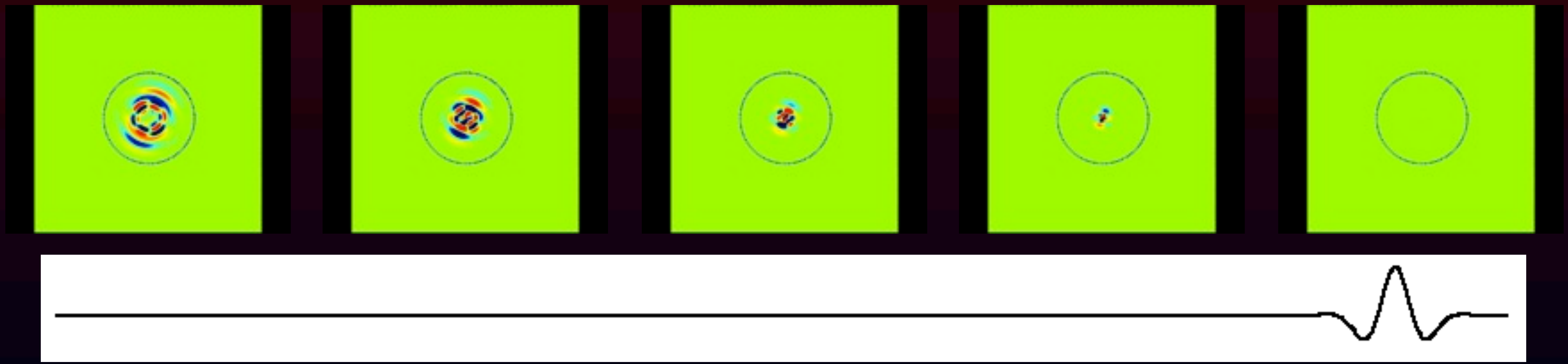
Reconstructing a wave field within an enclosed volume



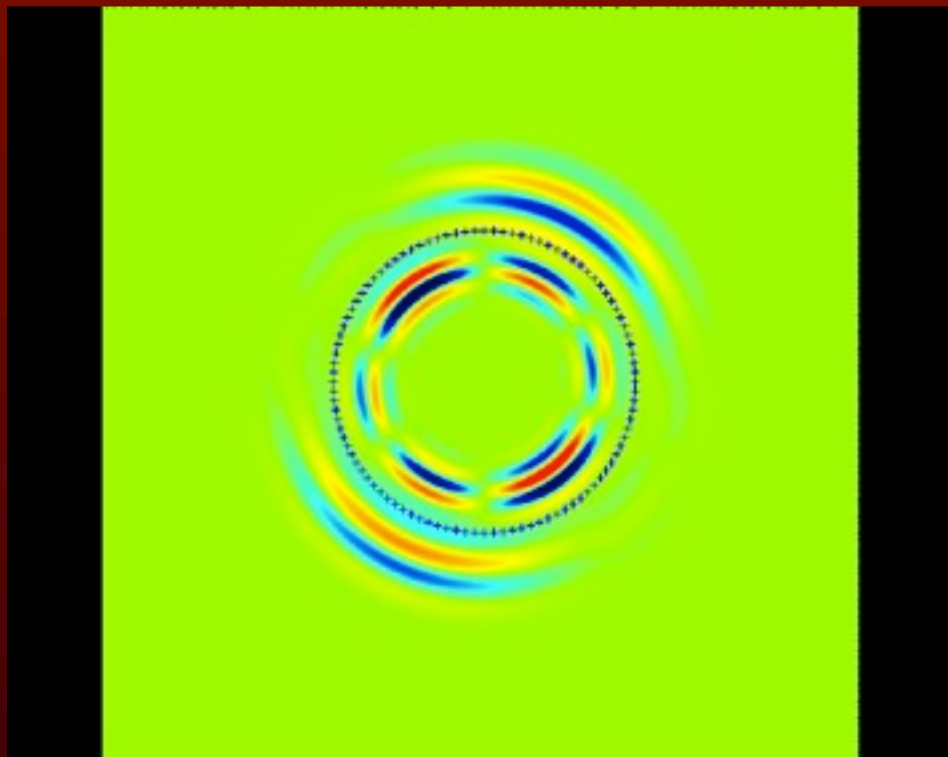
Source

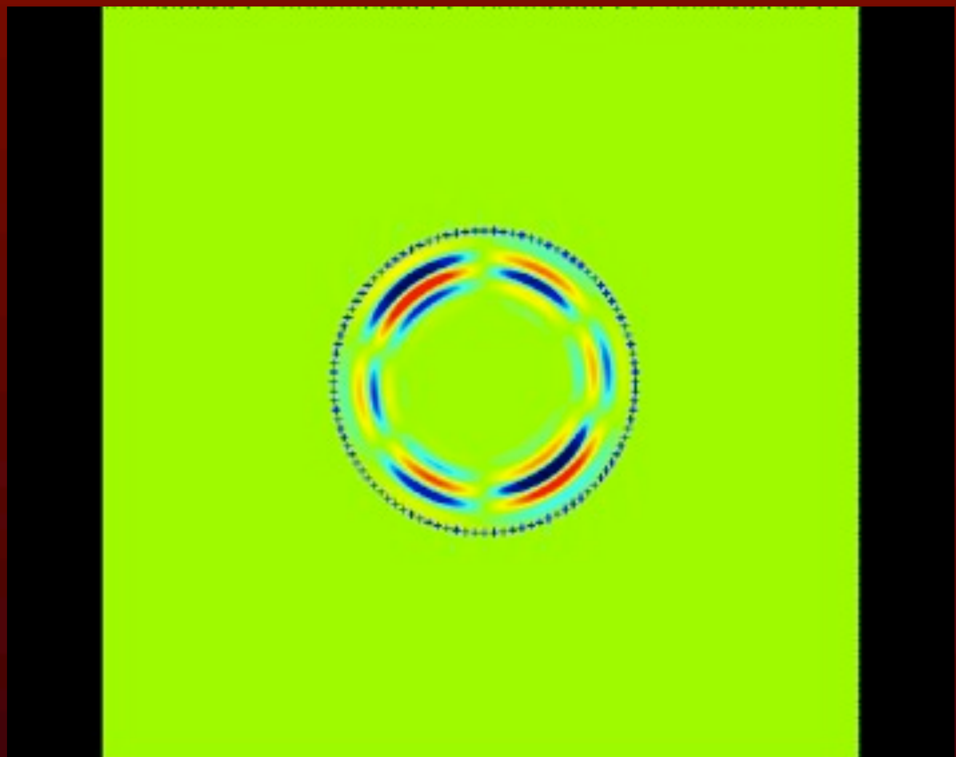


Sink



Initial Value





$$\mathbf{M}_e \frac{\mathbf{u}^{t+\Delta t} - 2\mathbf{u}^t + \mathbf{u}^{t-\Delta t}}{\Delta t^2} + \mathbf{K}_e \mathbf{u} = \mathbf{f}$$

$$\mathbf{u}^{t+\Delta t} = \Delta t^2\, \mathbf{M}_e^{-1} \left(\mathbf{f} - \mathbf{K}_e \mathbf{u}\right) + 2\mathbf{u}^t - \mathbf{u}^{t-\Delta t}$$

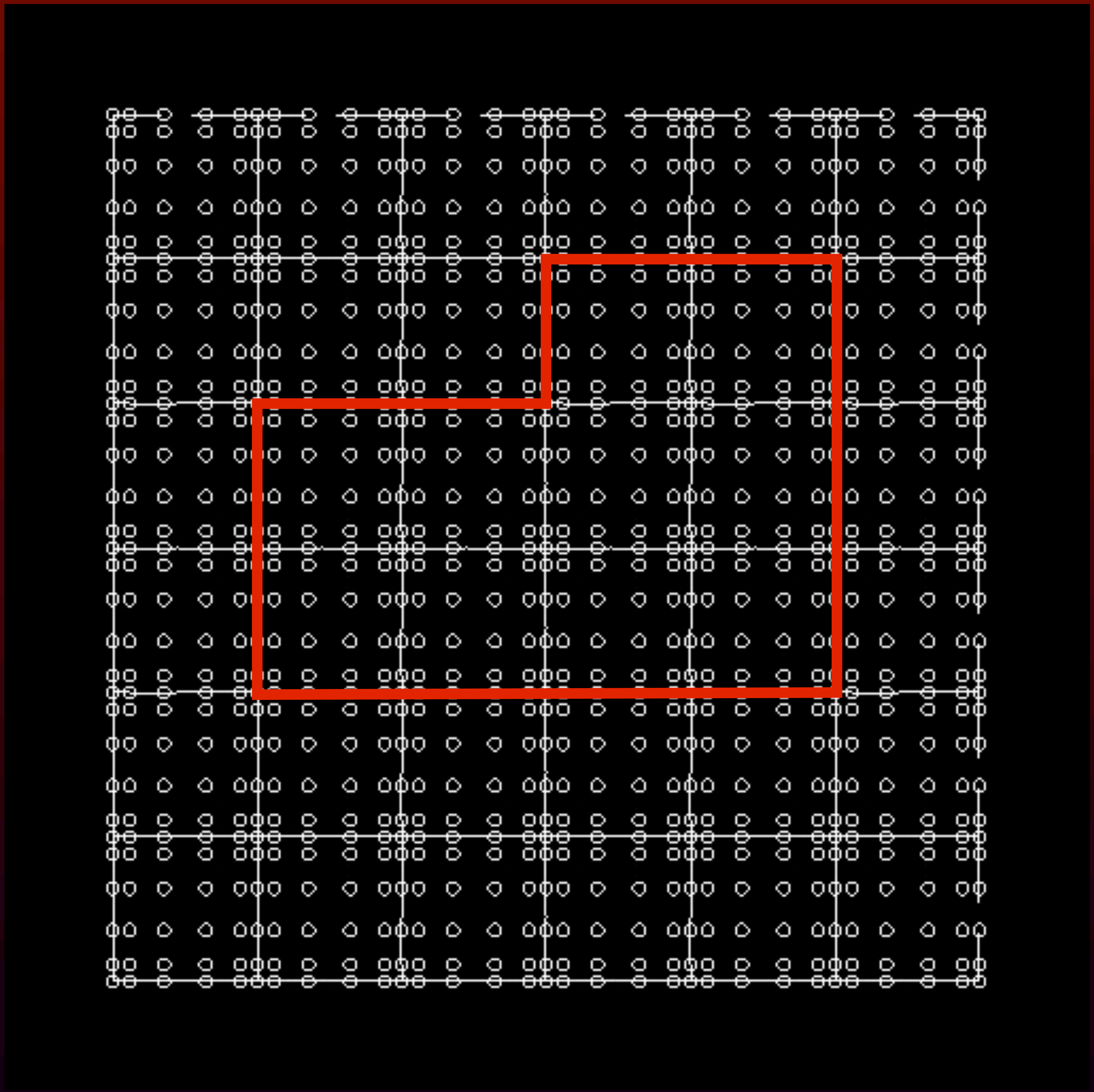
Forward

$$\mathbf{u}^{t-\Delta t} = \Delta t^2\, \mathbf{M}_e^{-1} \left(\mathbf{f} - \mathbf{K}_e \mathbf{u}\right) + 2\mathbf{u}^t - \mathbf{u}^{t+\Delta t}$$

Backward

! Initial value might need to be reseted periodically when attenuation is present !

