3rd QUEST workshop
May 22, 2012, Slovakia

University of East Anglia

Earth's normal modes: an introduction

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University of East Anglia
Norwich, UK

Normal modes
Session outline

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11:20 **Lentas:** Finite source model determinations for large magnitude earthquakes using long-period normal mode data

11:40 **Koelemeijer:** Observability of lower mantle structures in Earth's free oscillation data

12:30 Lunch

15:00 **Nader-Nieto:** Rotational long-period signals: from ring laser data to large seismic networks array derived rotations

15:20 **Gualtieri:** Modeling seismic noise by normal mode summation

Outline

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- What are the Earth's normal modes? Overview
- Normal mode theory: forward modelling
- Normal mode studies: successes
- Current directions and challenges

Data - seismograms

Mw ~9.2 2004 26 December, Sumatra

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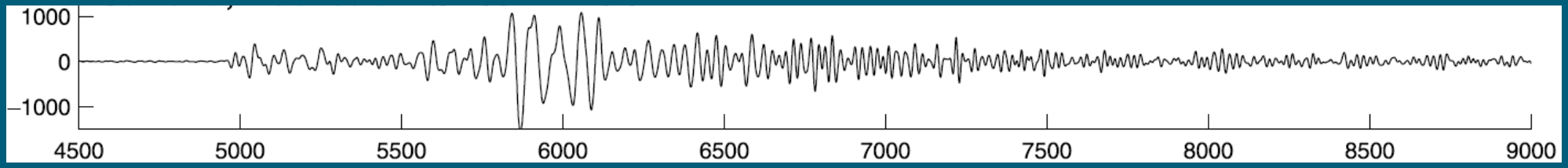
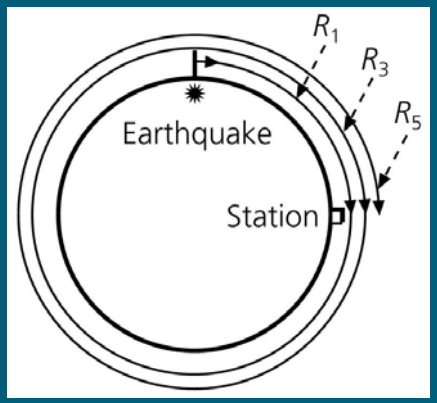
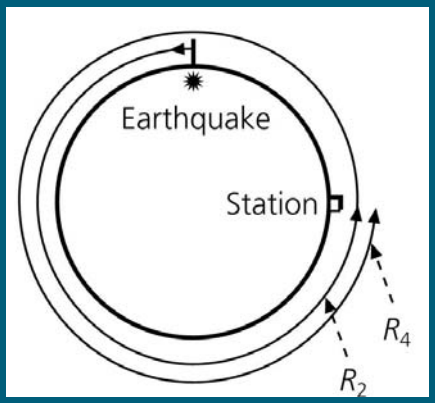
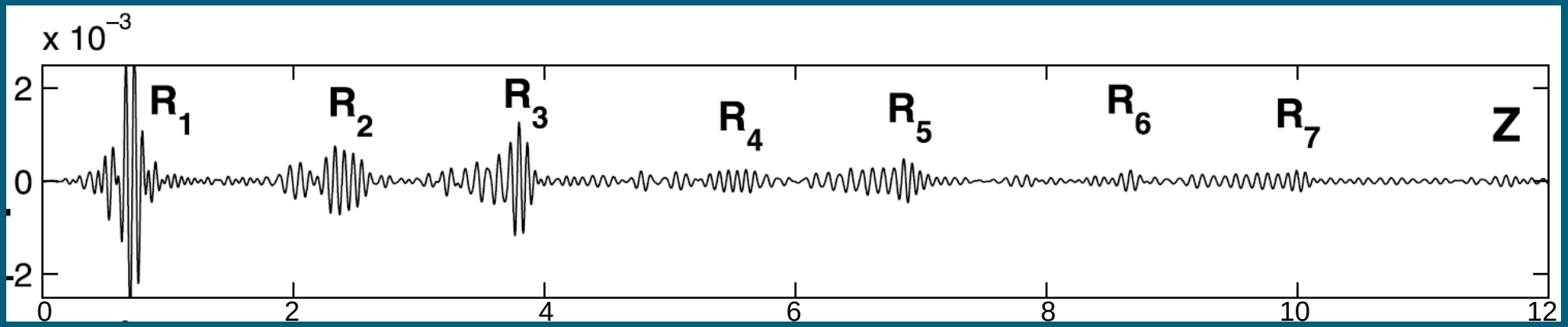


Time (s)







Time (h)

Data - normal mode spectra

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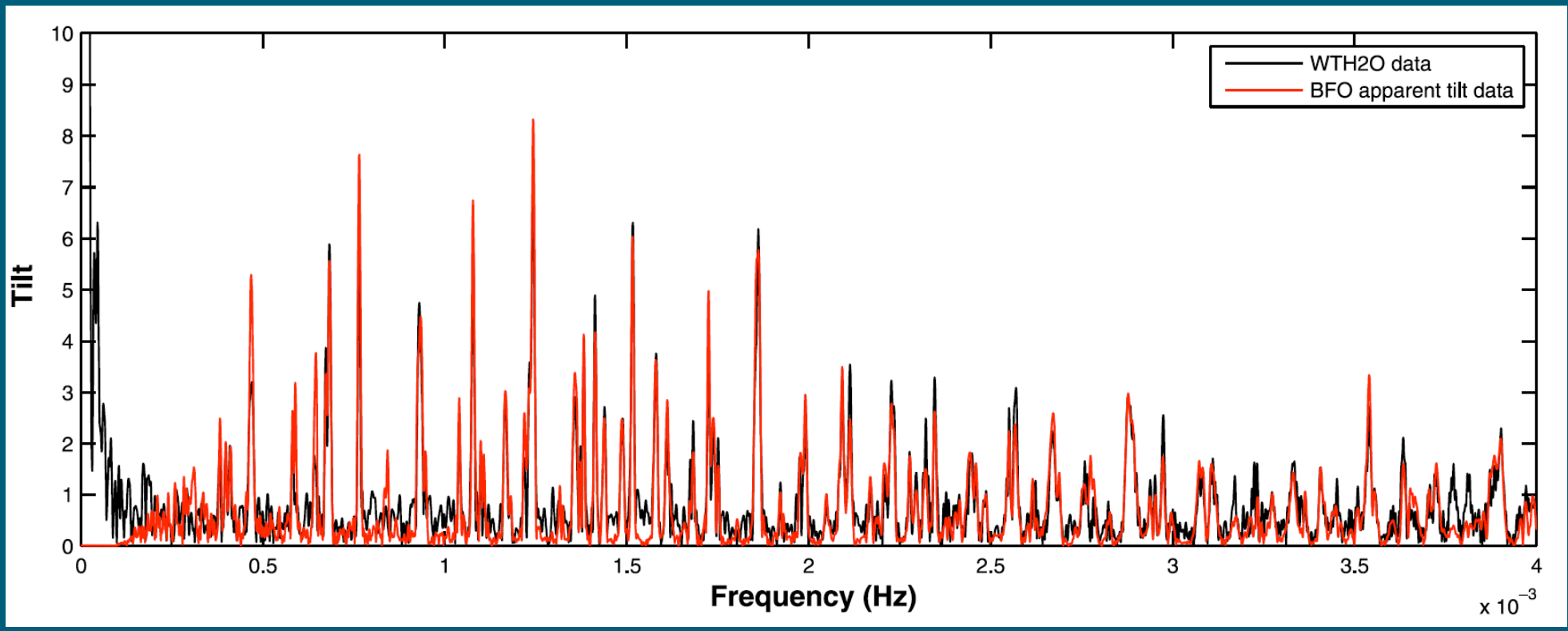
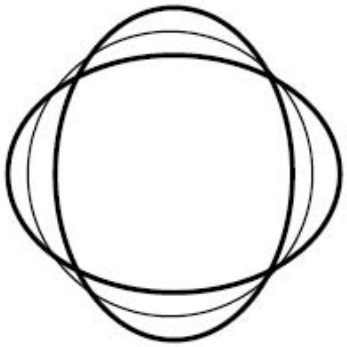
December 26, 2004 Mw ~9.2 Sumatra earthquake



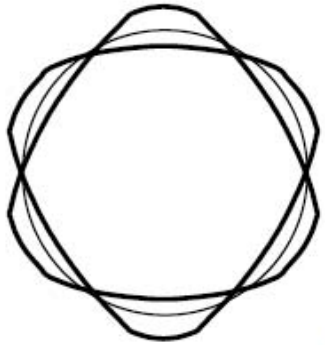
72hr of tilt data

Ferreira et al., JGR, 2006

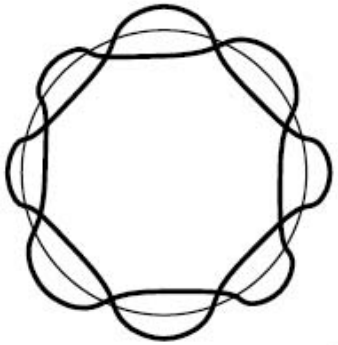
Surface patterns



$_0S_2$



$_0S_3$



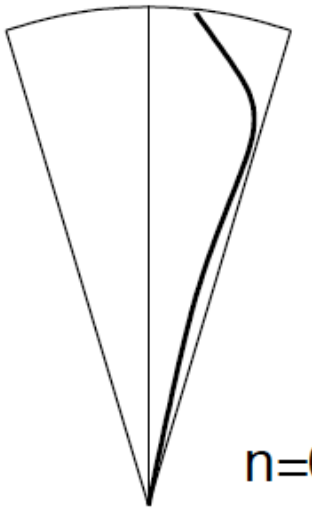
$_0S_4$

$_nS_l$

n: Overtone nr

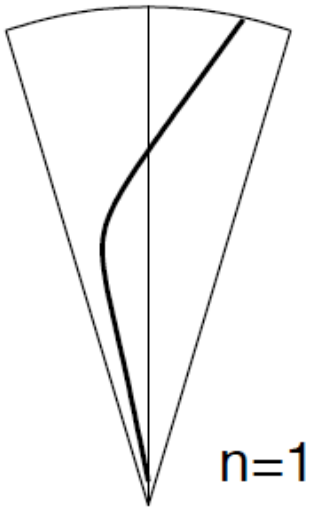
l: Angular order

Radial patterns



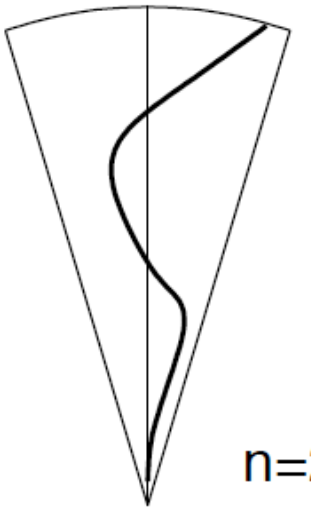
n=0

Fundamental



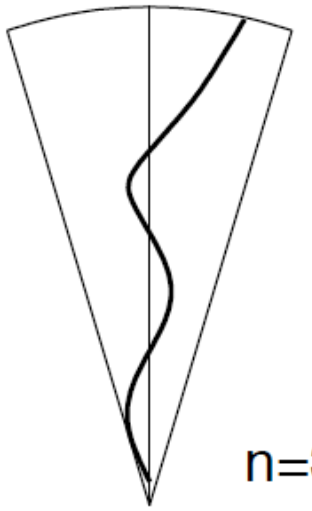
n=1

First Overtone



n=2

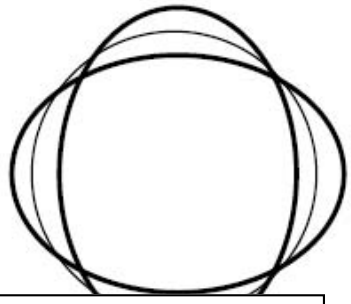
Second Overtone



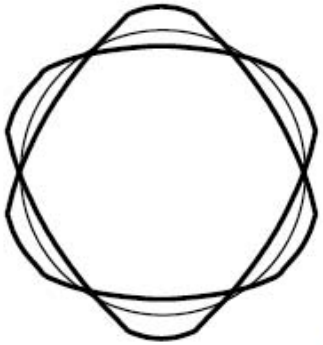
n=3

Third Overtone

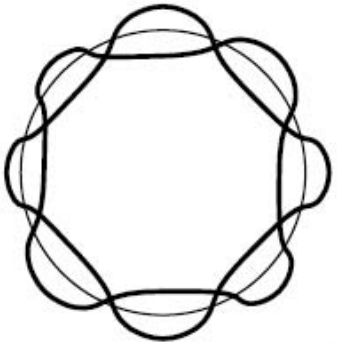
Surface patterns



$_0S_2$



$_0S_3$



$_0S_4$

$_n^mS_l$

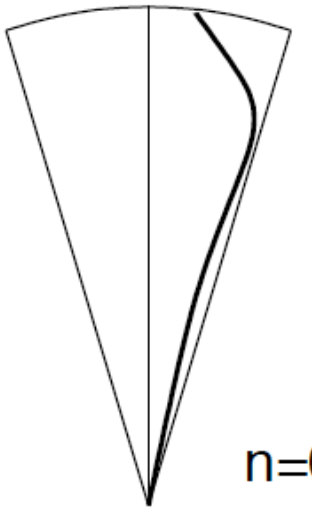
n: Overtone nr

l: Angular order

m: Azimuthal order

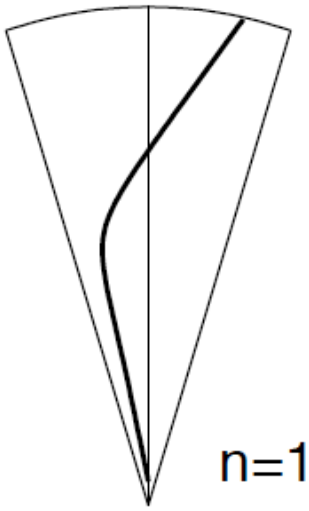
(*2l+1*)

Radial patterns



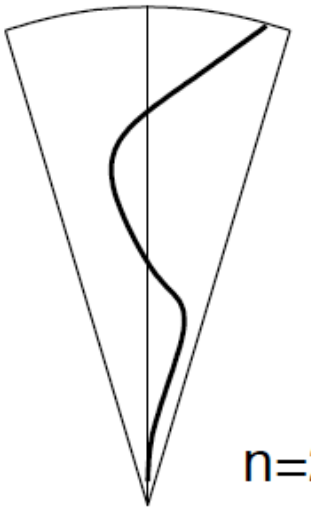
n=0

Fundamental



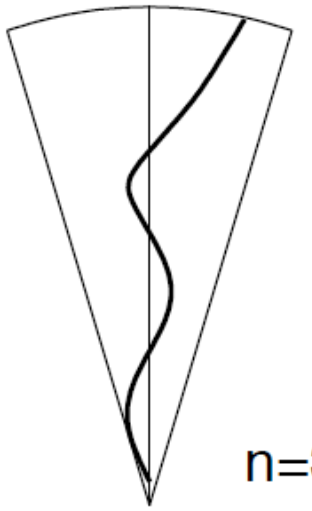
n=1

First Overtone



n=2

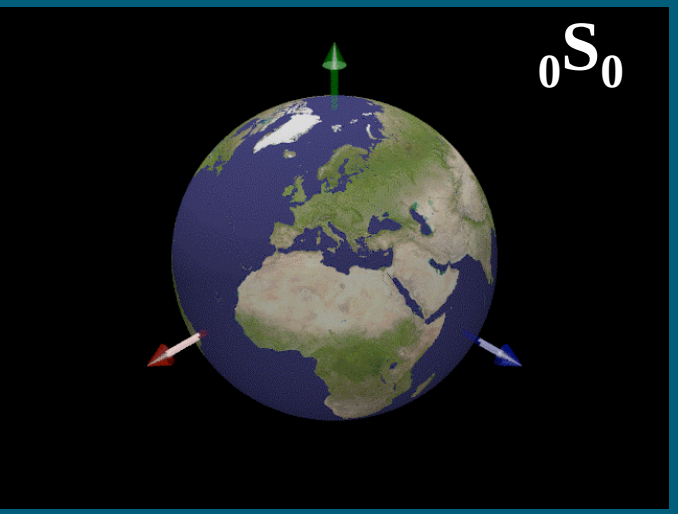
Second Overtone



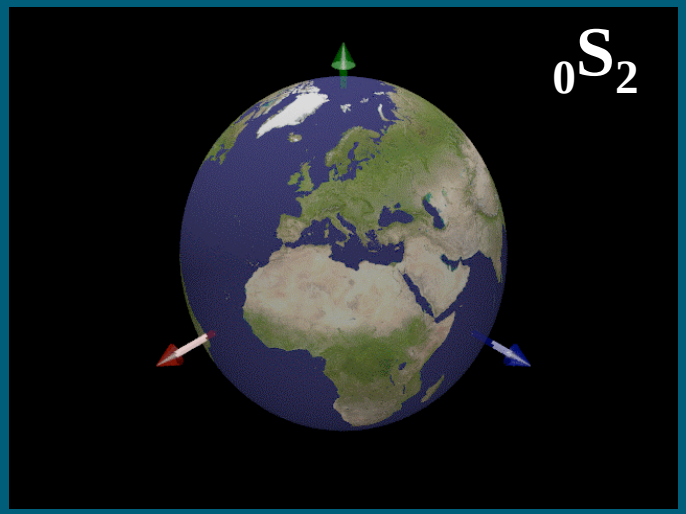
n=3

Third Overtone

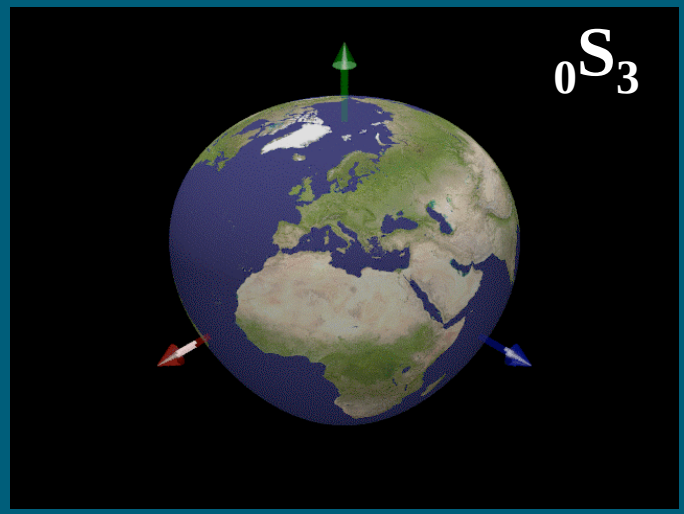
Earth's normal modes (m=0)



$_0S_0$ - T ~ 20 minutes



$_0S_2$ - T ~ 54 minutes



$_0S_3$ – T ~ 36 minutes

http://icb.u-bourgogne.fr/Nano/manapi/saviot/terre/index.en.html

Earth's normal modes (m=0)

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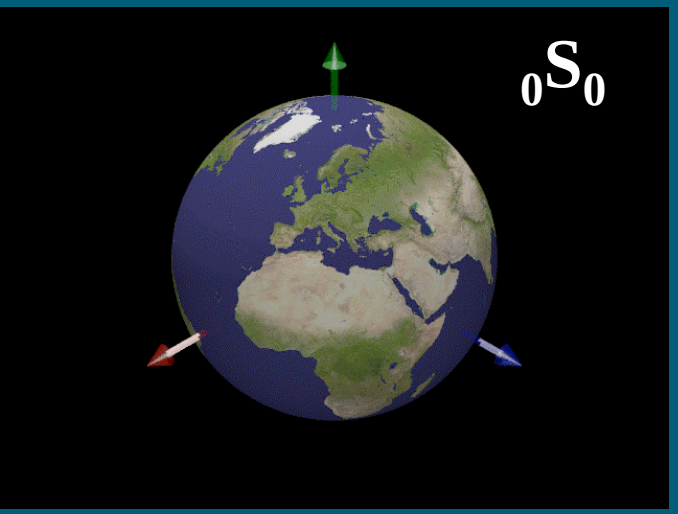
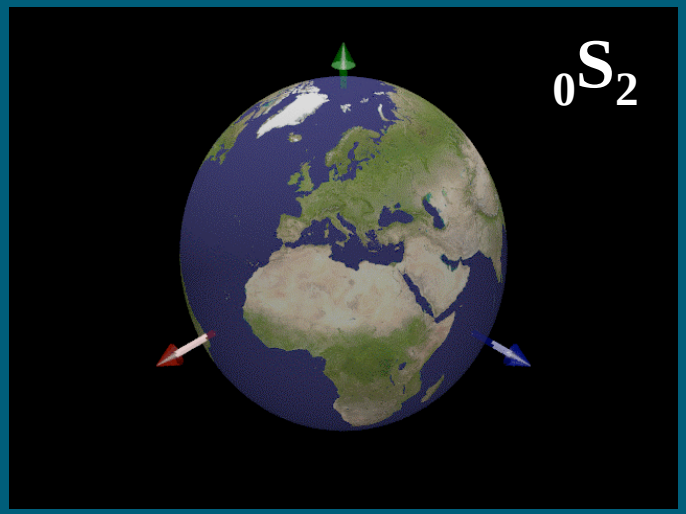
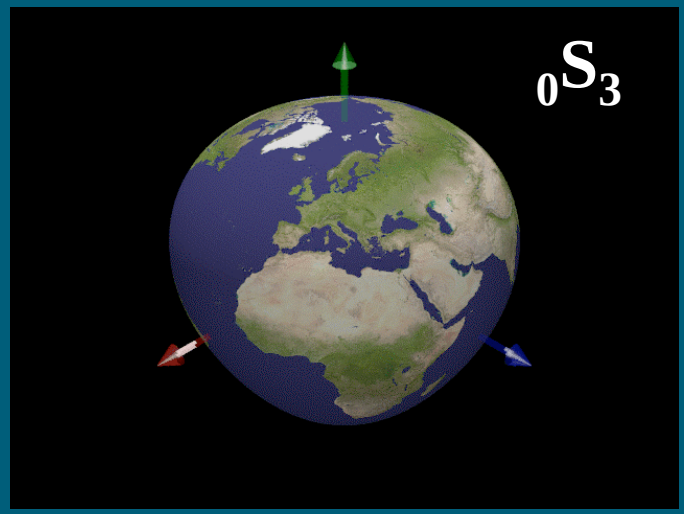


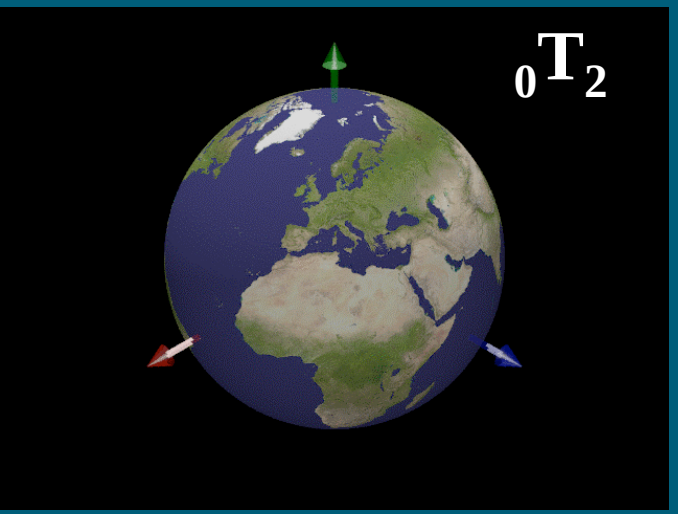
$_0S_0$ - T ~ 20 minutes



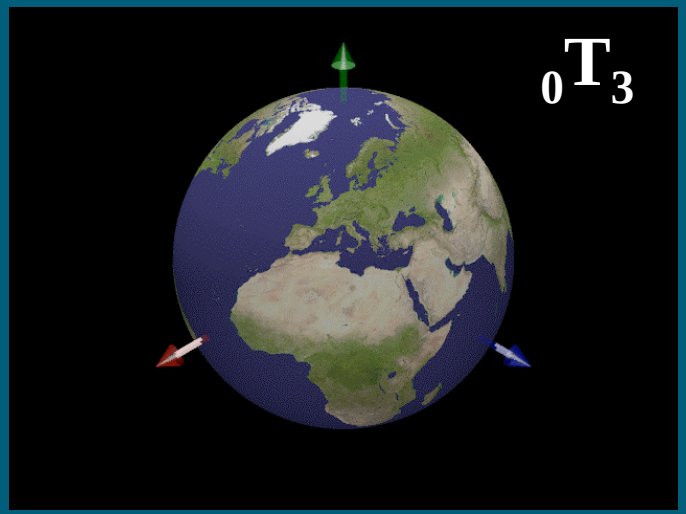
$_0S_2$ - T ~ 54 minutes



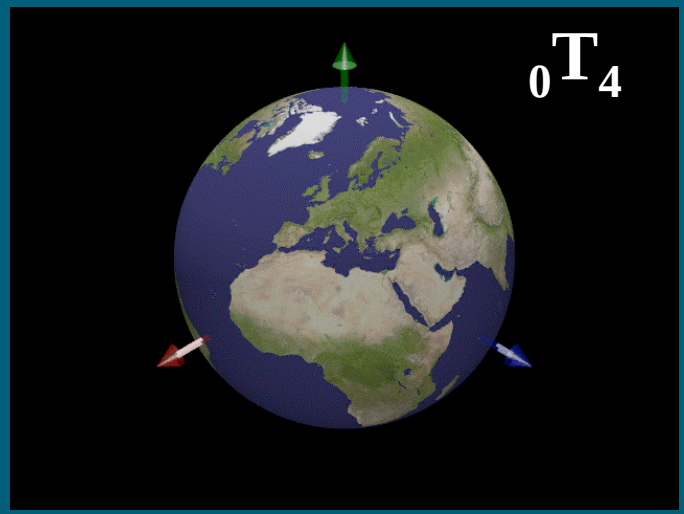
$_0S_3$ - T ~ 36 minutes



$_0T_2$ - T ~ 44 minutes



$_0T_3$ - T ~ 28 minutes



$_0T_4$ T ~ 22 minutes

Mode splitting

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If SNREI (Spherical Non Rotating Elastic Isotropic) Earth:

Degeneracy: for n and l, same frequency for $-l < m < l$

No more degeneracy if no more spherical symmetry:
- Rotation
- Ellipticity
- 3D Earth structure

Different frequencies for each m

Mode splitting

University of East Anglia

If SNREI (Spherical Non Rotating Elastic Isotropic) Earth:

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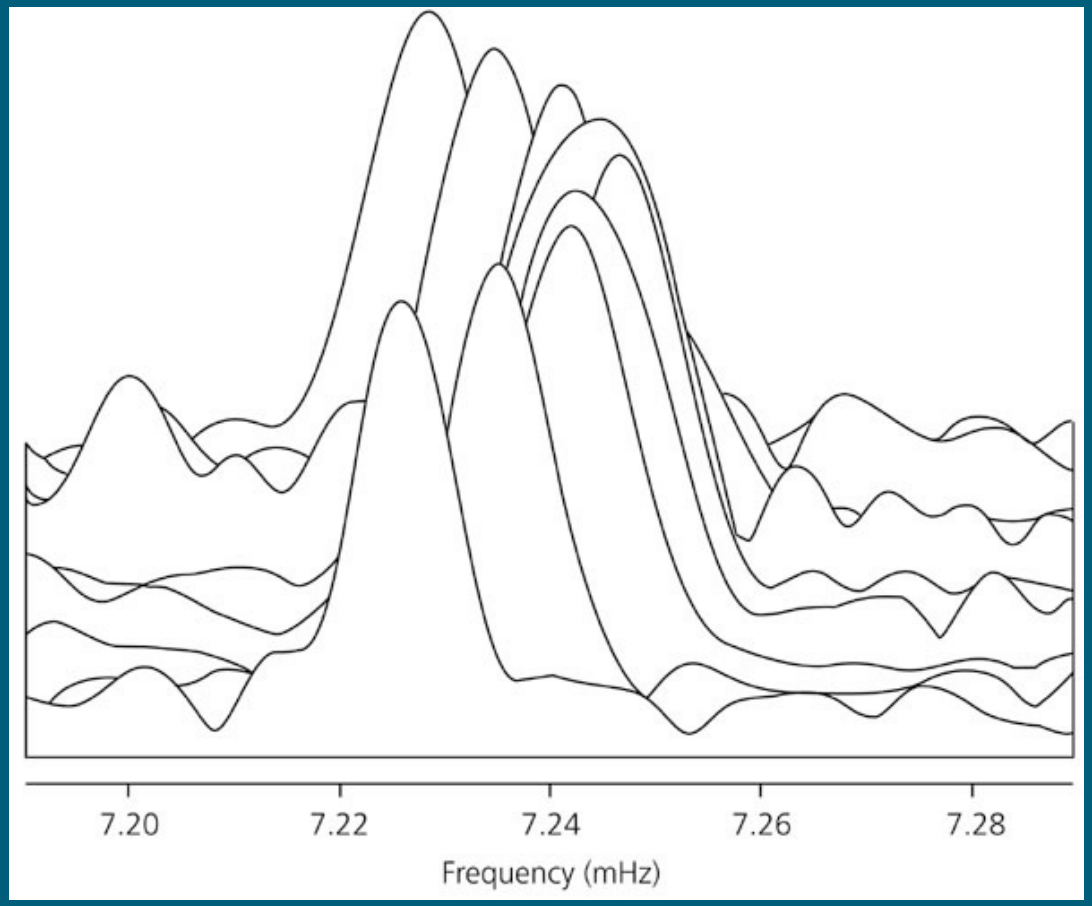
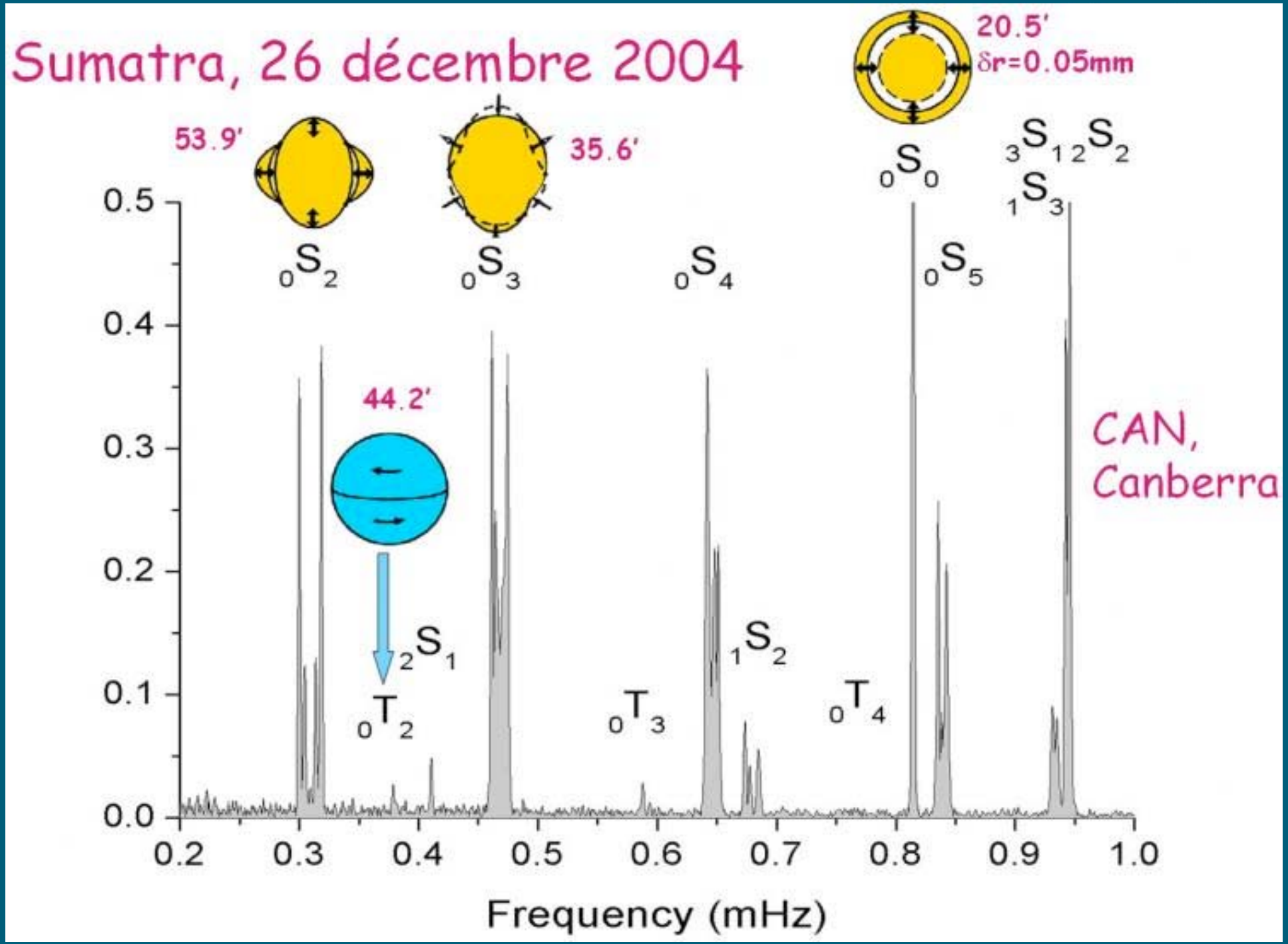
Different frequencies for each *m*