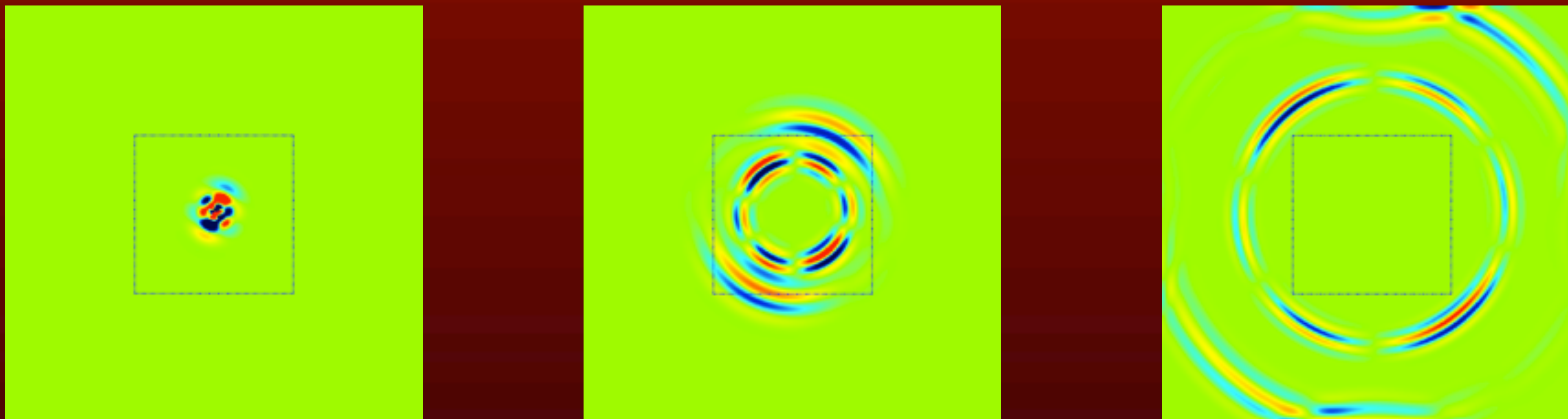
“Rigid” elastic mirror



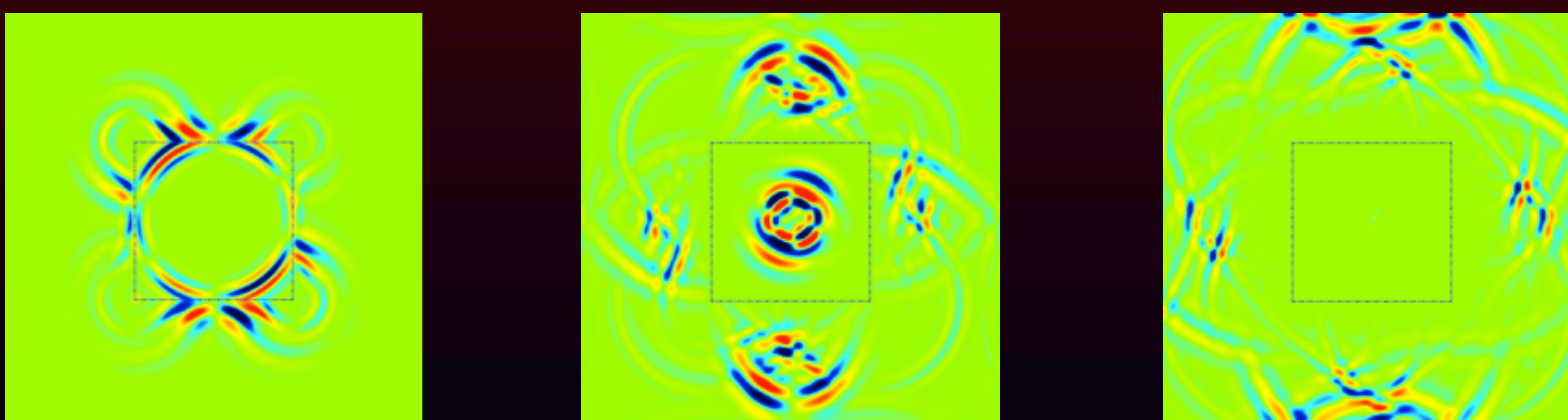
A) Record displacement at all grid nodes on the mirror

B) Force the displacement to match the recorded values

Direct



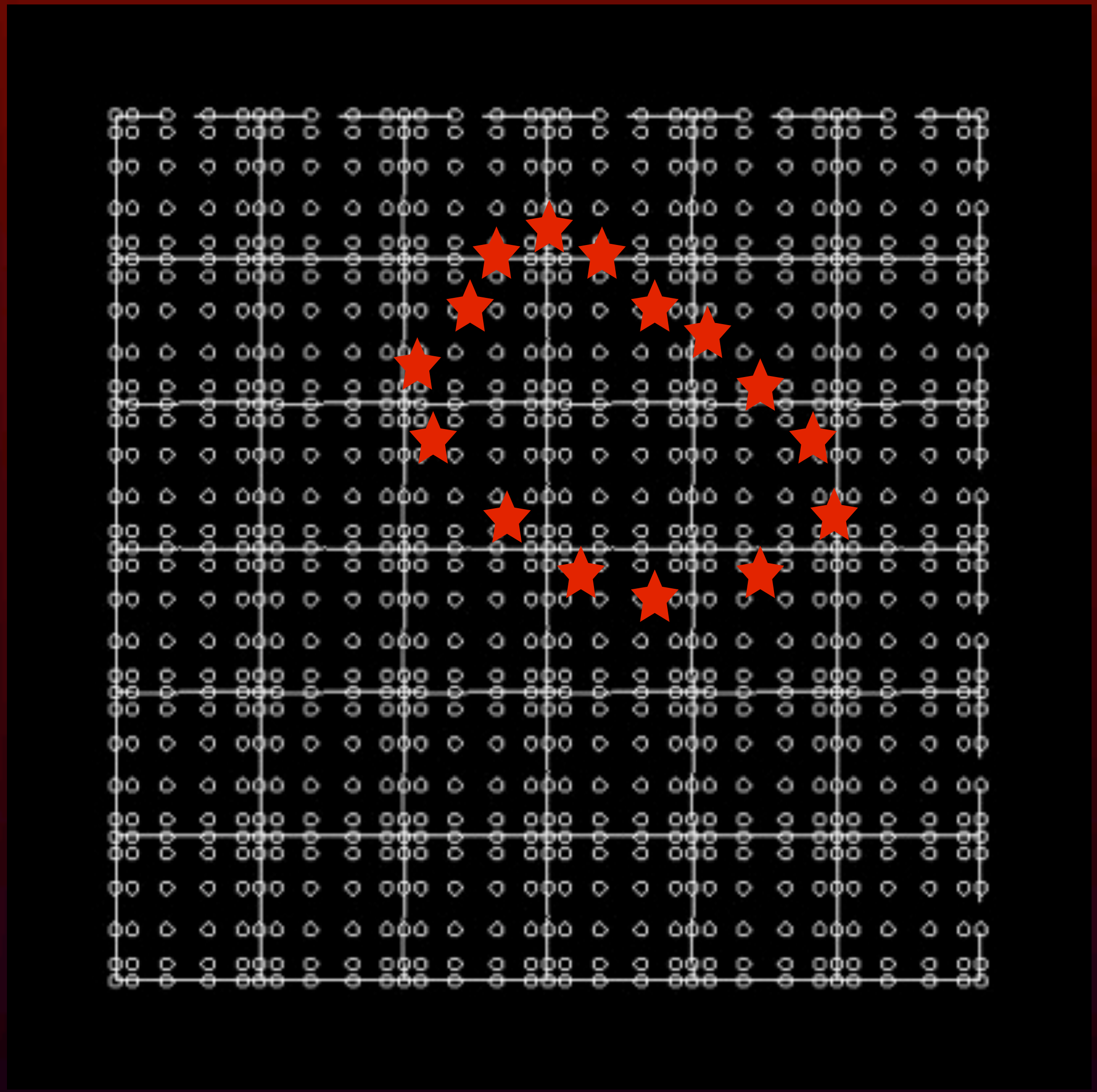
Time Reversed



Summary for the rigid mirror :

- Perfect accuracy
- Mirror must match the elements border
- Mirror is not transparent
- We need to store $N = 3Sh^2$

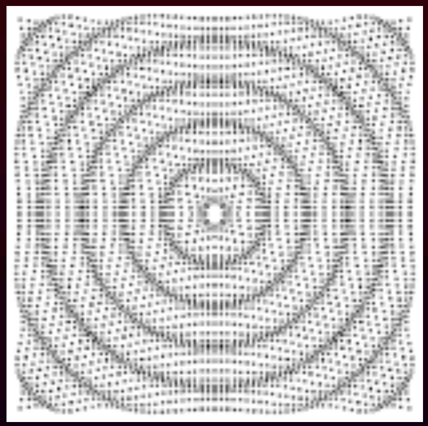
Transparent mirror using a set of point sources