Mode singlets for the split multiplet ${}_{18}S_4$



Widmer et al., 1992

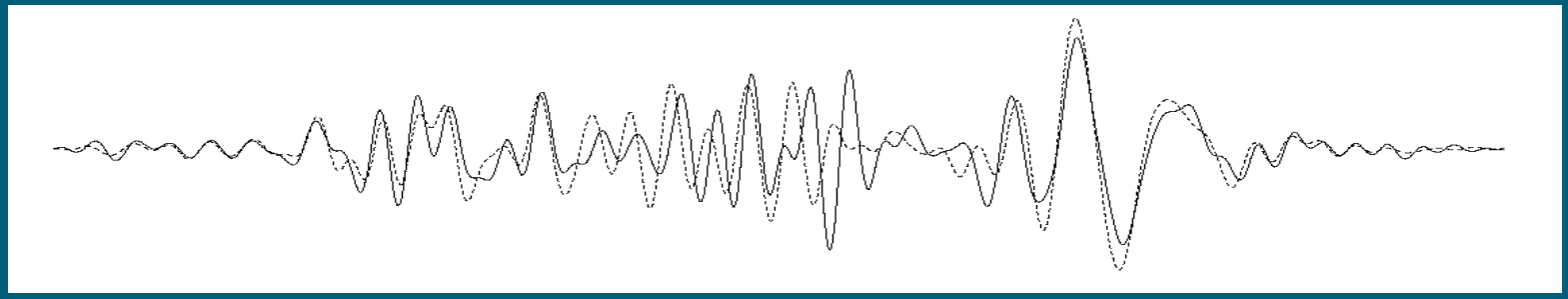


Park et al., 2005

120 h spectra

Forward modelling
Equations of motion

University of East Anglia



$$(\mathcal{H} + \rho^0 \partial_t^2)\mathbf{u} = \mathbf{F}$$

Linear integro-differential operator incorporating the elastodynamic and gravitational eqs

Initial density

Equivalent body force distribution of the seismic source

Forward modelling
Equations of motion

University of East Anglia

Taking the Fourier transform in time:

$$(\mathcal{H} - \rho^0 \omega^2)\overline{\mathbf{u}} = \overline{\mathbf{F}}$$

Forward modelling
Equations of motion

University of East Anglia

Taking the Fourier transform in time:

$$(\mathcal{H} - \rho^0\omega^2)\overline{\mathbf{u}} = \overline{\mathbf{F}}$$

F=0

$$\mathcal{H}\mathbf{s}_k = \rho^0\omega_k^2\mathbf{s}_k$$

Forward modelling
Equations of motion

University of East Anglia

Taking the Fourier transform in time:

$$(\mathcal{H} - \rho^0 \omega^2)\overline{\mathbf{u}} = \overline{\mathbf{F}}$$

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Set of eigenvalues

Set of eigenfunctions

Forward modelling
Equations of motion

University of East Anglia

Taking the Fourier transform in time:

$$(\mathcal{H} - \rho^0 \omega^2)\overline{\mathbf{u}} = \overline{\mathbf{F}}$$

F=0

$$\mathcal{H}\mathbf{s}_k = \rho^0 \omega_k^2 \mathbf{s}_k$$

Set of eigenvalues

Set of eigenfunctions

$$\mathbf{u}(\mathbf{x}, t) = e^{i\omega_k t}\mathbf{s}_k$$

Forward modelling
Equations of motion

University of East Anglia

Taking the Fourier transform in time:

$$(\mathcal{H} - \rho^0 \omega^2)\overline{\mathbf{u}} = \overline{\mathbf{F}}$$

F=0

$$\mathcal{H}\mathbf{s}_k = \rho^0 \omega_k^2 \mathbf{s}_k$$

Set of eigenvalues

Set of eigenfunctions

$$\mathbf{u}(\mathbf{x}, t) = \sum_k [1 - \cos \omega_k (t - t_s)] M_{ij} e_{ij}^{(k)*}(\mathbf{x}_s)\mathbf{s}_k(\mathbf{x})$$

Seismic moment tensor

Strain tensor of k^{th} mode

Forward modelling

Eigenfunctions/eigenfrequencies: the spherical Earth

University of East Anglia

$$\frac{d\mathbf{y}}{dr} = \mathbf{A}(r)\mathbf{y}$$

Forward modelling

Eigenfunctions/eigenfrequencies: the spherical Earth

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$$\frac{d\mathbf{y}}{dr} = \mathbf{A}(r)\mathbf{y}$$

- The equations for (U,V) are decoupled from those for W:

Two types of modes:

Spheroidal (W=0)
Toroidal (U=V=0)

Forward modelling

Eigenfunctions/eigenfrequencies: the spherical Earth

University of East Anglia

$$\frac{d\mathbf{y}}{dr} = \mathbf{A}(r)\mathbf{y}$$

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- The equations are independent of m - the eigenfrequency w_k is $(2l+1)$-fold degenerate

Forward modelling

Eigenfunctions/eigenfrequencies: the spherical Earth

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$$\frac{d\mathbf{y}}{dr} = \mathbf{A}(r)\mathbf{y}$$

- The equations for (U,V) are decoupled from those for W:

Two types of modes:

Spheroidal (W=0)
Toroidal (U=V=0)

- The equations are independent of m - the eigenfrequency w_k is $(2l+1)$-fold degenerate

- Modes can be identified according to mode type q (S, T), radial order n and angular order l: ${}_nq_l$

Forward modelling
Equations of motion: the spherical Earth

University of East Anglia

$$\frac{d\mathbf{y}}{dr} = \mathbf{A}(r)\mathbf{y}$$

$$\mathbf{u}(\mathbf{x},t) = \sum_k [1 - \cos\omega_k(t - t_s)] M_{ij} e_{ij}^{(k)*}(\mathbf{x}_s)\mathbf{s}_k(\mathbf{x})$$

Normal mode summation synthetic seismograms

Toroidal normal modes
Eigenfrequencies

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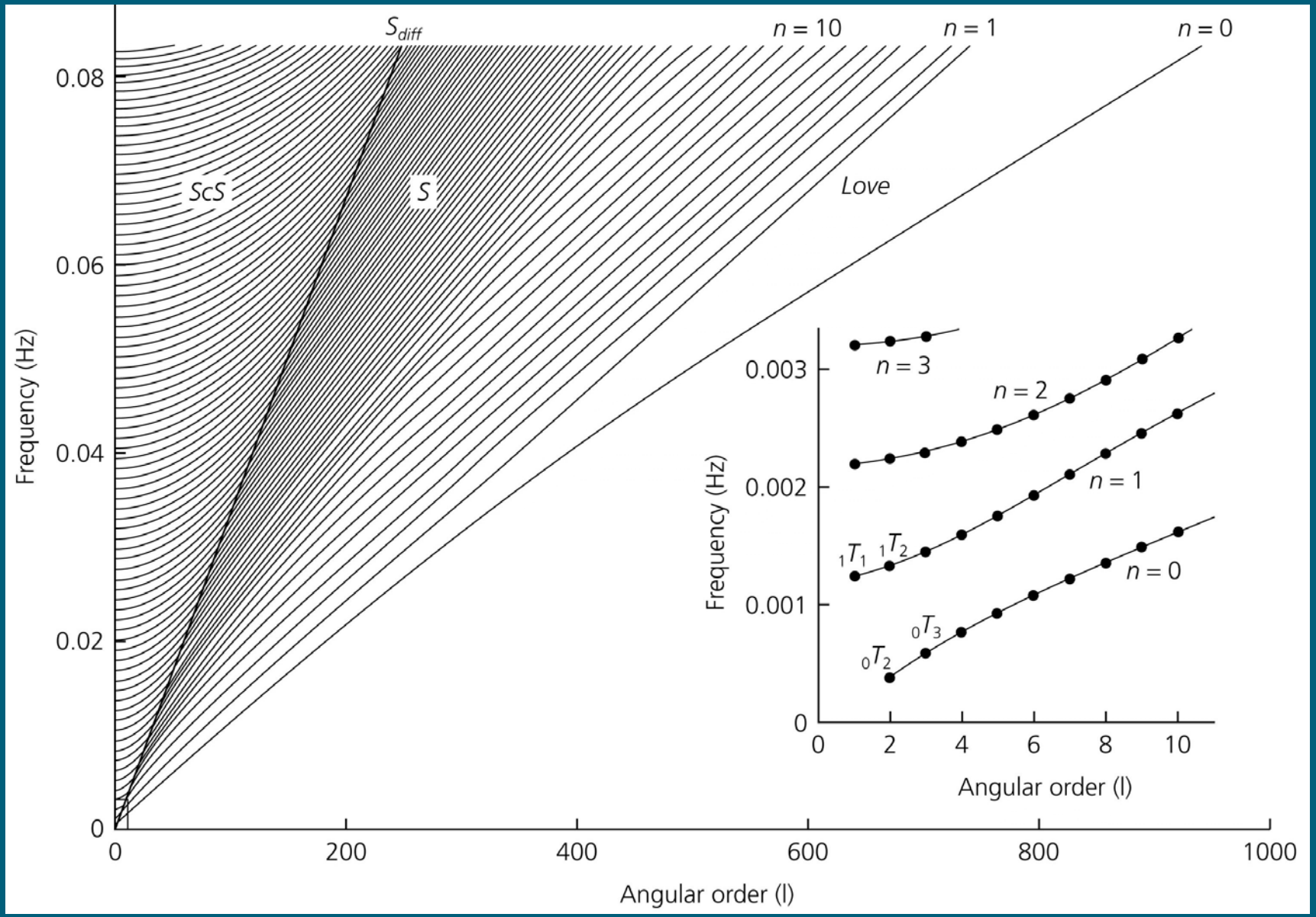

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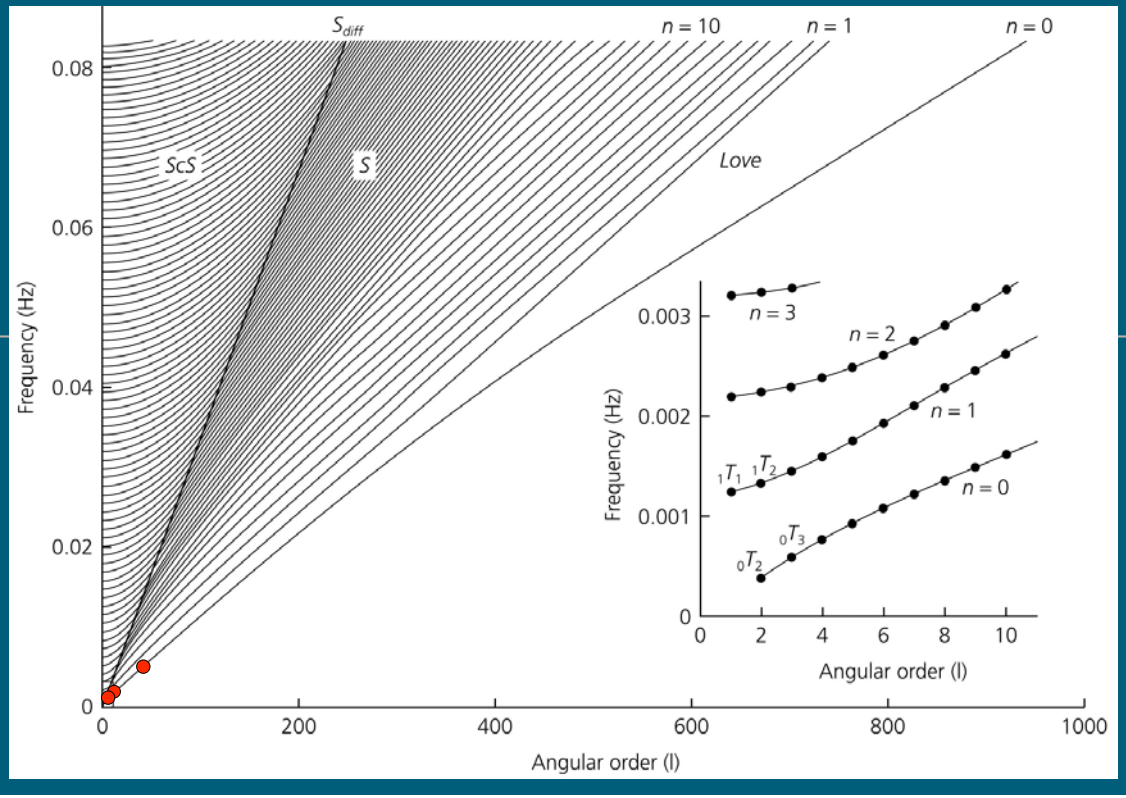
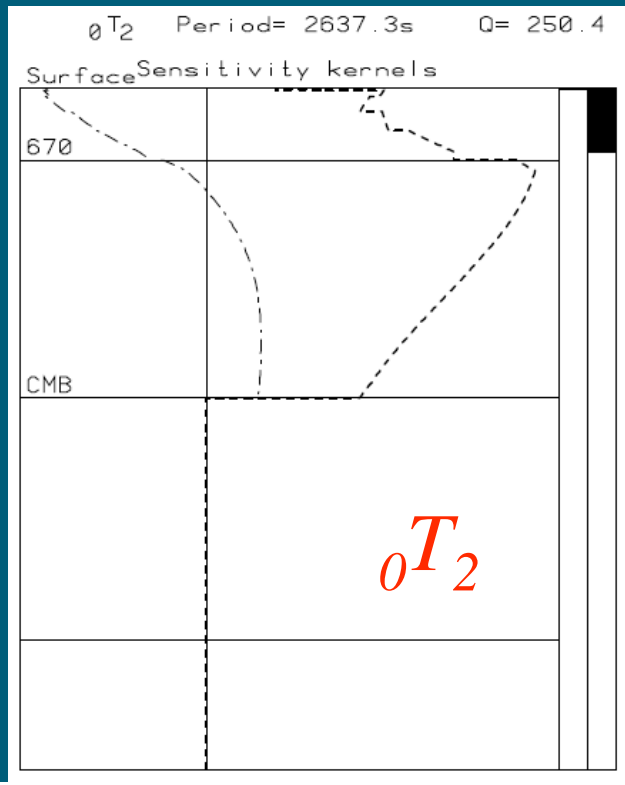
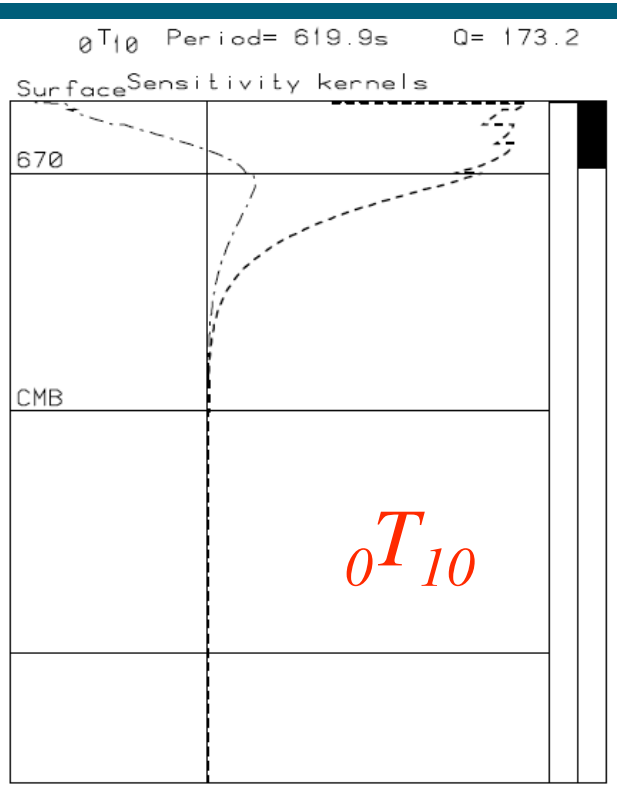