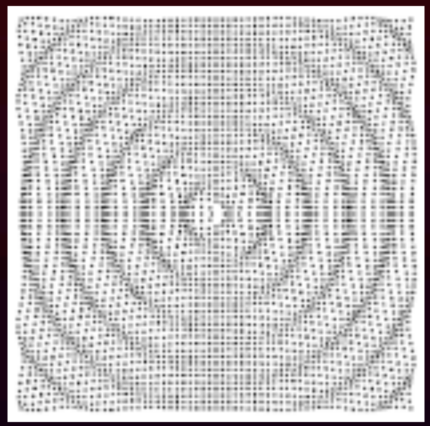
Representation theorem

$$
\begin{aligned}
u_i(\mathbf{x}) \quad &= \int_V G_{in}(\mathbf{x},\mathbf{x}') f_n(\mathbf{x}') dV' \\
&+ \oint_S \left\{ G_{in}(\mathbf{x},\mathbf{x}') n_j c_{njkl} \partial'_k u_l(\mathbf{x}') - u_n(\mathbf{x}') n_j c_{njkl} \partial'_k G_{il}(\mathbf{x},\mathbf{x}') \right\} dS'
\end{aligned}
$$

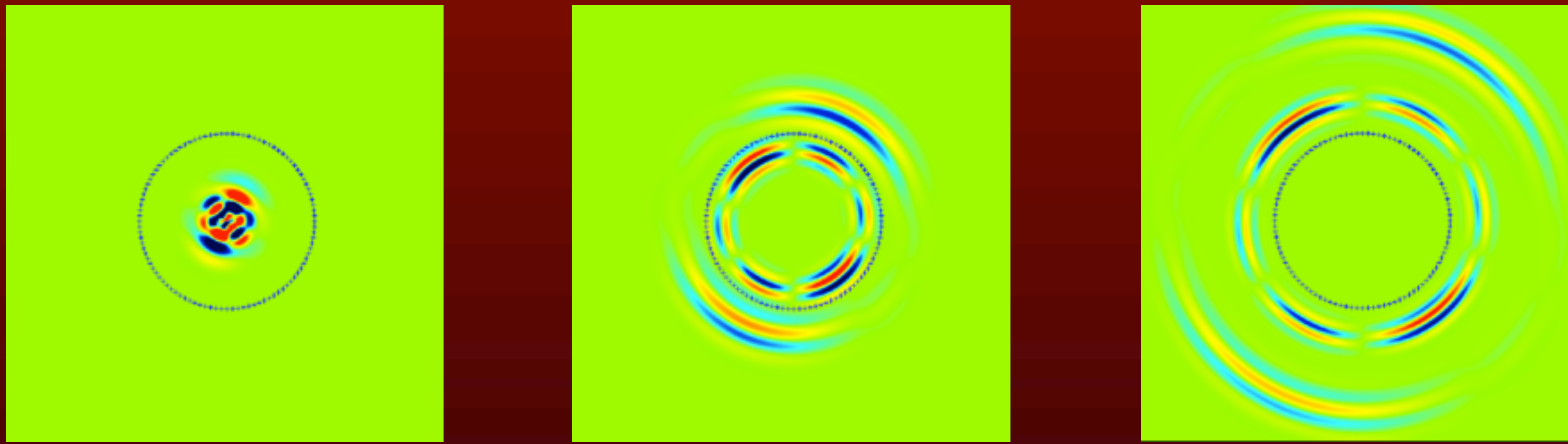
Monopole source



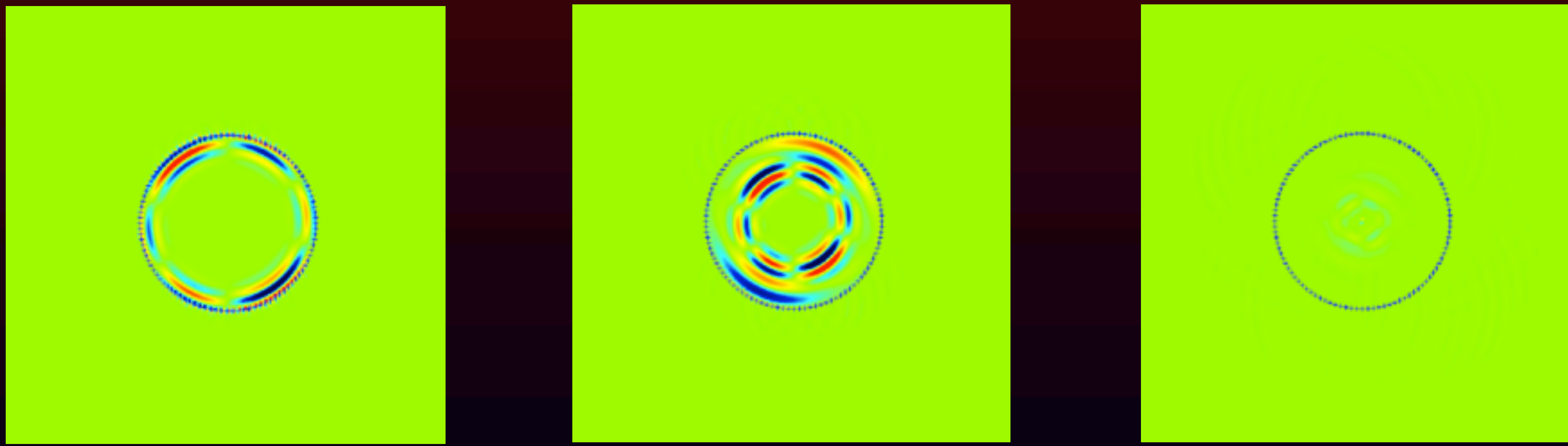
Dipole source



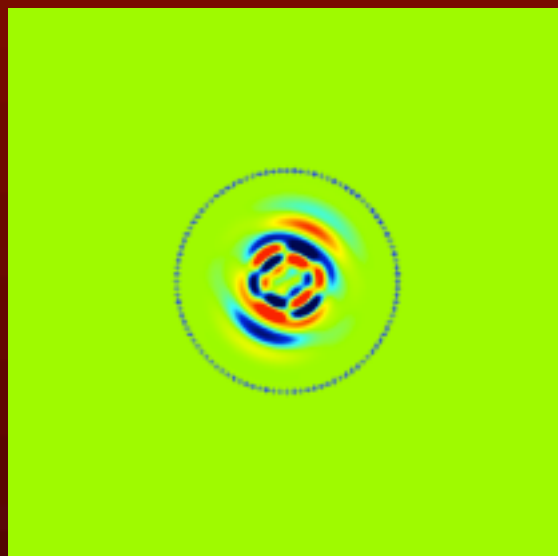
Direct



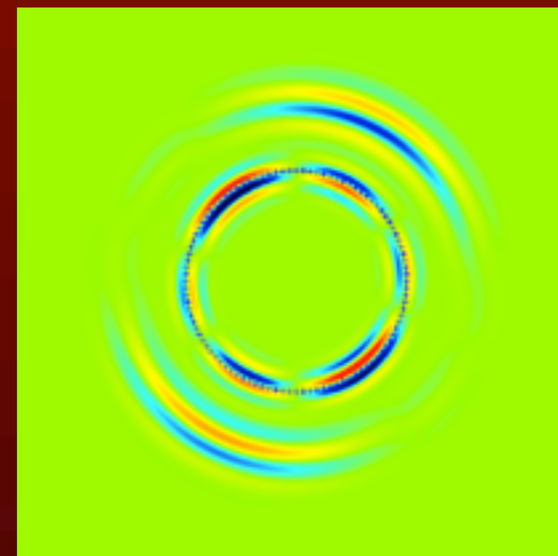
Time Reversed



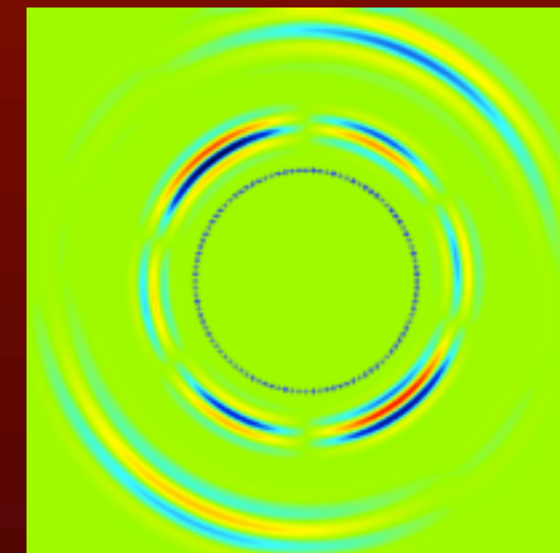
Direct



Pure initial value

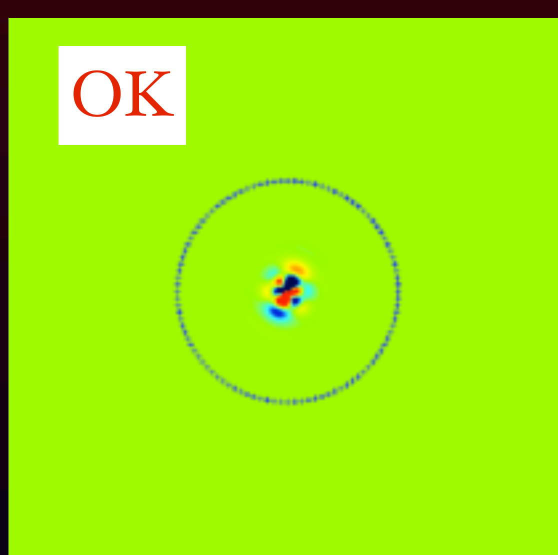


Mixed initial / boundary value



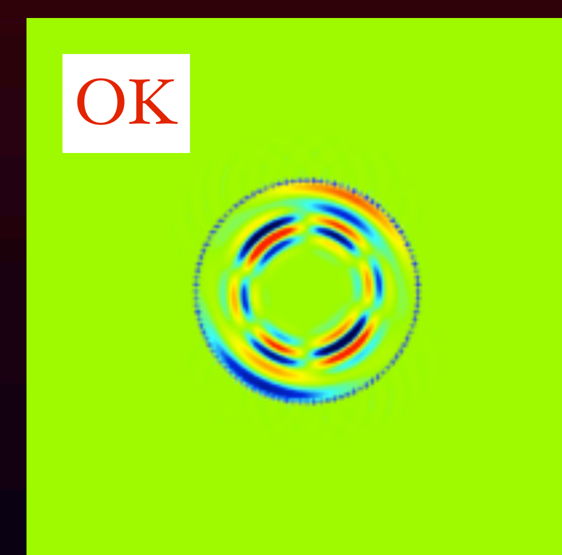
Pure boundary value

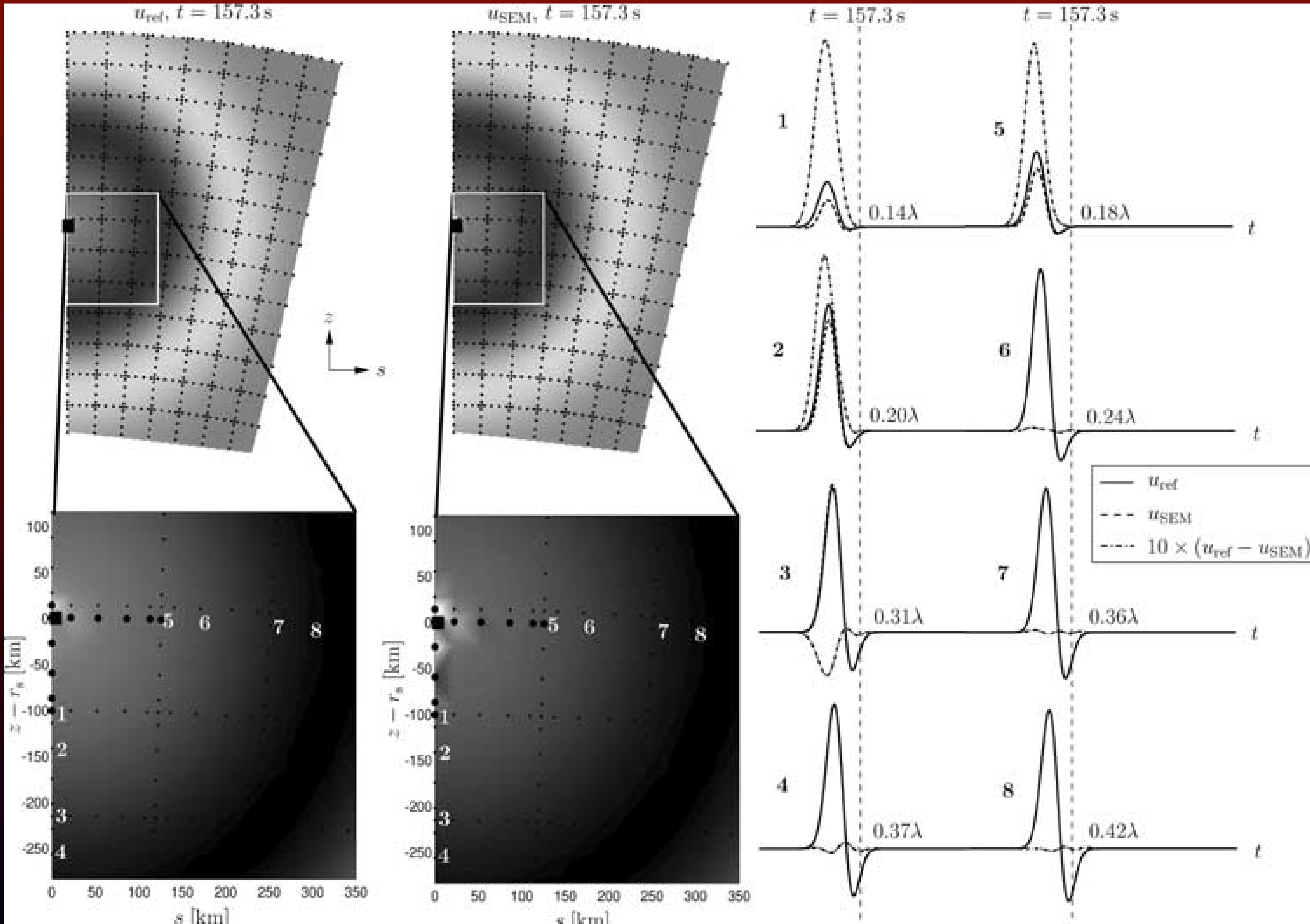
Time Reversed



Time Reversed

Time Reversed





(Figure form Nissen-Meyer et al 2007)

“Standard method”

Express the Dirac delta function in the spectral element basis to get the **f** coefficients

$$\mathbf{M}_e\ddot{\mathbf{u}} + \mathbf{K}_e\mathbf{u} = \mathbf{f}$$

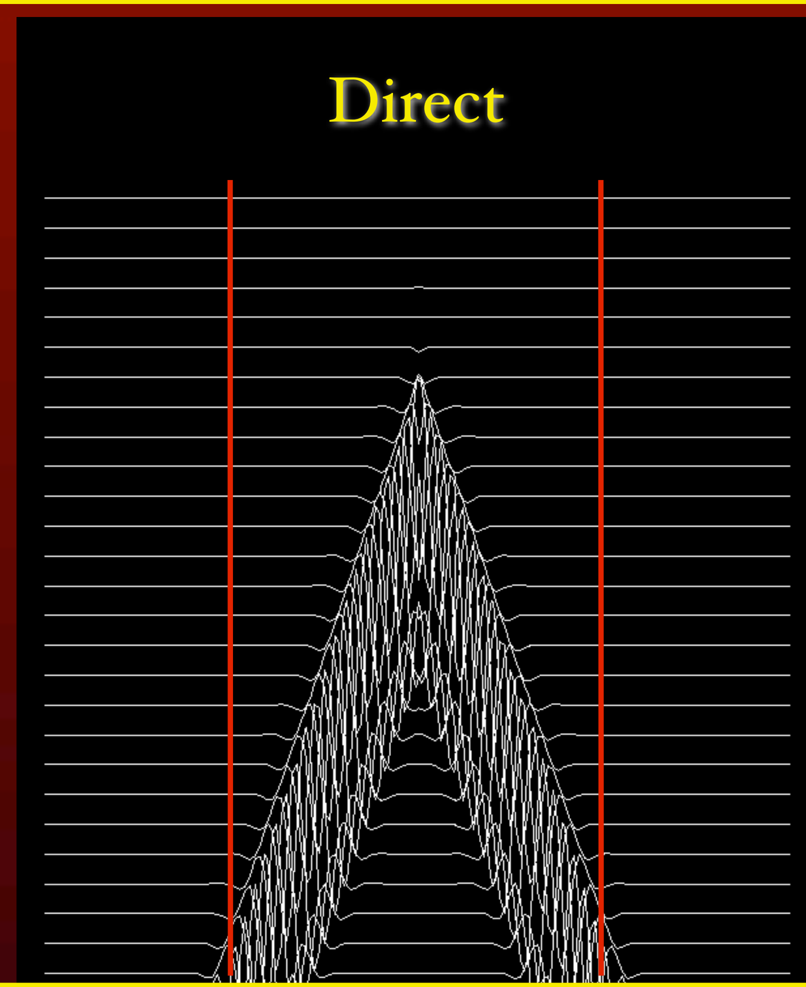
$$\mathbf{f} = -\mathbf{M} \,.\, \nabla\delta(\mathbf{x} - \mathbf{x}_s)S(t)$$

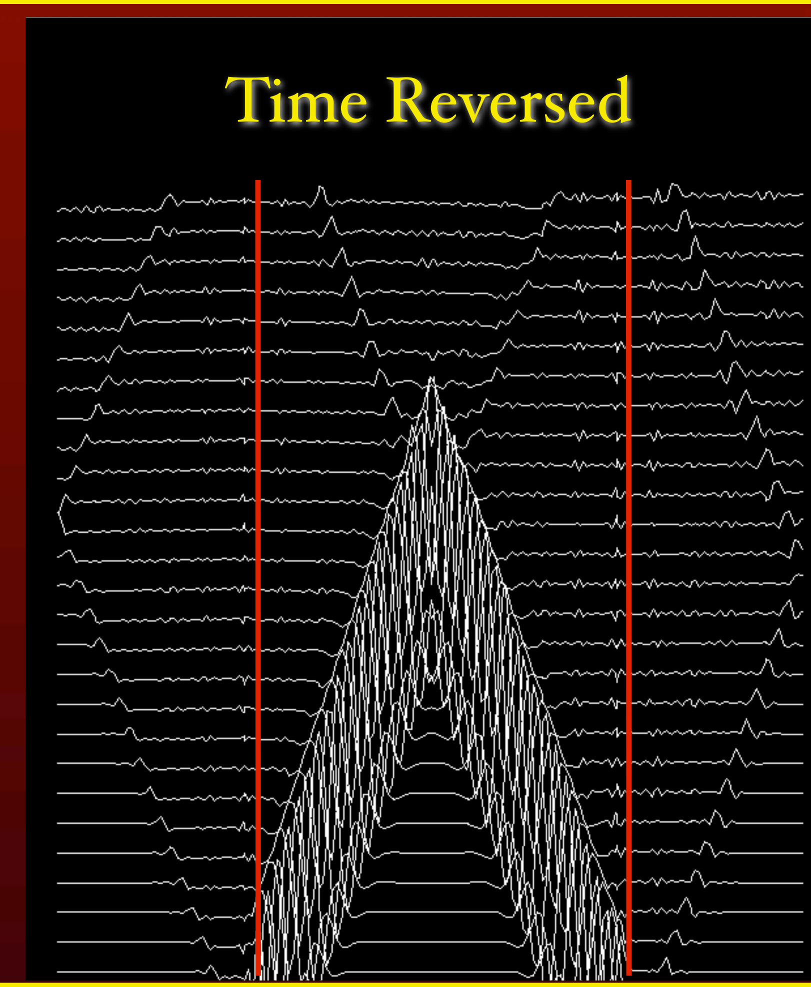
$$u_n = \mathbf{M}_{pq} * \mathbf{G}_{np,q}$$