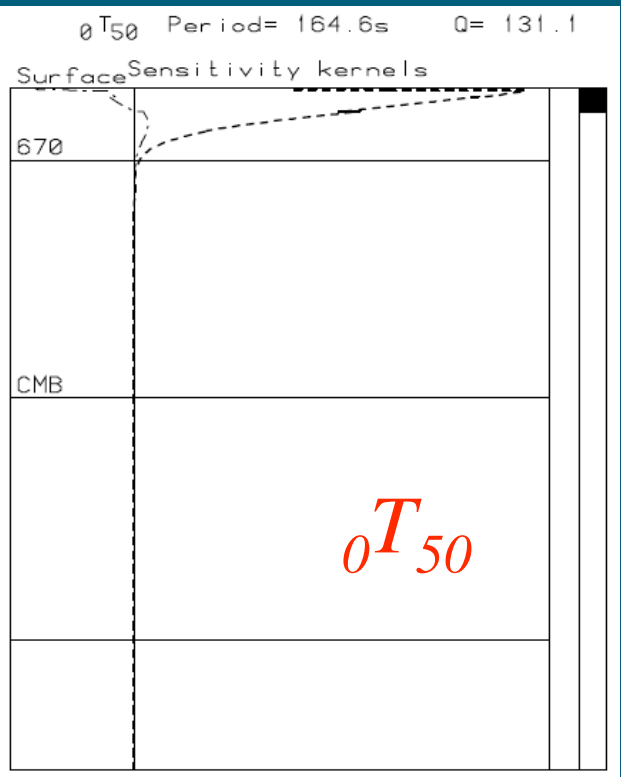
- - Vs

- . Density

Woodhouse, 1996

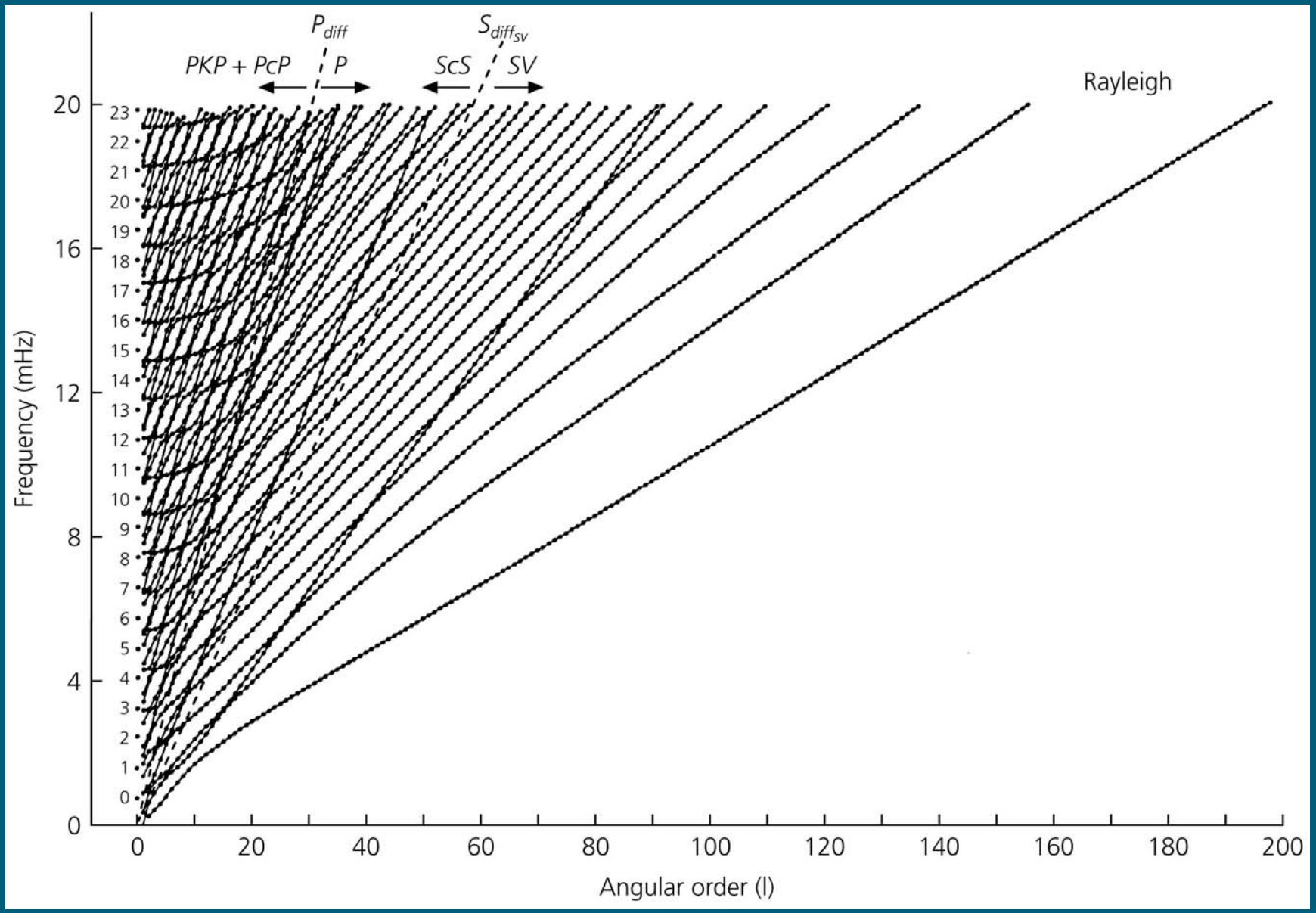






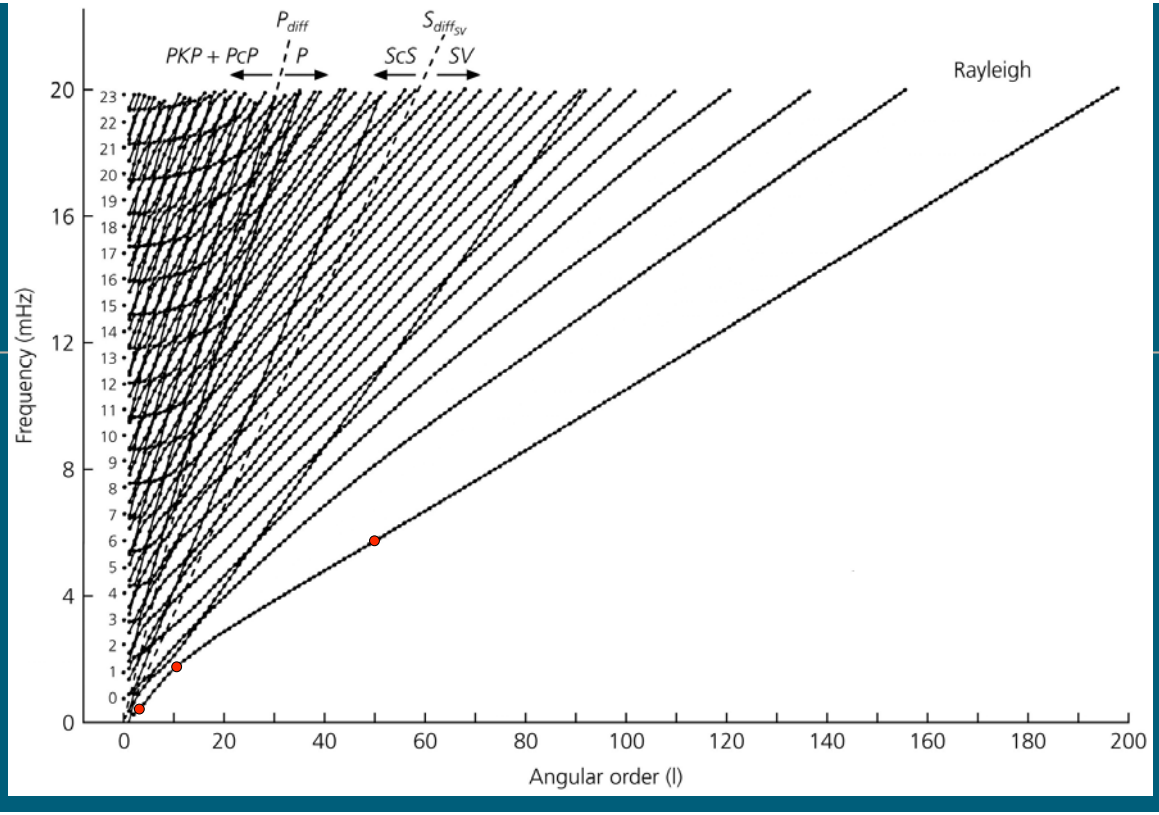
Spheroidal normal modes
Eigenfrequencies

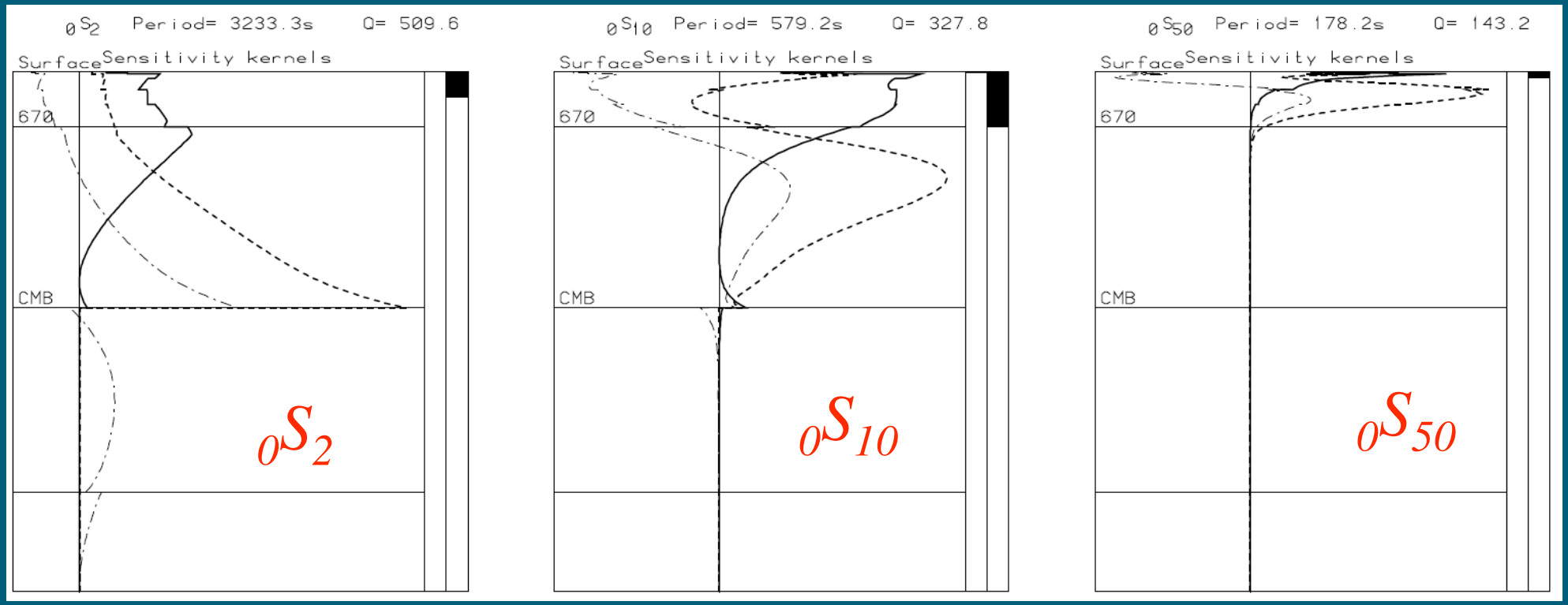
University of East Anglia



University of East Anglia

- - Vs
- Vp
- . Density





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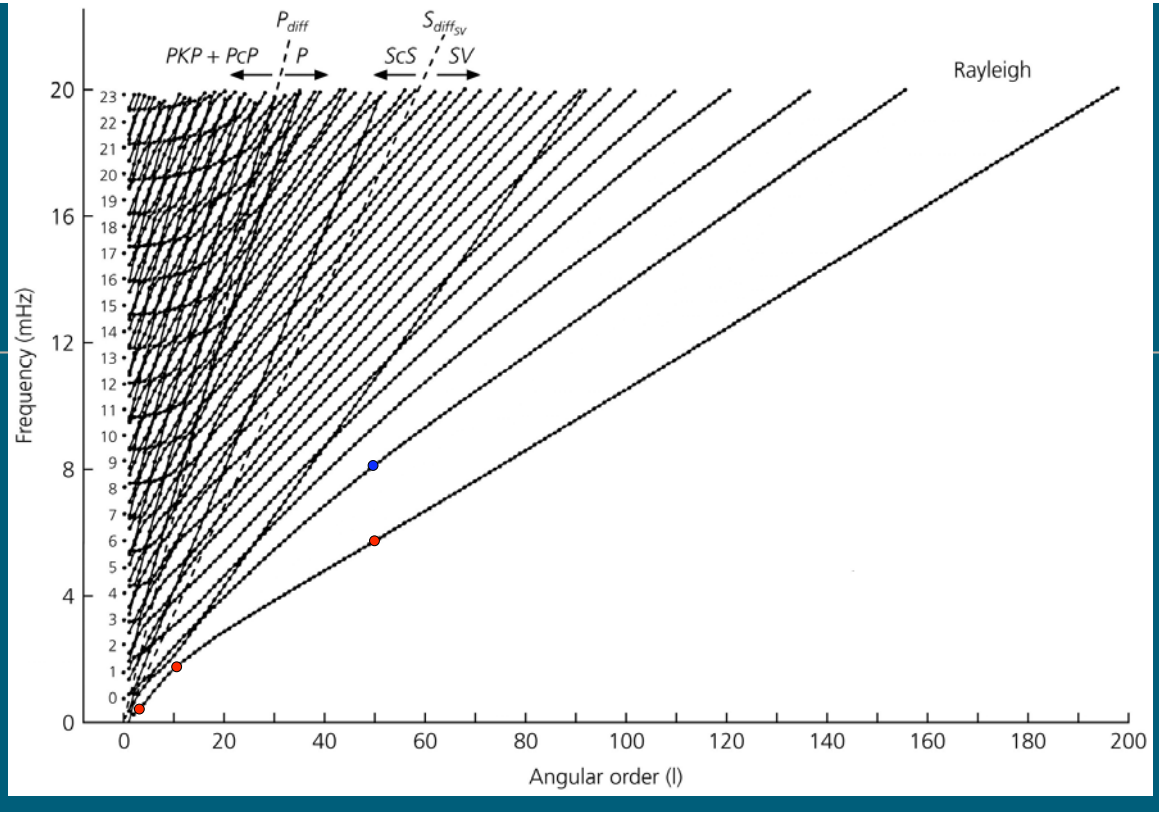
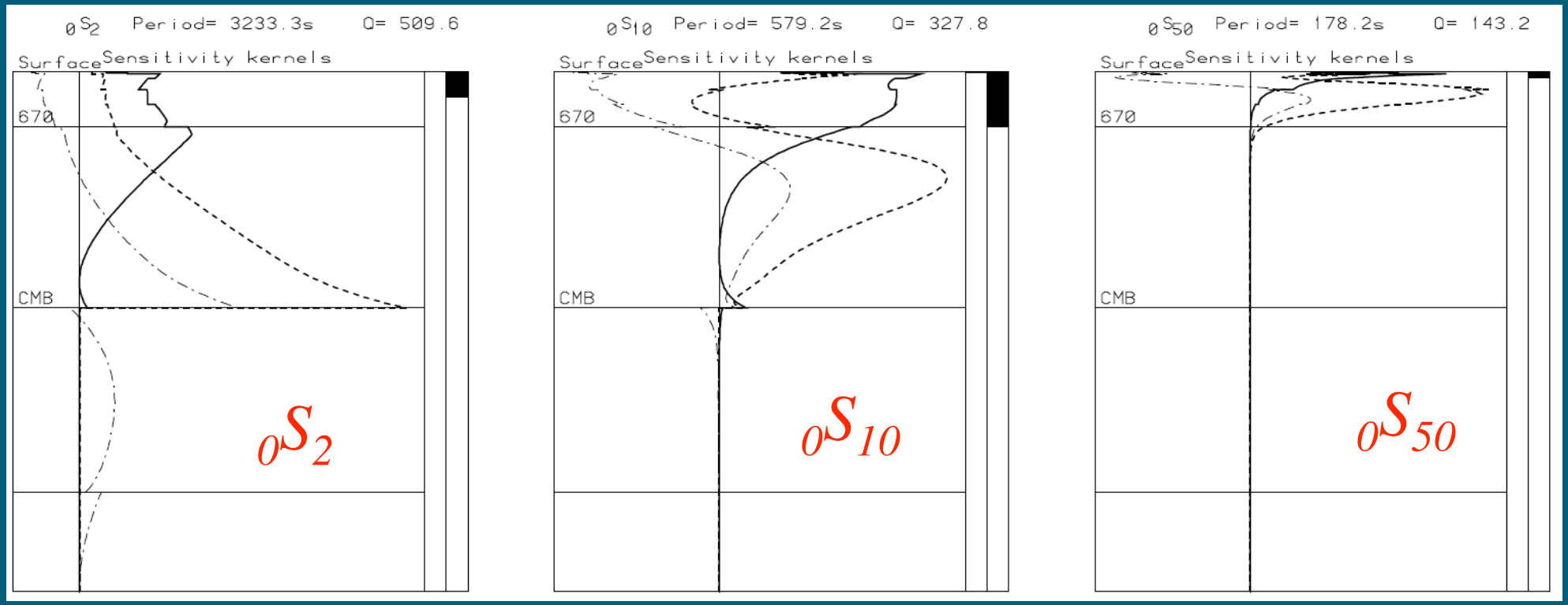
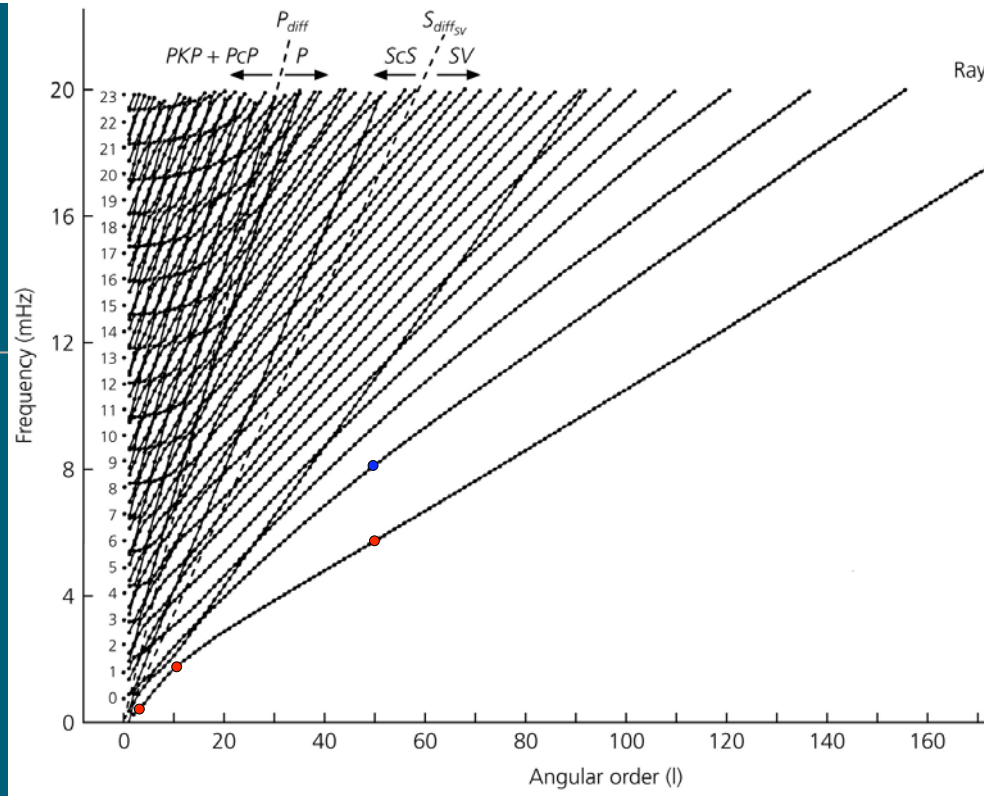
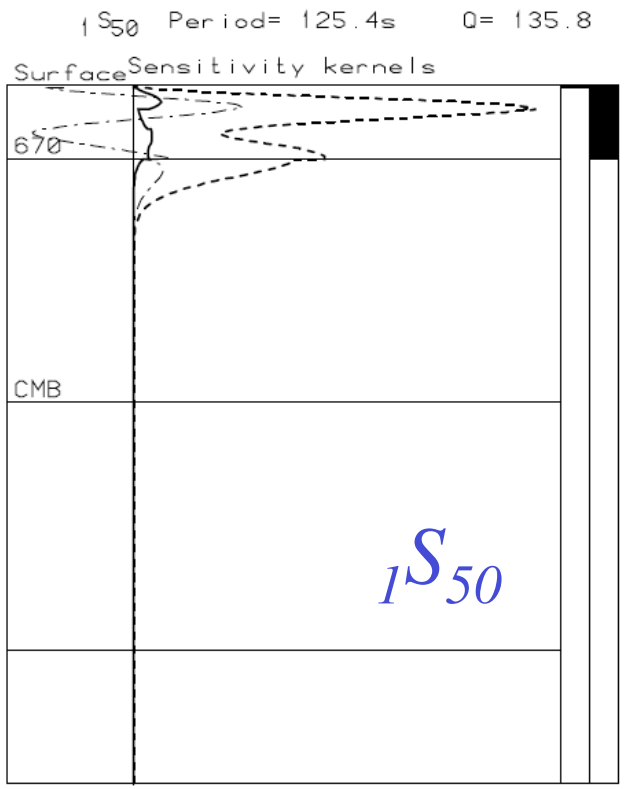
- -- Vs
- - Vp
- -. Density