“Perfect Source”

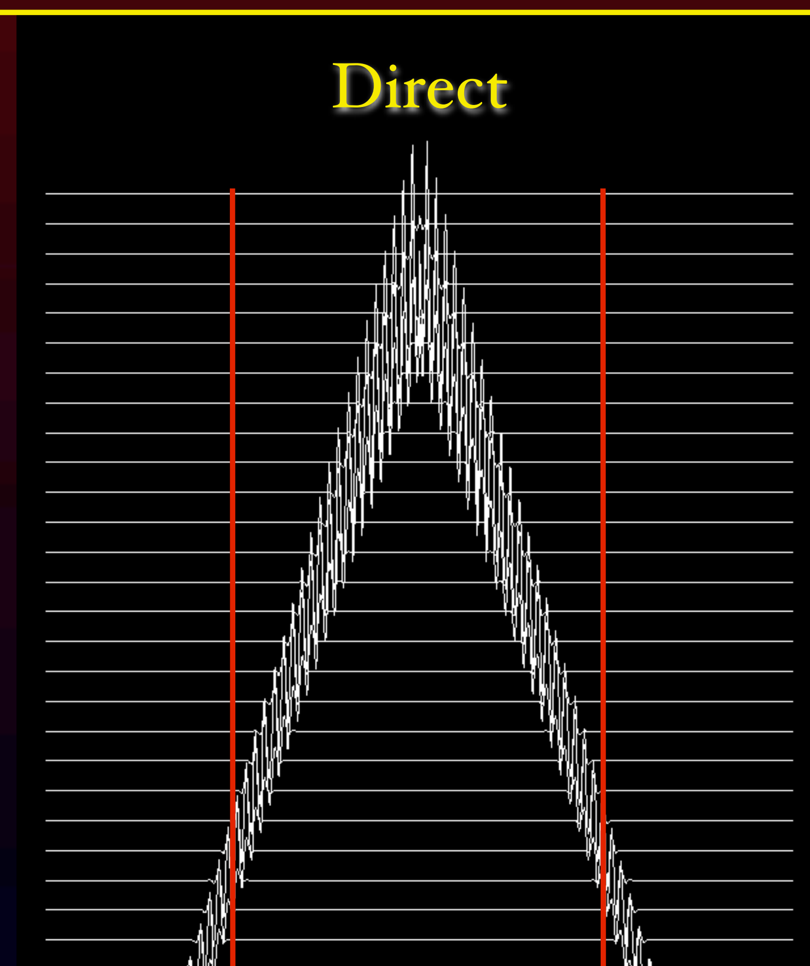
Substitute the displacement with an analytical solution to get the **f** coefficients

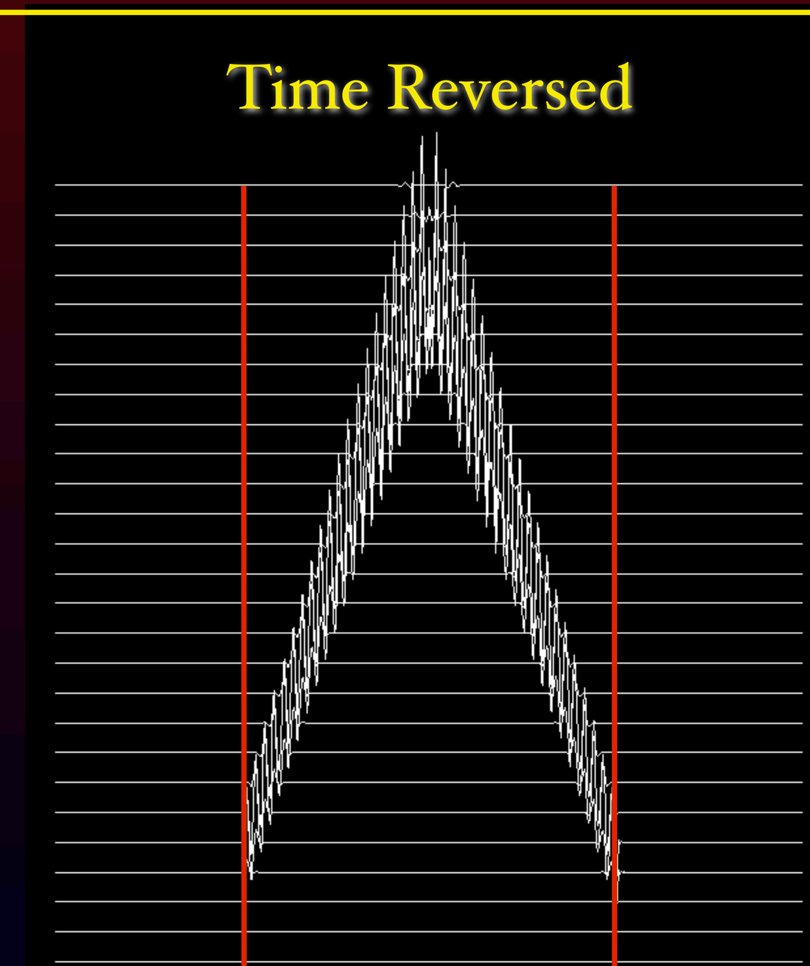
“Standard”
Source





“Perfect”
Source

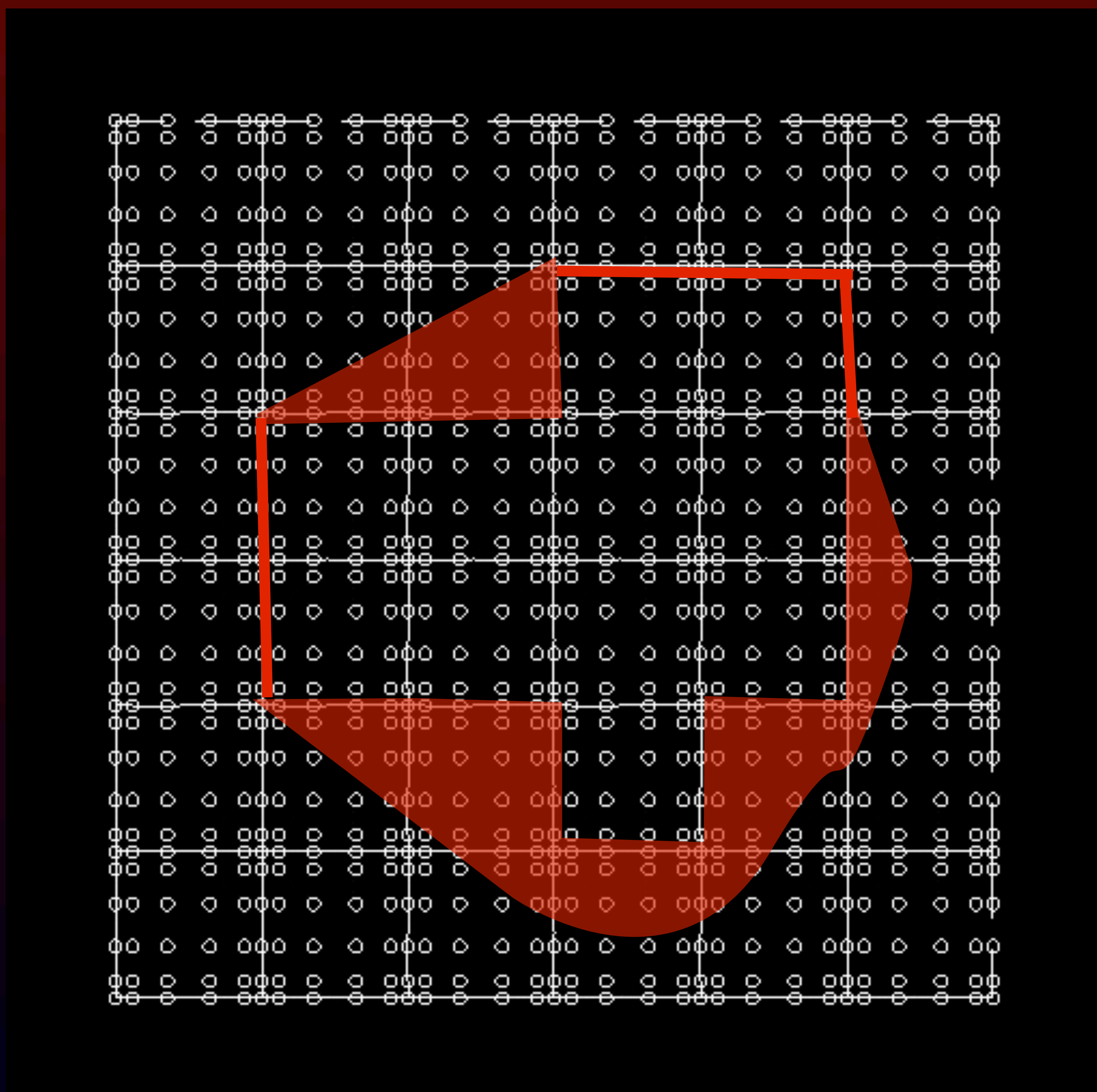




Summary for the point source mirror :

- Variable accuracy depending on the number of sources
- Can handle arbitrary geometry (at the expense of adding more sources)
- Mirror is transparent
- We need to store $N = 6Sh_s^2$

Perfect transparent mirror using discrete differences



Forward simulation

$$\mathbf{M}_e \ddot{\mathbf{u}} + \mathbf{K}_e \mathbf{u} = 0$$

Compute mirror source coefficients by differentiating the windowed wavefield

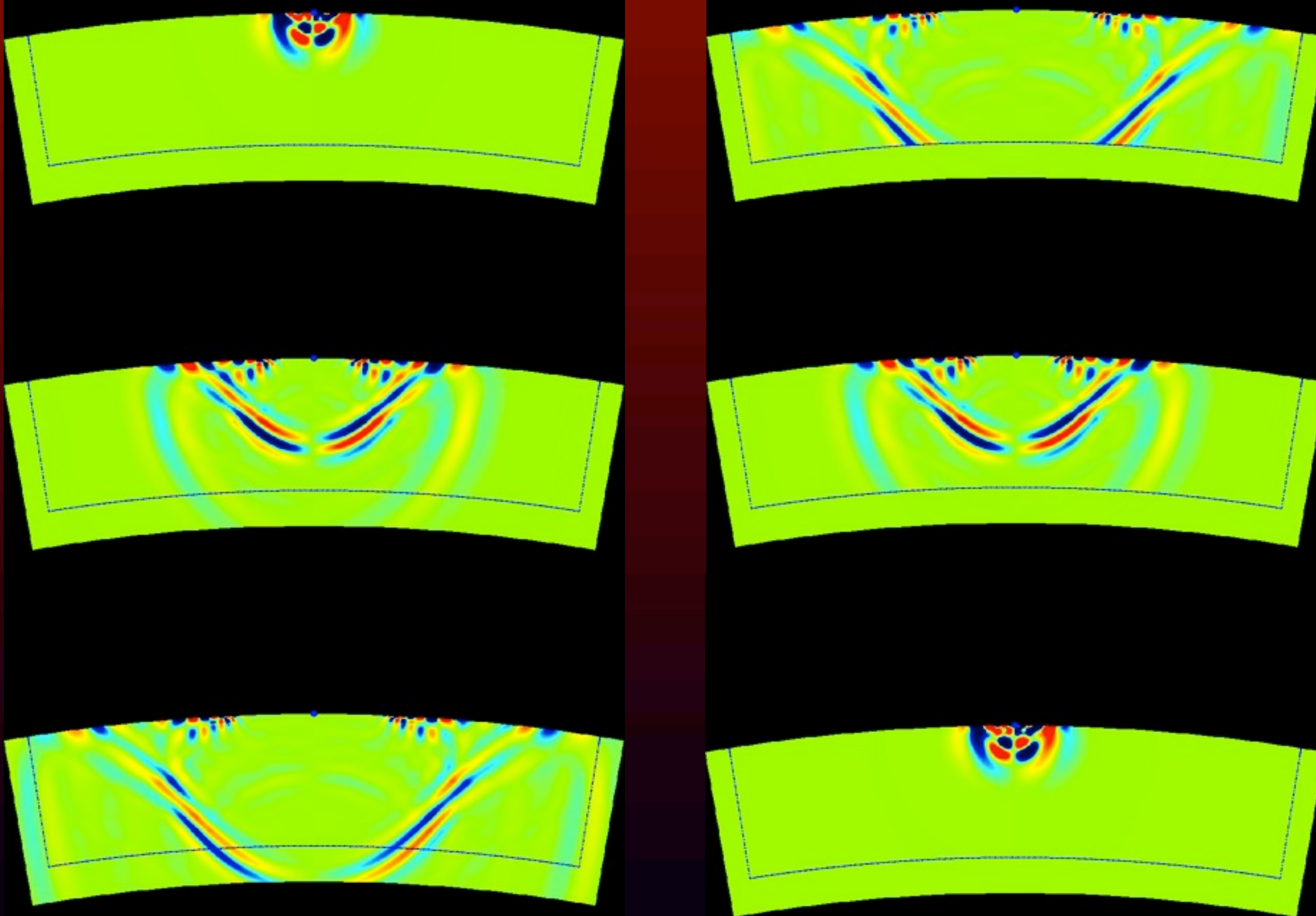
$$w = 0 \quad \textit{ouside the mirror}$$

$$w = 1 \quad \textit{inside the mirror}$$

$$\mathbf{M}_e(w\ddot{\mathbf{u}}) + \mathbf{K}_e(w\mathbf{u}) = \mathbf{f}_{Mirror}$$

Similar to grid injection when used with finite differences (see Robertsson 2000)

Forward | Time reversed



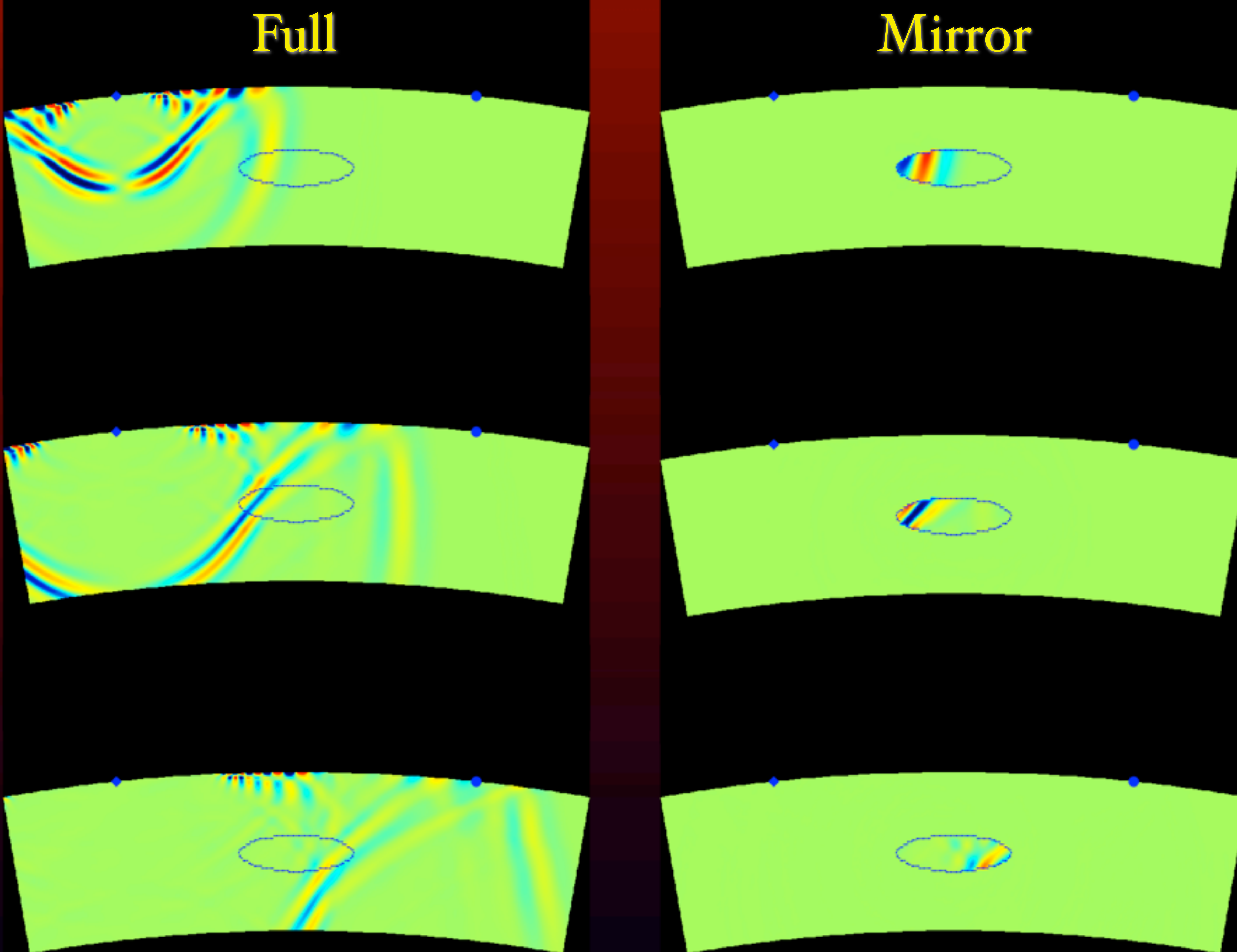
Summary for the mirror based on discrete differences :

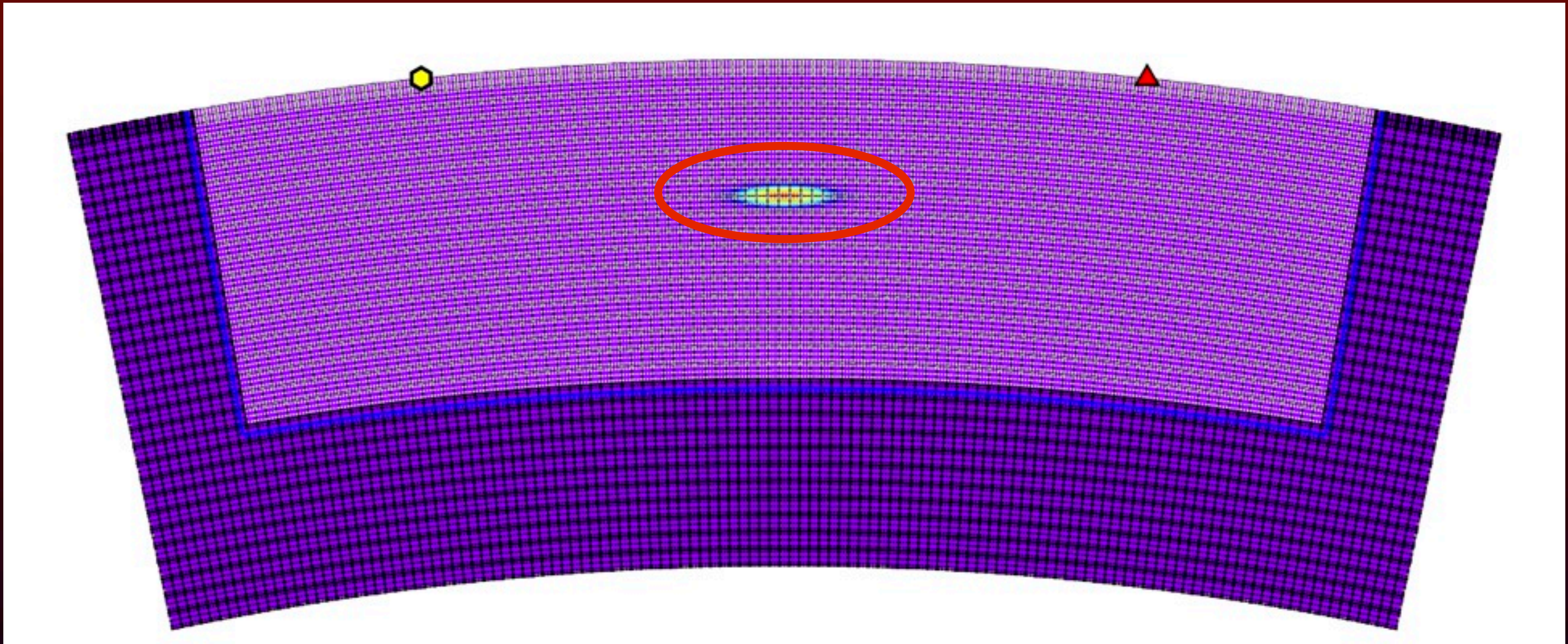
- Perfect accuracy
- Can handle arbitrary geometry
- Mirror is transparent
- We need to store