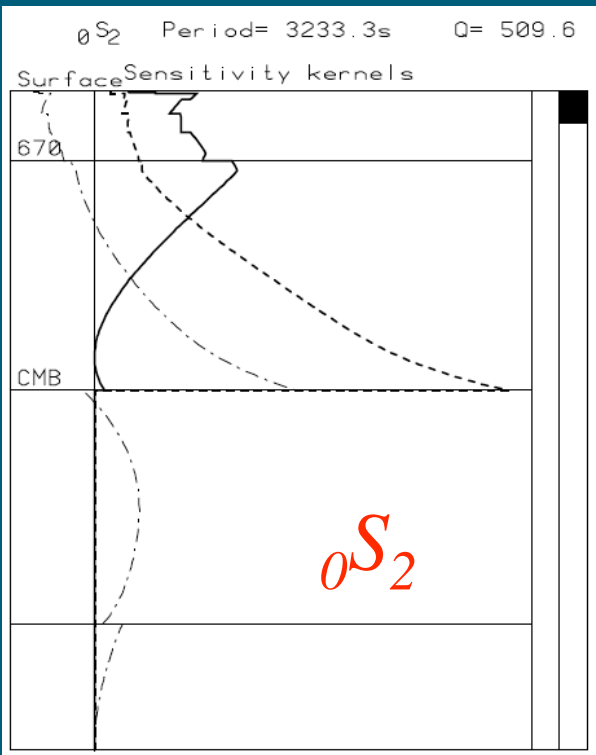


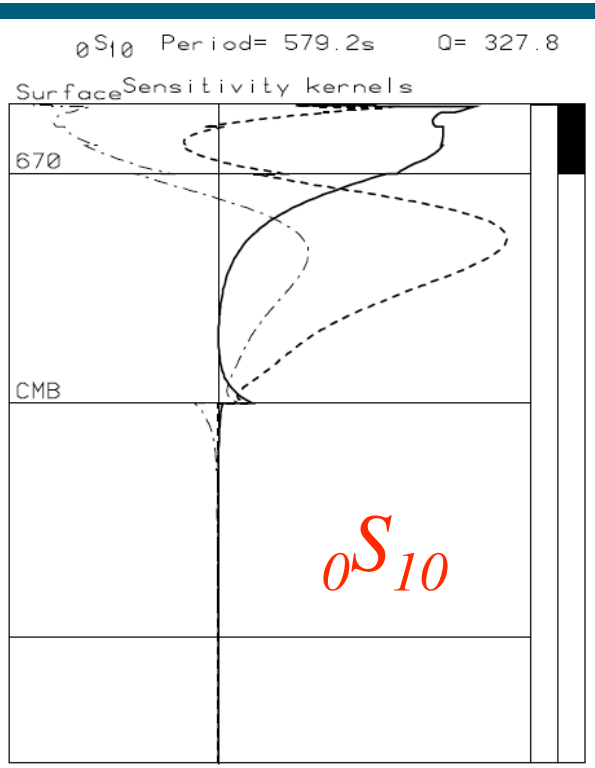


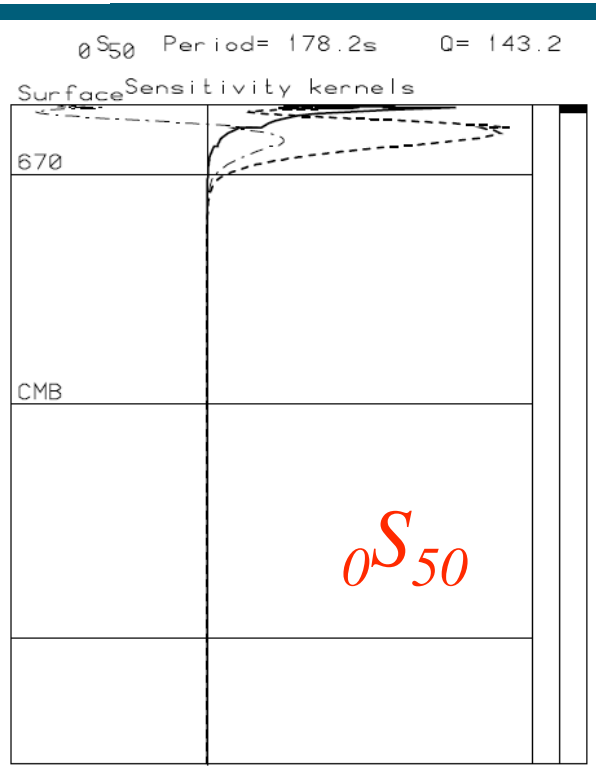
- - Vs
  - Vp
- . Density

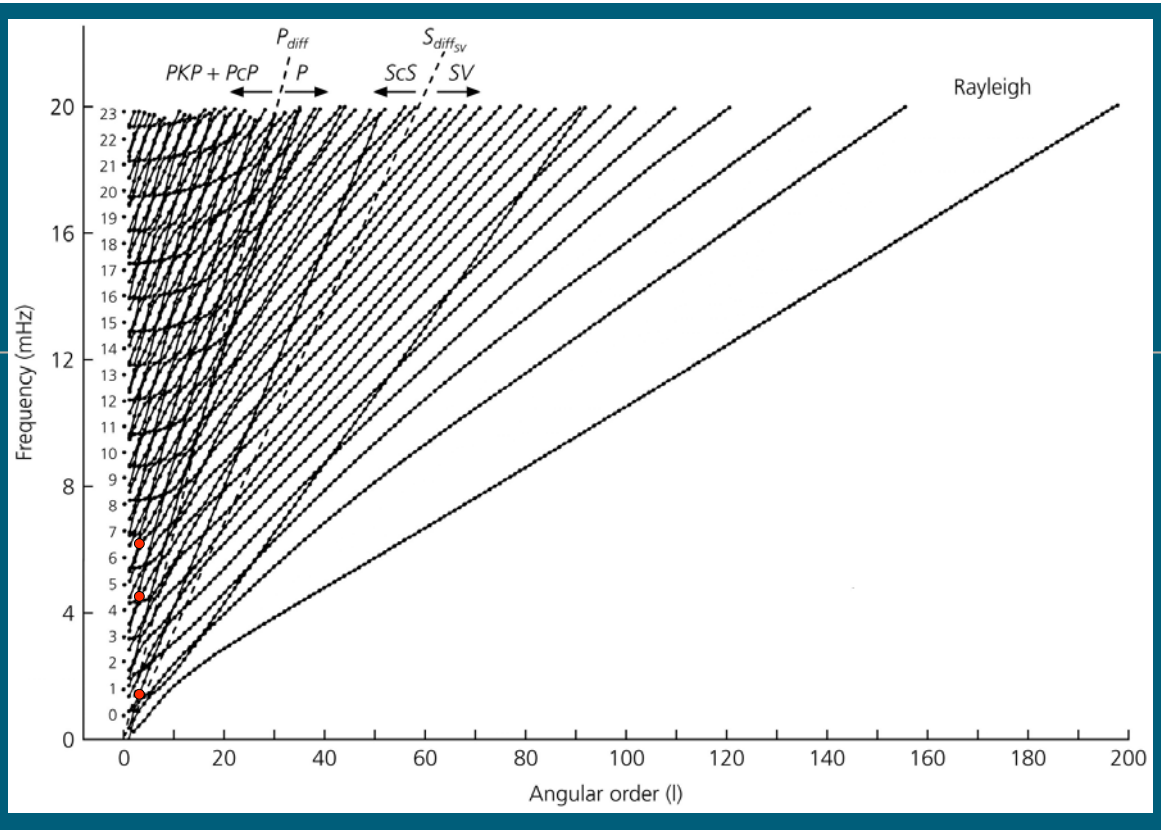












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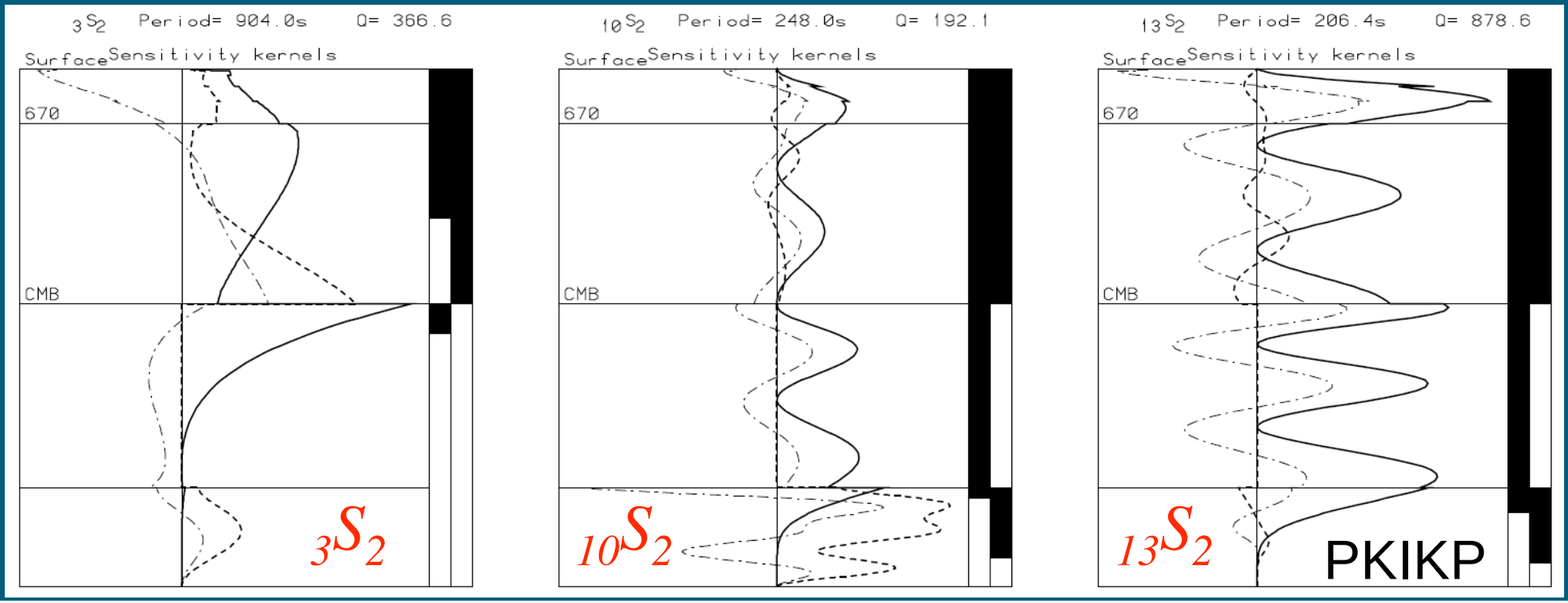
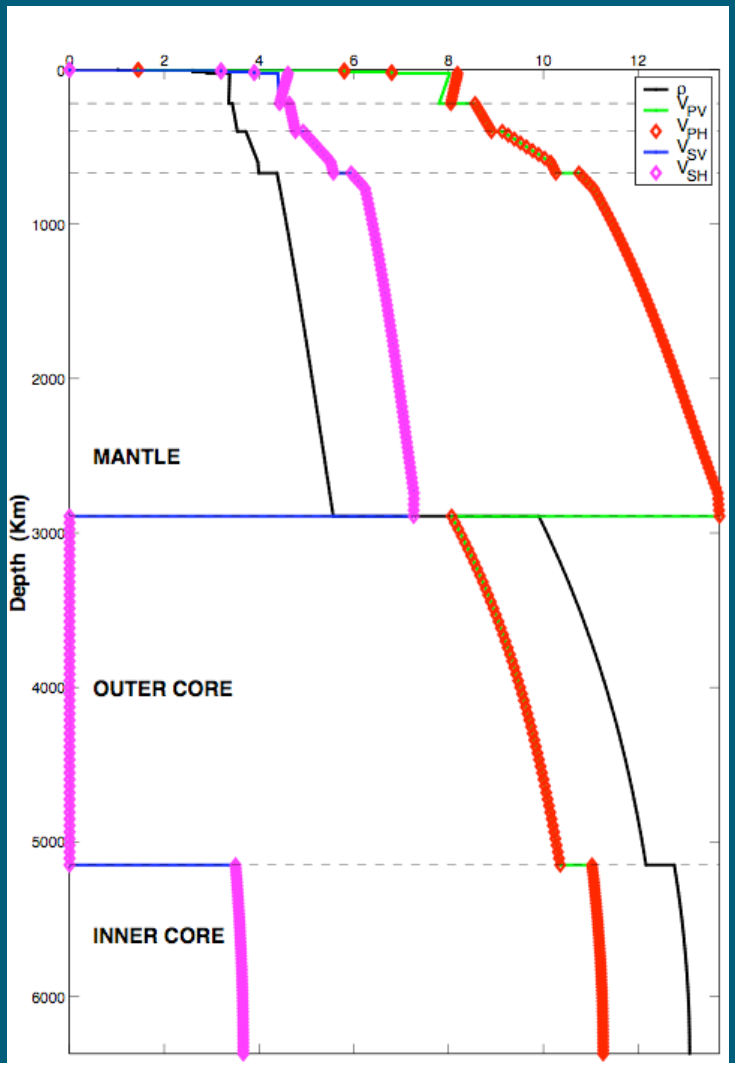
- -- Vs
- - Vp
- -. Density



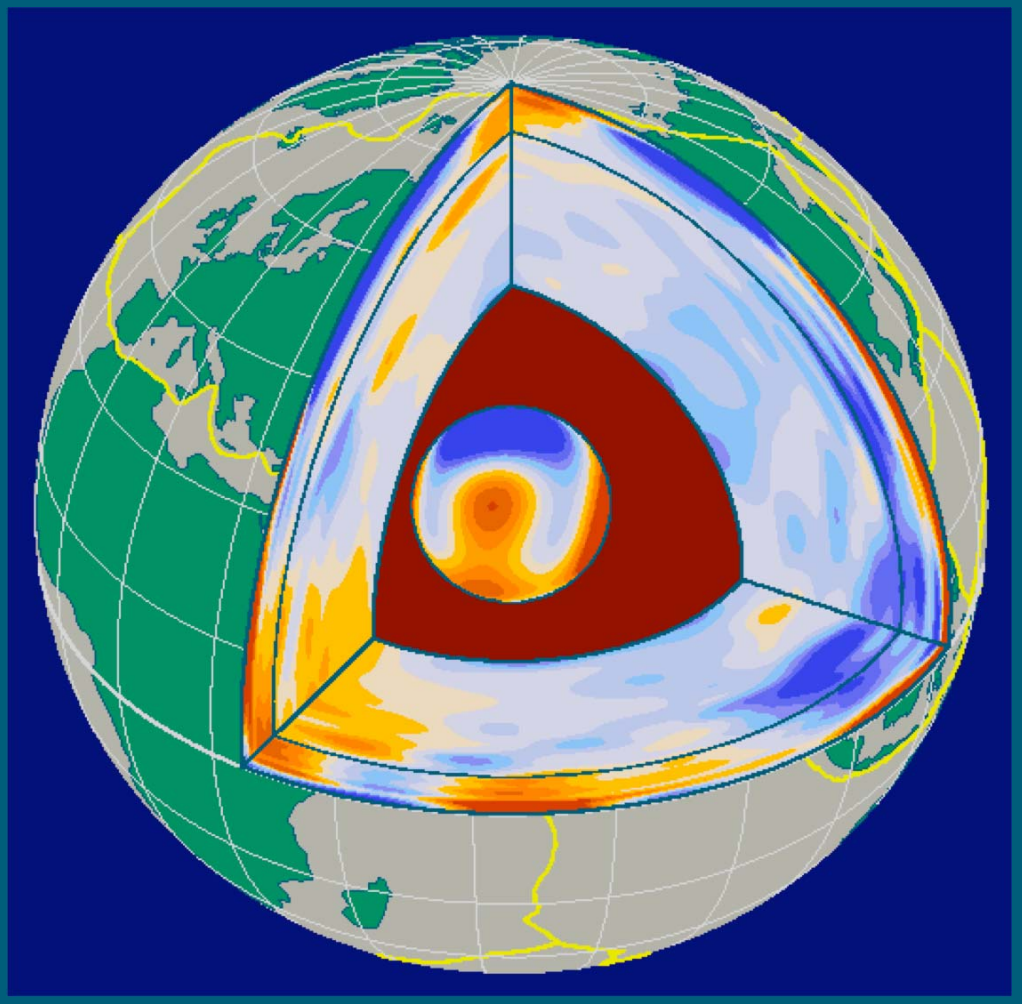
Normal modes - successes
Global seismic tomography

University of East Anglia

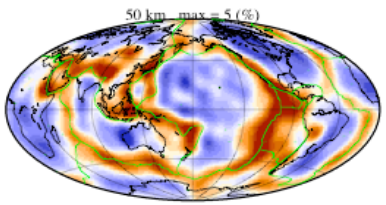
PREM: ~1000 mode frequencies
Dziewonski and Anderson, 1981

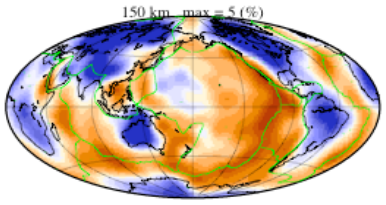


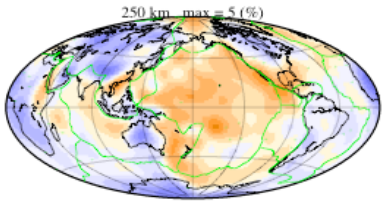
S20RTS - Mode splitting measurements
Ritsema et al., 1999

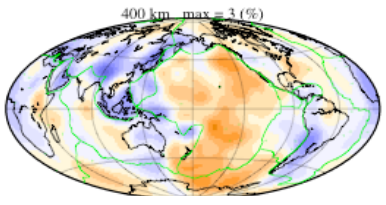


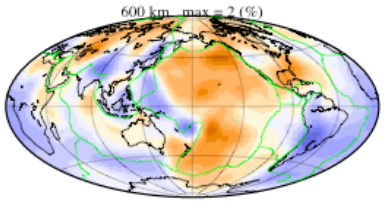
$\frac{\delta V_S}{V_S}$

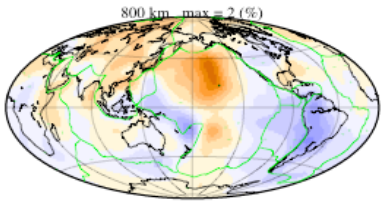




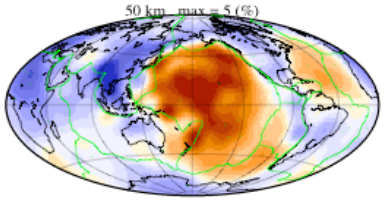


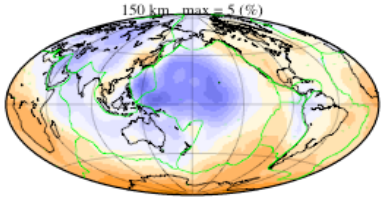


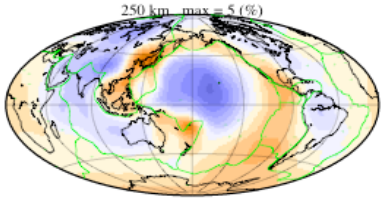


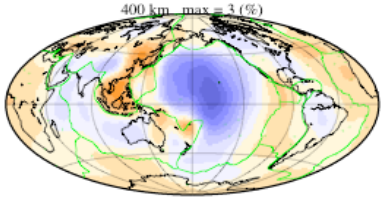


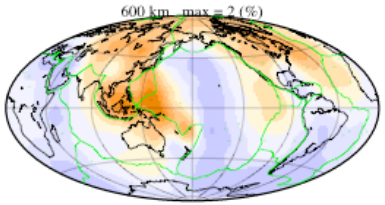
$\frac{V_{SH}-V_{SV}}{V_S}$

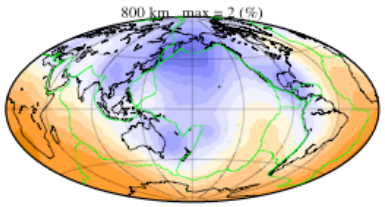












50 km

150 km

250 km

400 km

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800 km

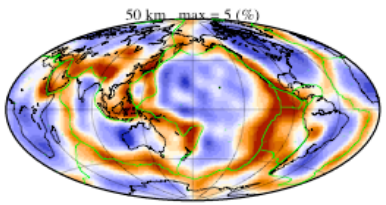
University of East Anglia

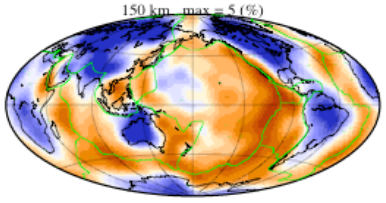
Global seismic tomography: radial anisotropy

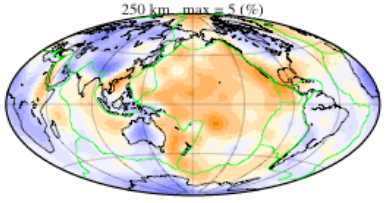
The influence of the crust

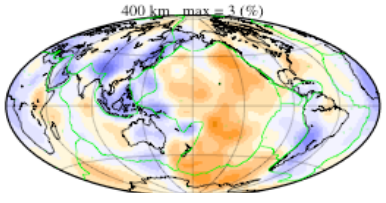
Ferreira et al., JGR, 2010

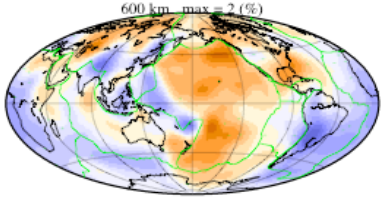
$\frac{\delta V_S}{V_S}$

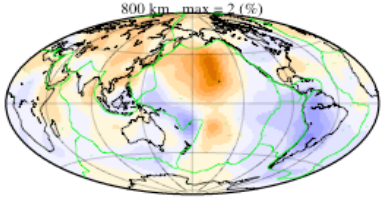




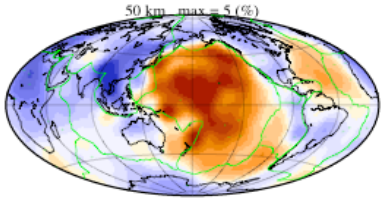


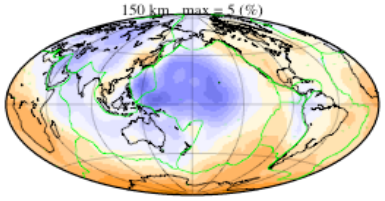


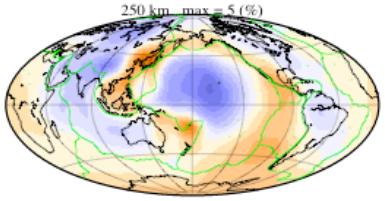


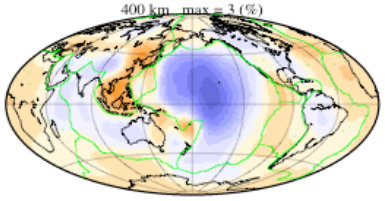


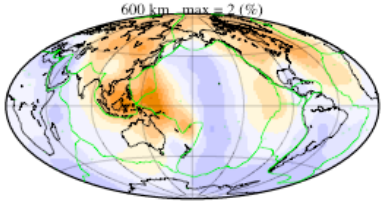
$\frac{V_{SH}-V_{SV}}{V_S}$

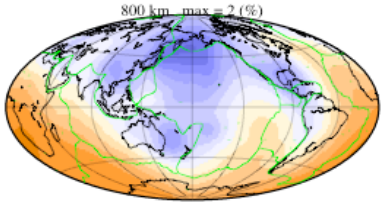












50 km

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University of East Anglia

Global seismic tomography: radial anisotropy

The influence of the crust

See poster :
S.-J. Chang (thursday)