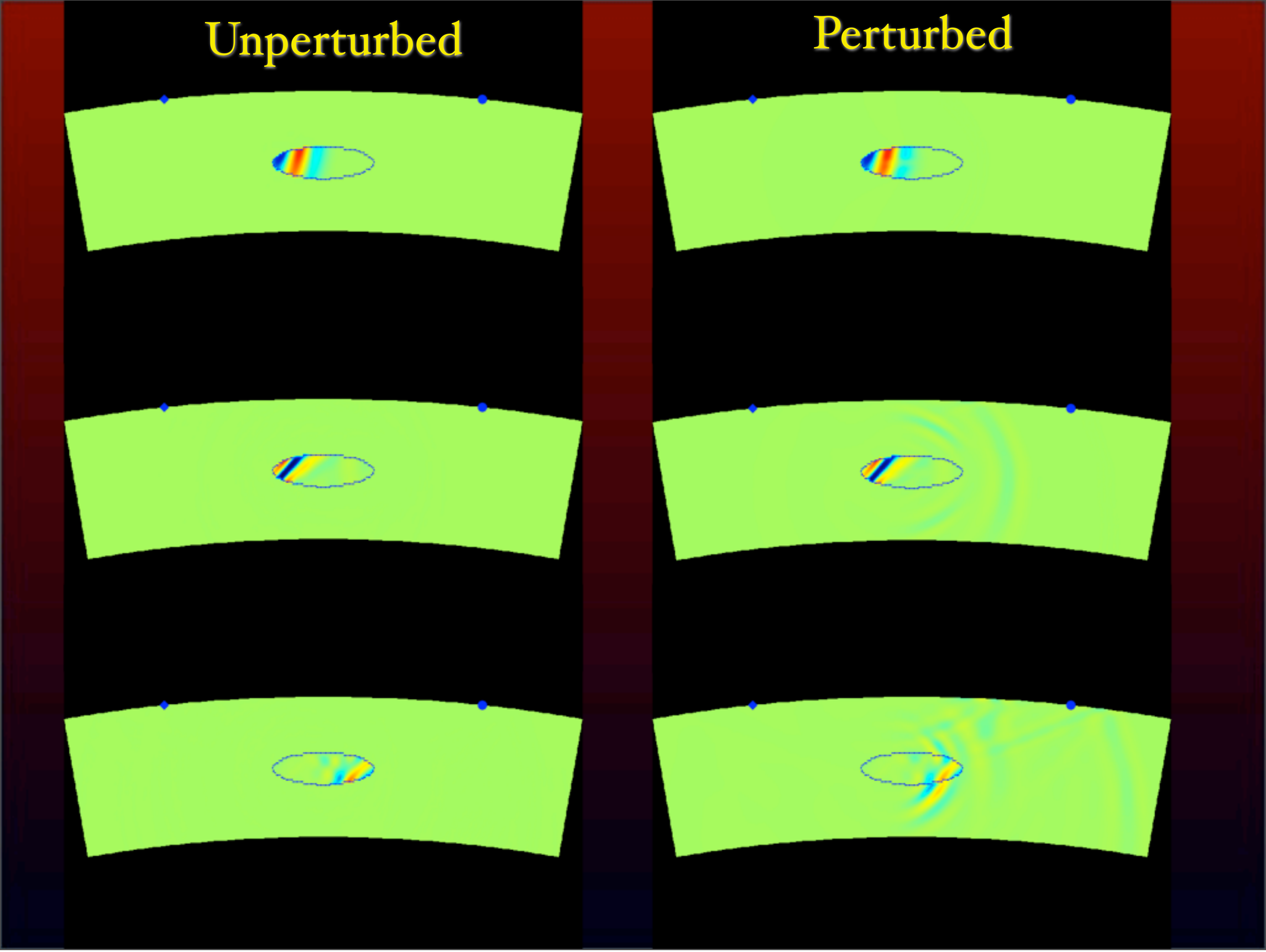
$$N = 3\frac{p}{2}Sh^2 \quad General$$

$$N = 3Sh^2 \quad Best$$

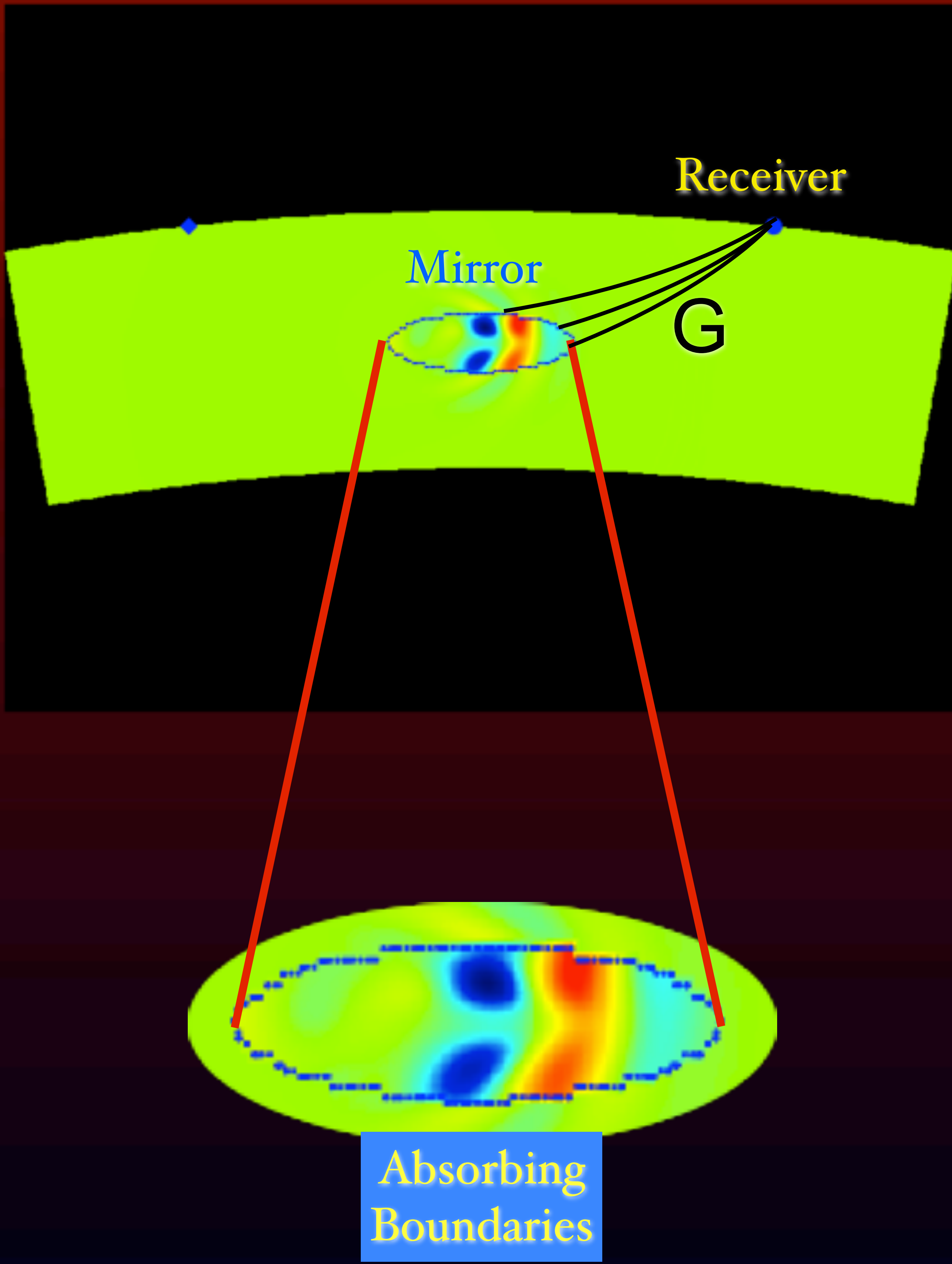
Why transparency matters ...