Ferreira et al., JGR, 2010

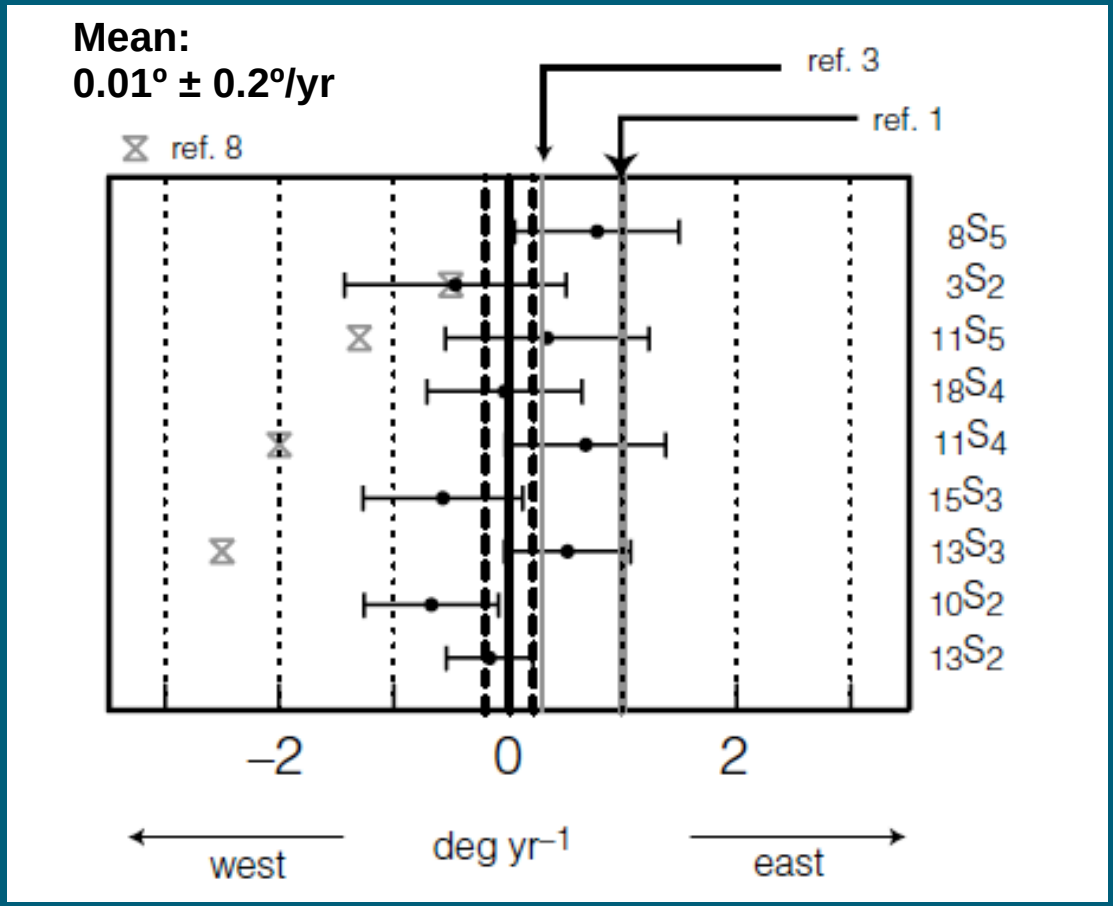
Normal modes - successes

Inner core structure and rotation

University of East Anglia

Inner-core rotation rates

Mean:
0.01° ± 0.2°/yr

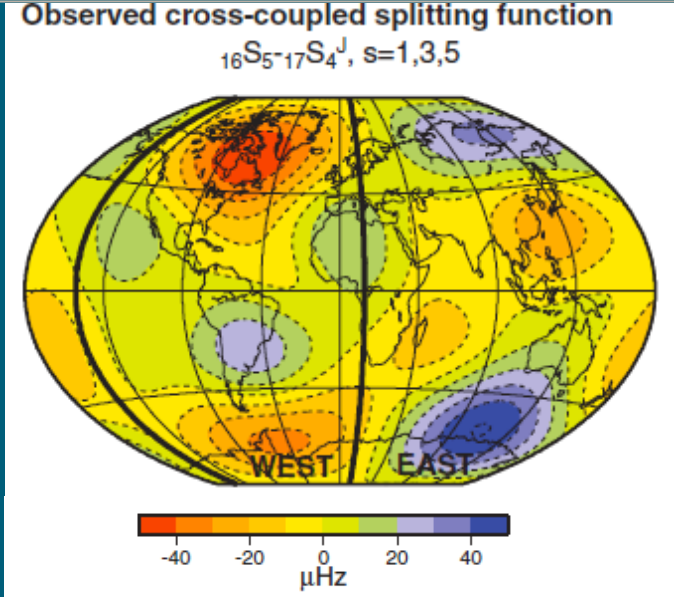


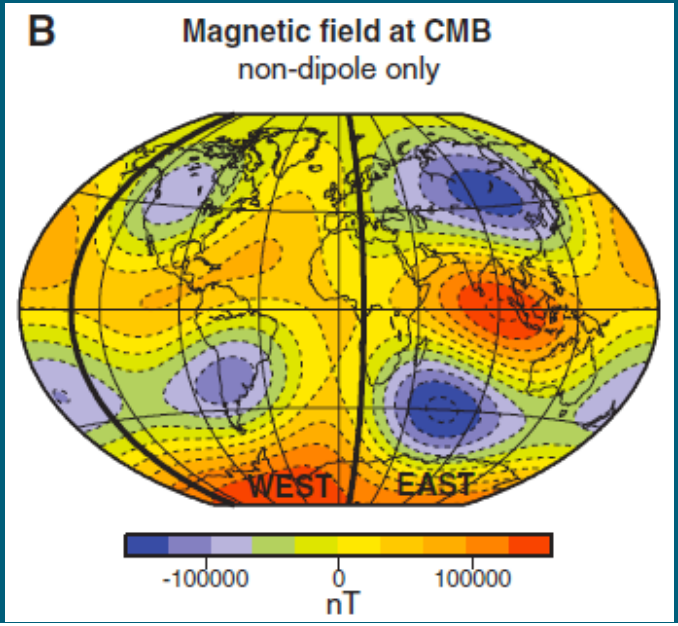
Laske & Masters, 1999

Normal modes - successes

Inner core structure and rotation

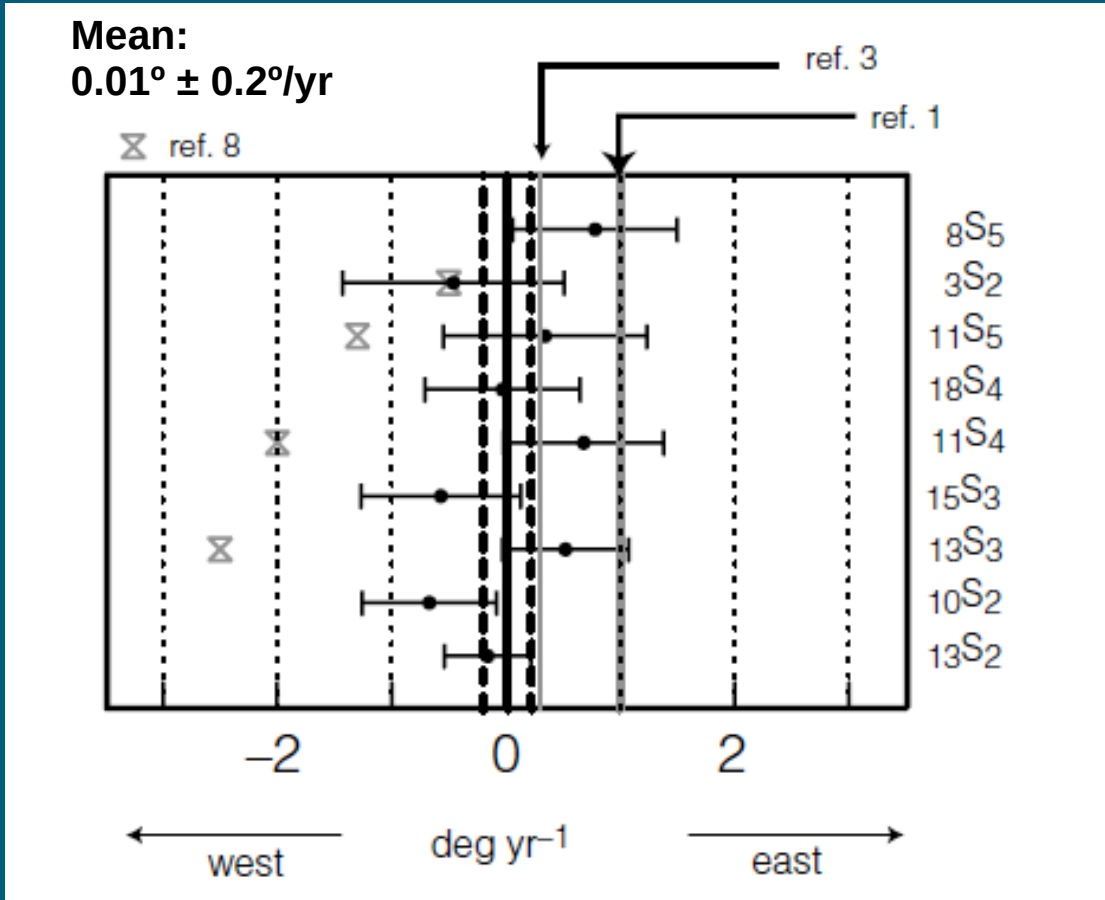
University of East Anglia





Deuss et al., 2010

Inner-core rotation rates



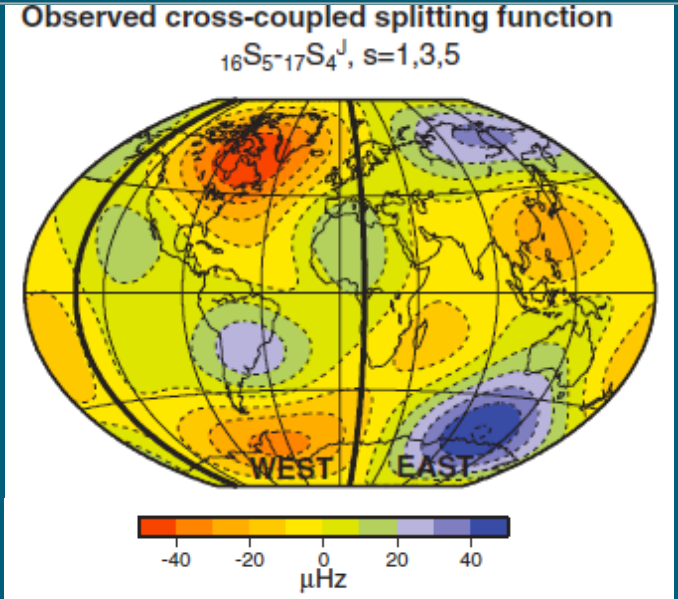
Laske & Masters, 1999

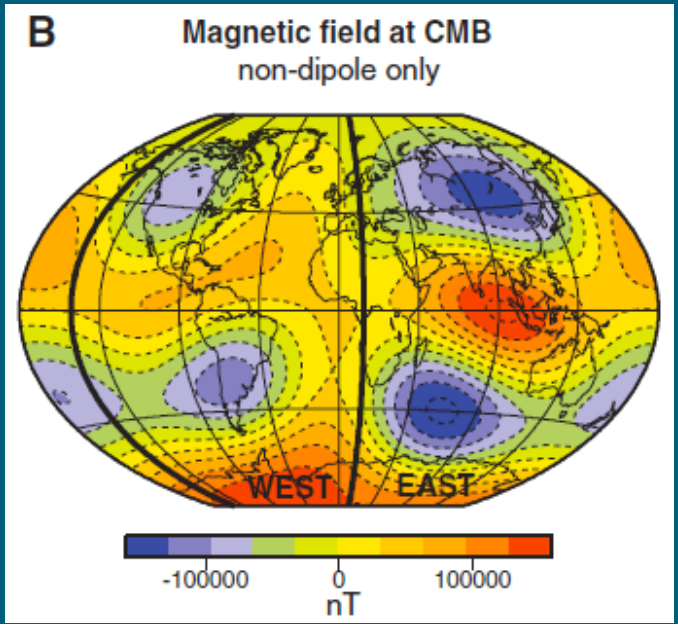
Common origin for radial magnetic field and IC anisotropy? Freezing-in of crystal alignment during solidification or texturing by Maxwell stress.

Normal modes - successes

Inner core structure and rotation

University of East Anglia





Deuss et al., 2010

See talk :
P. Koelemeijer (this session)

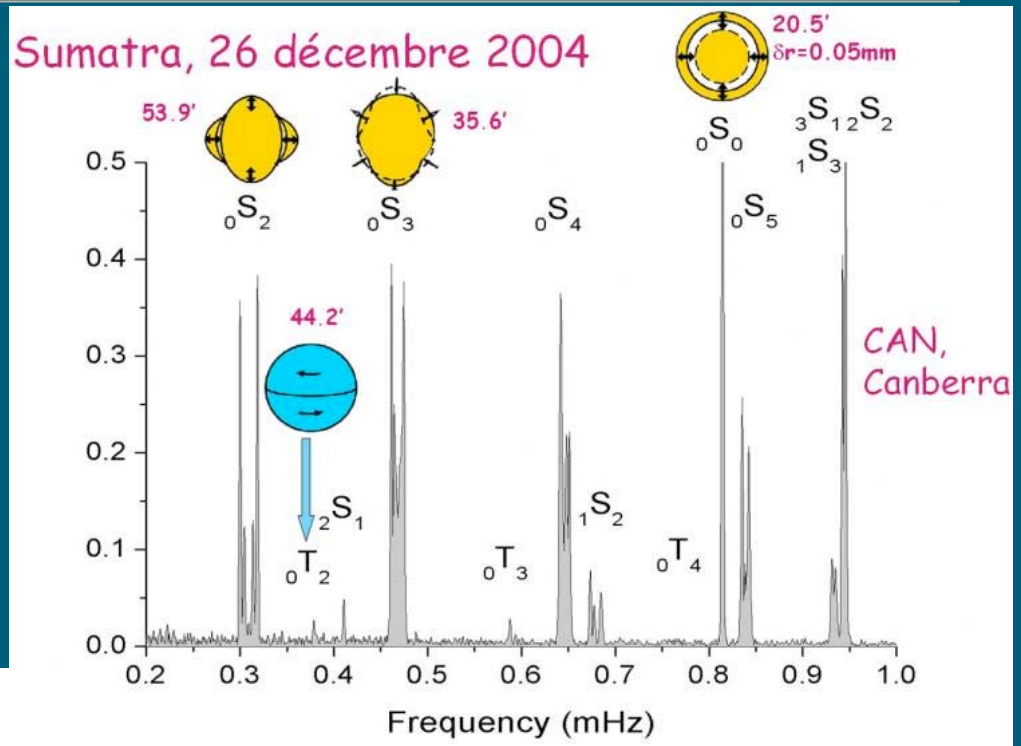
See poster:
A. Makinen (thursday)

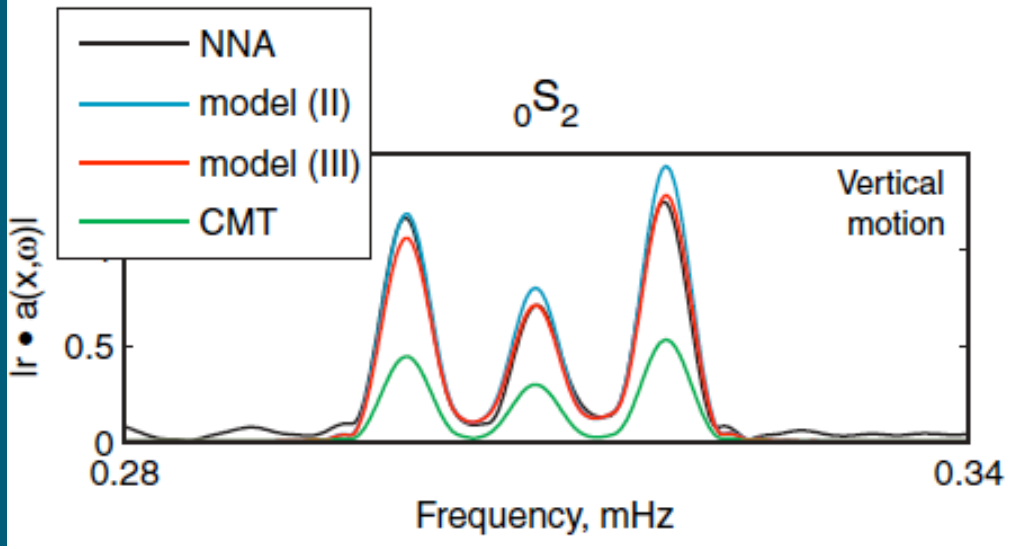
Normal modes - successes
Giant earthquake source studies

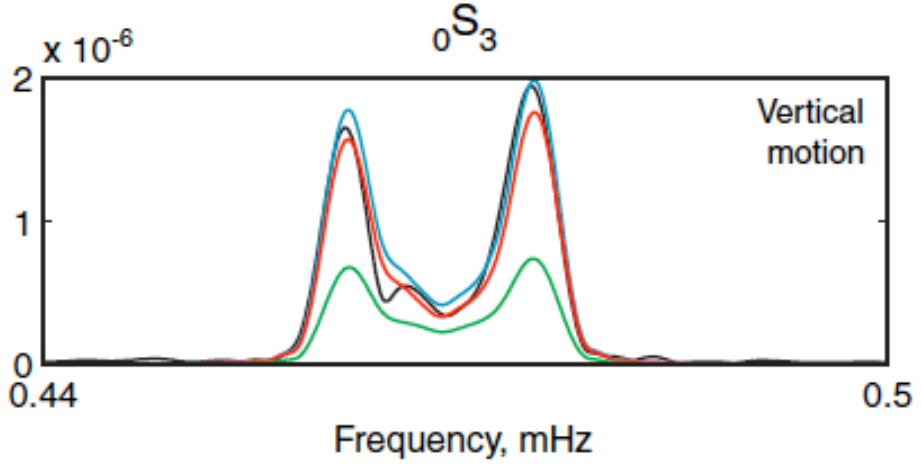
University of East Anglia

December 26, 2004
Mw ~9.2 Sumatra

Park et al., 2005





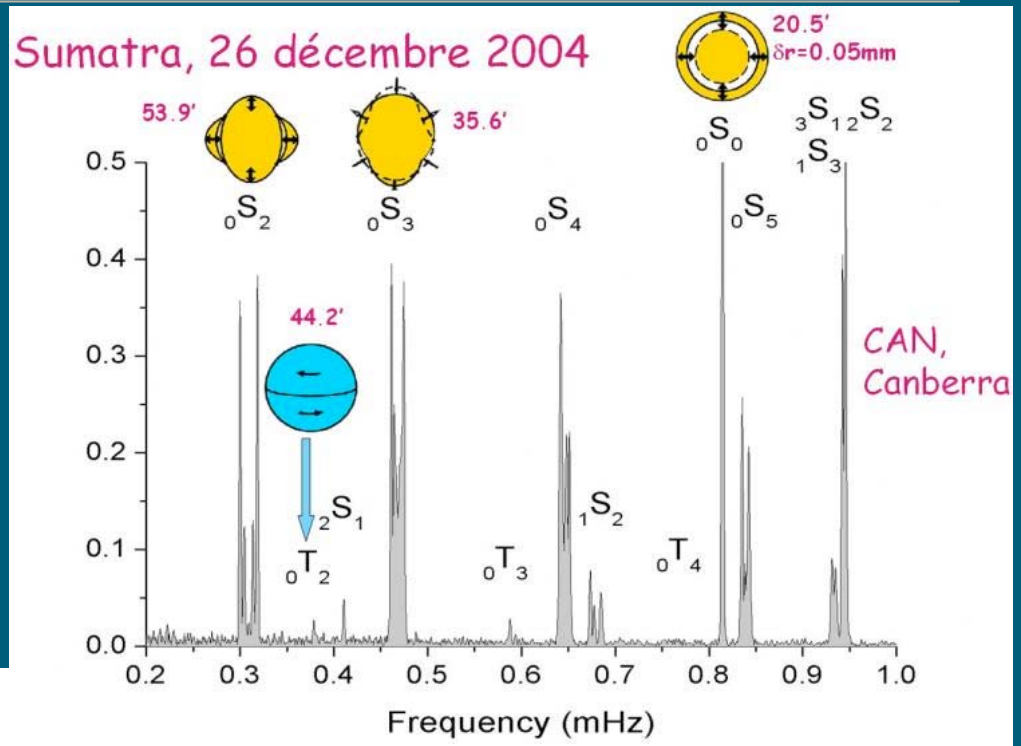


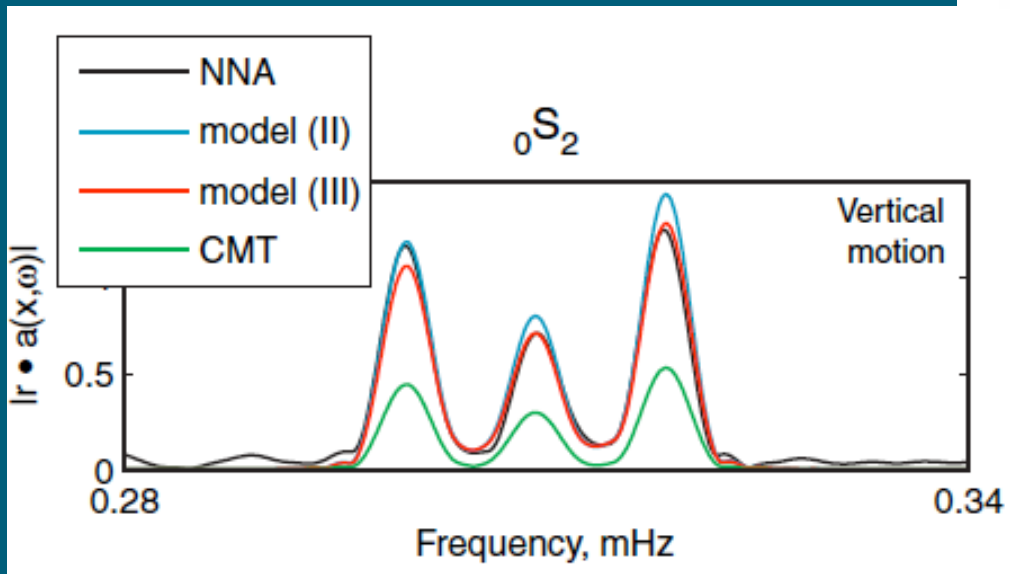
Normal modes - successes
Giant earthquake source studies