Full
Mirror





Unperturbed
Perturbed



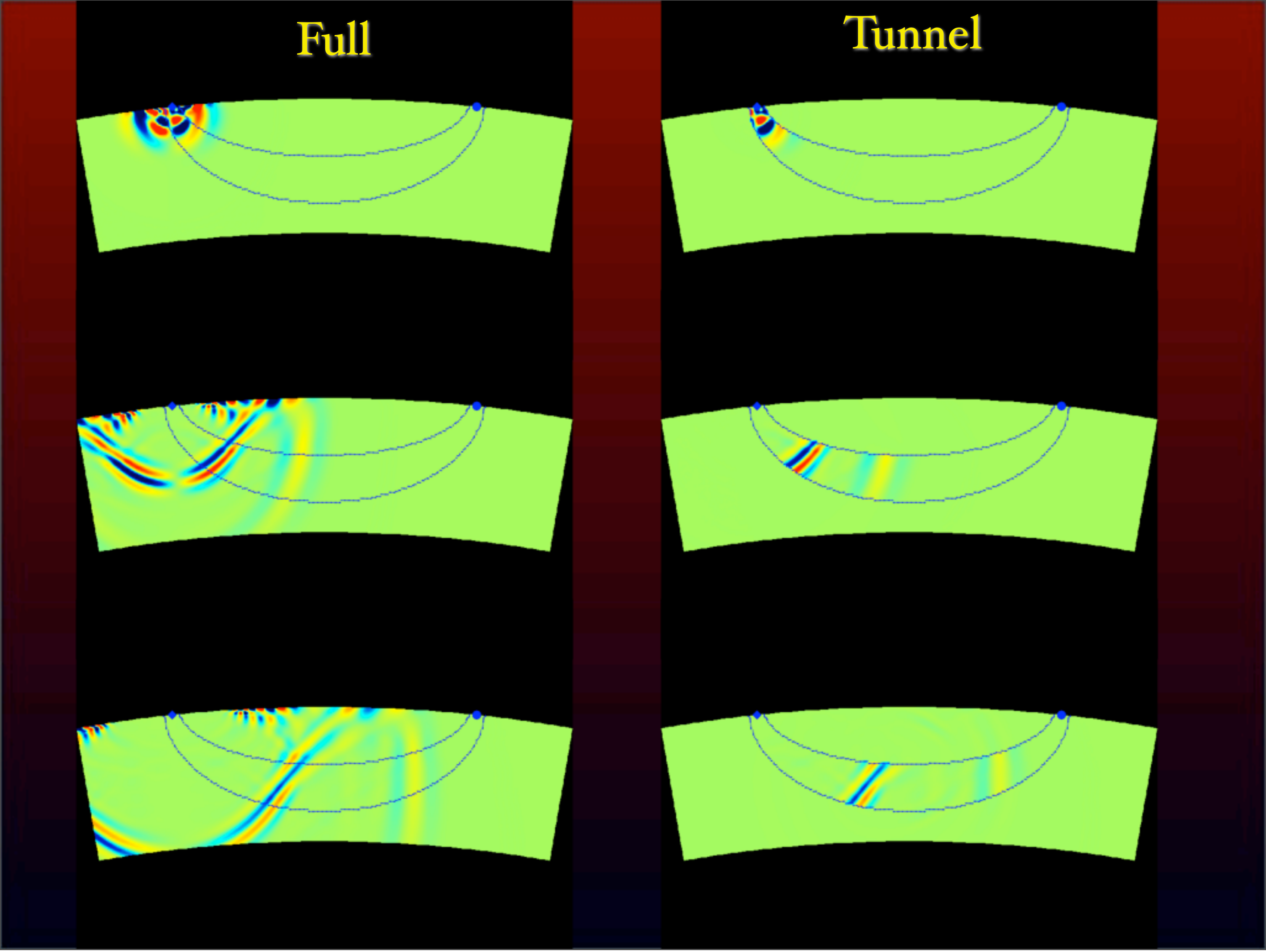
Local modeling

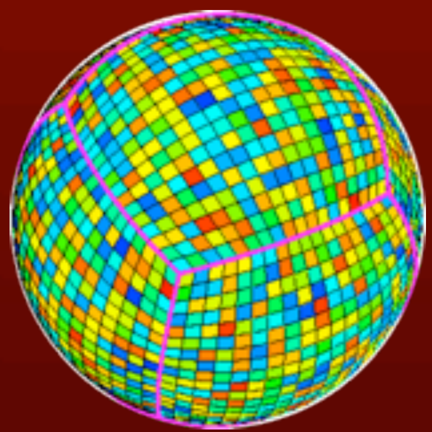
+

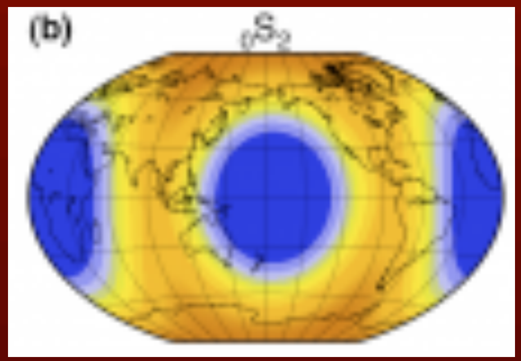
field extrapolation

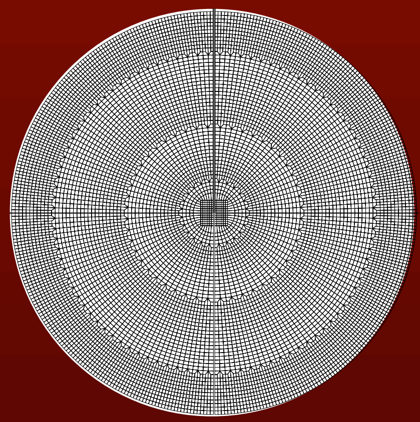
=

synthetic data at receivers


Full
Tunnel



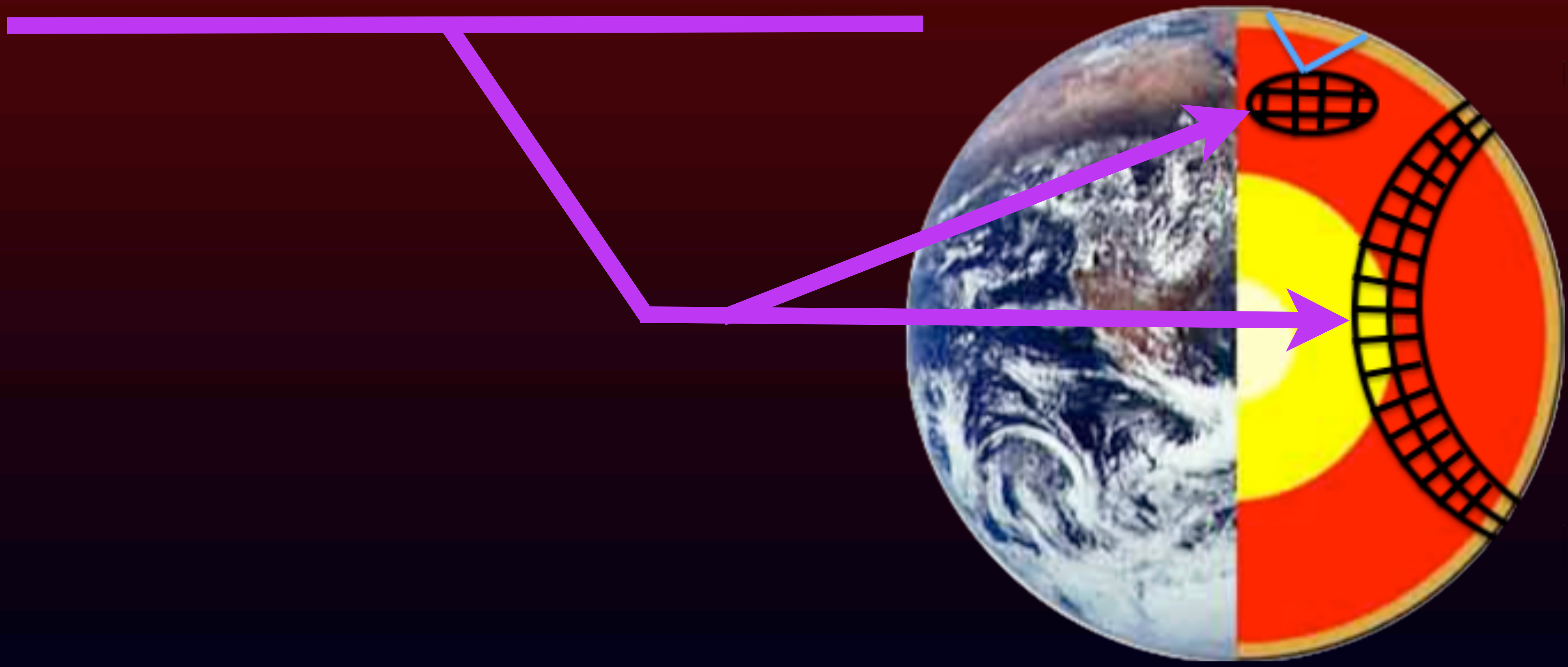




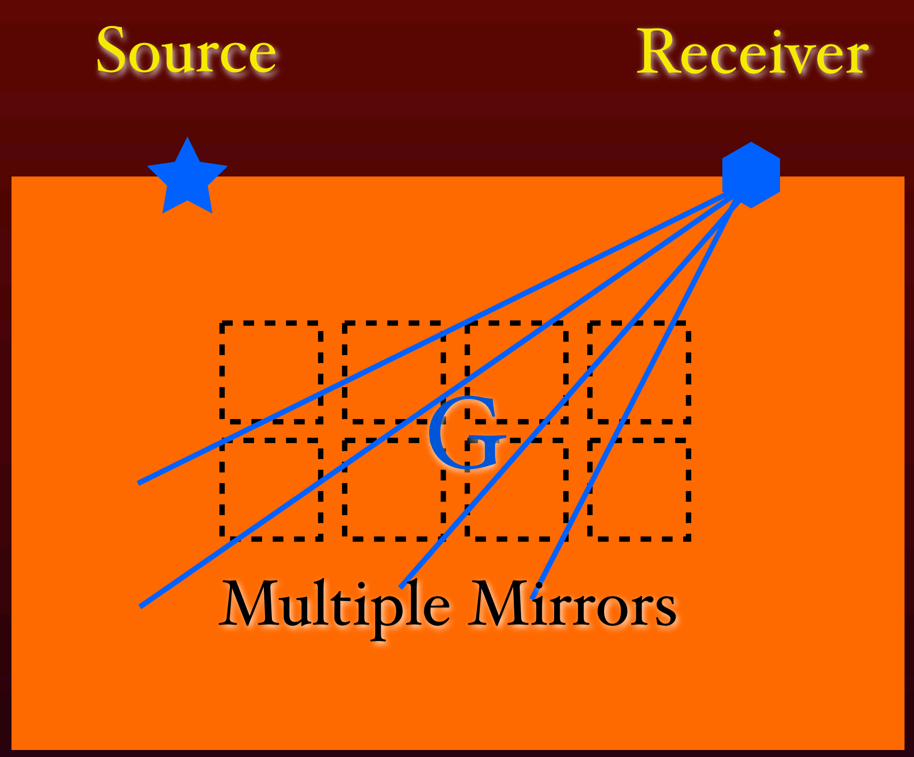
Exact
3D solution

Approximate
3D solution

1D Solution



Piecewise inversion ?



- A) Compute green functions from receivers to mirrors in the current model using global simulations
- B) Perform piecewise waveform tomography (Possibly one inversion per core)
- C) Assembly of the updated model
- Iterate...

Conclusion

- We introduced a perfectly transparent elastic mirror allowing to reconstruct the seismic wavefield within a volume enclosed in a surface of arbitrary shape
- Combined with field extrapolation, the mirror can be used to Perform Local waveform tomography using a global dataset
- to be continued...