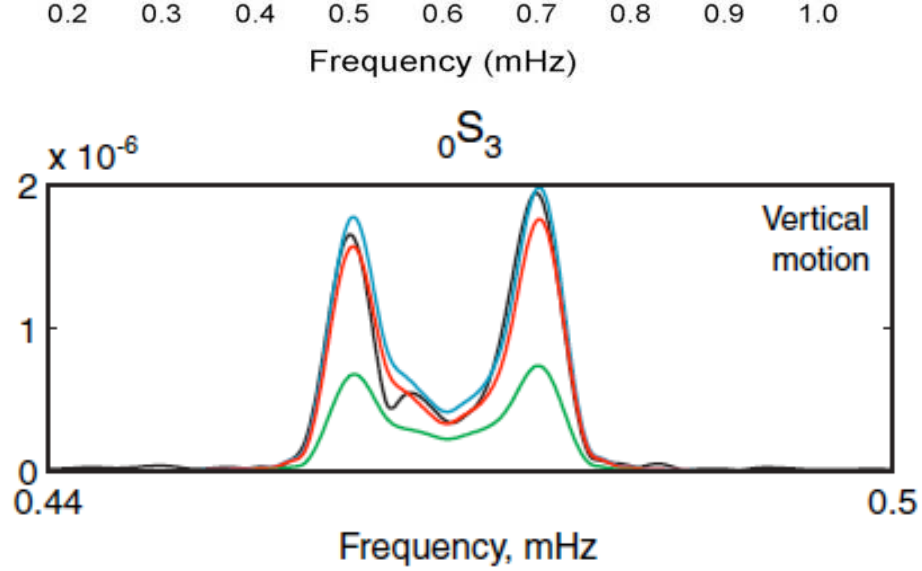
University of East Anglia

See talk :
K. Lentas (this session)

Park et al., 2005





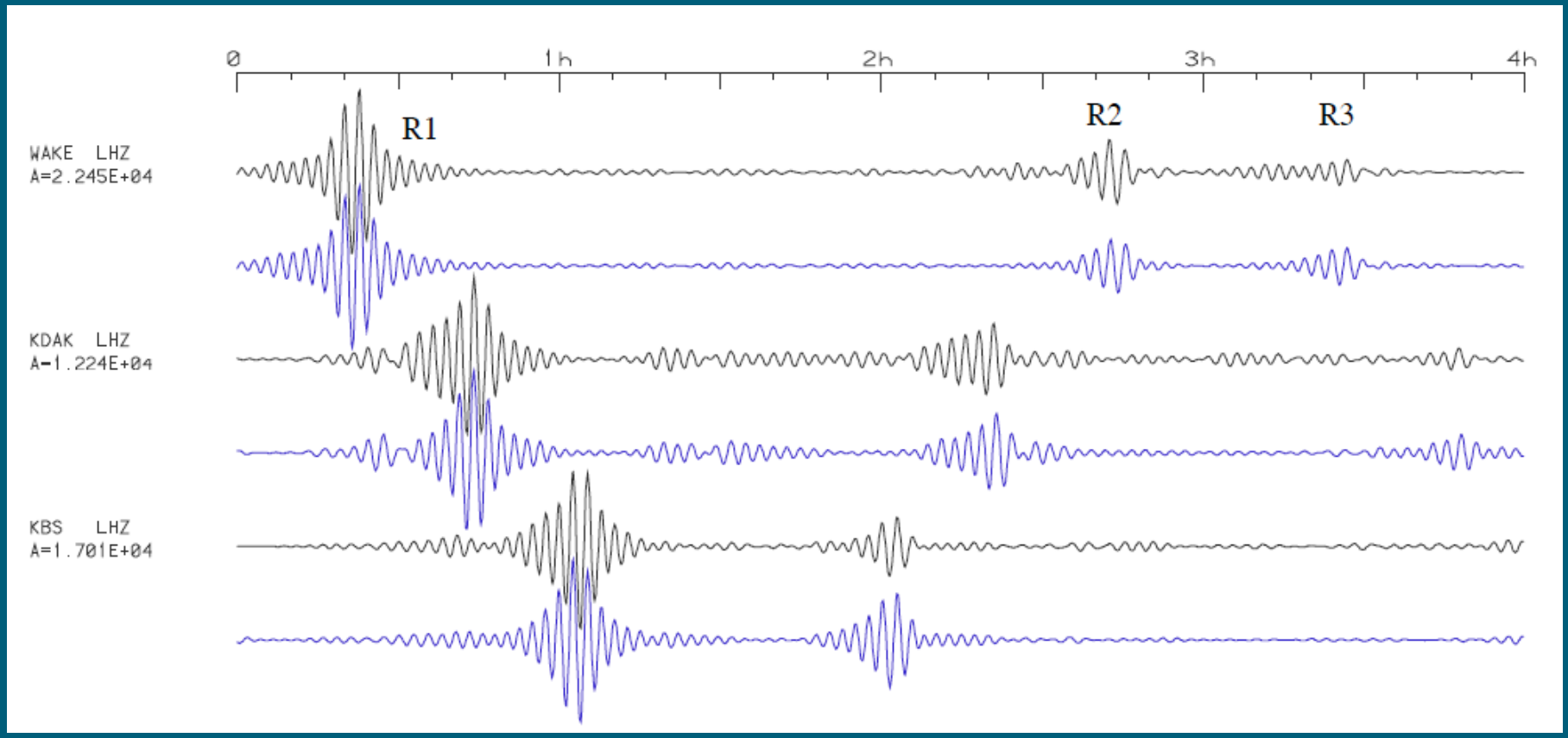


Normal modes - successes

Mode summation synthetics

University of East Anglia

Mw 7.4, January 4 1998, Loyalty Islands earthquake



Rayleigh waves, T~135-155s

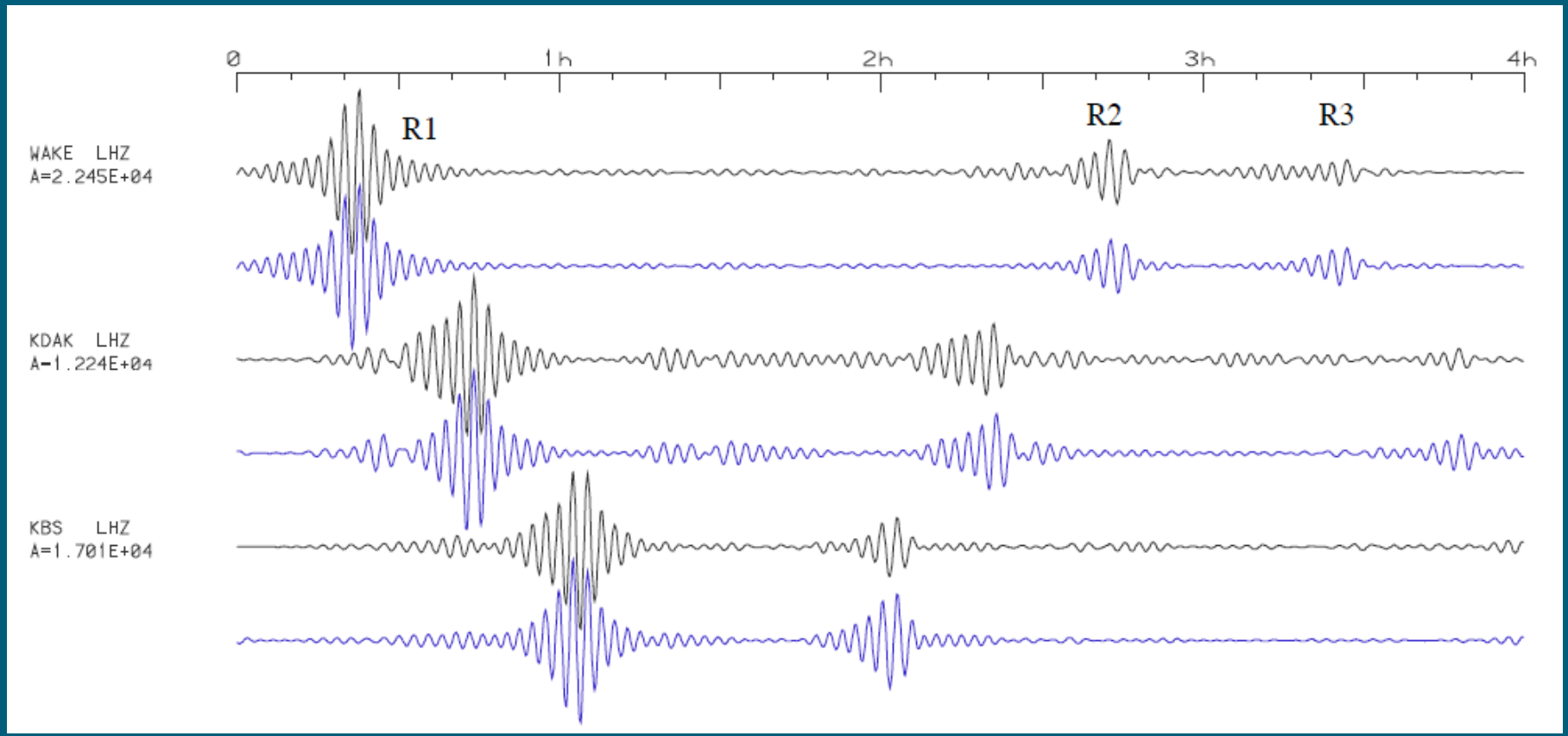
- Baseline for many approximate forward modelling methods

Normal modes - successes

Mode summation synthetics

University of East Anglia

Mw 7.4, January 4 1998, Loyalty Islands earthquake

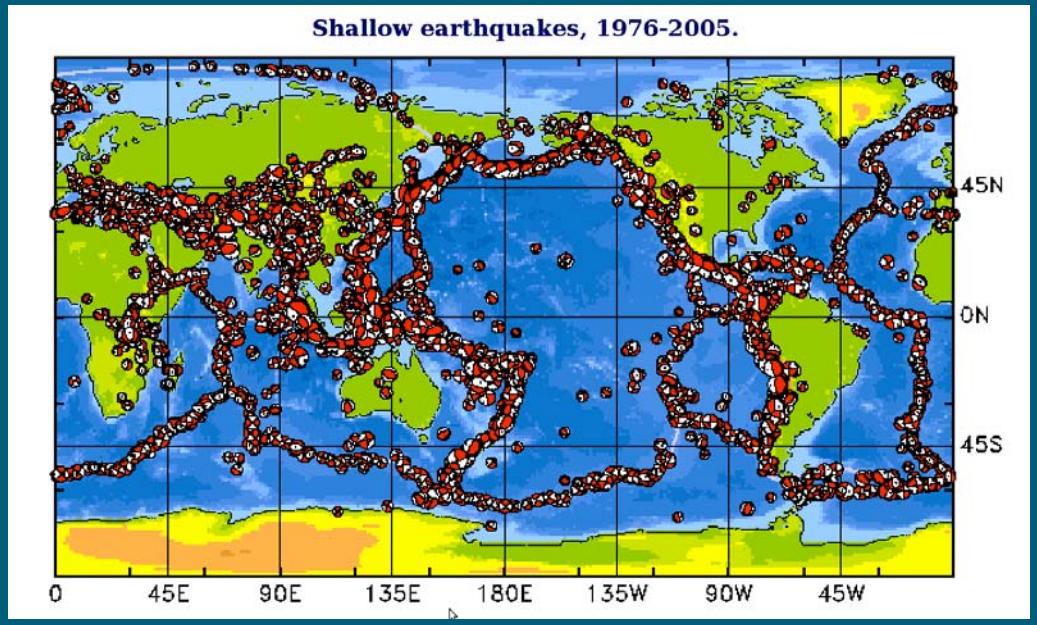


See talk : L. Gualtieri (this session)
See poster: L. Parisi (wednesday)

Normal modes - successes

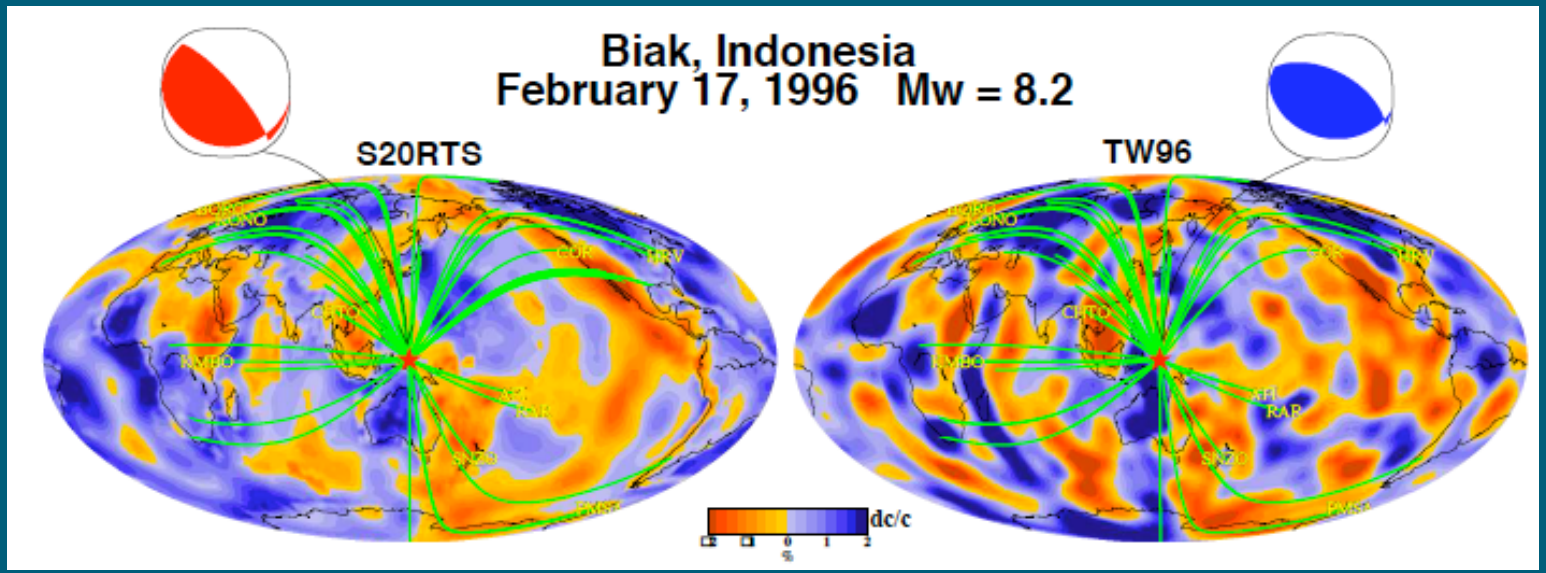
Mode summation synthetics - source invs

University of East Anglia



Global CMT

www.globalcmt.org



Source inversions using 3D Earth models

Ferreira & Woodhouse, 2006

Normal modes - current directions

Challenges

University of East Anglia

- Build robust images of:
  - Anisotropy: mantle flow patterns
  - Attenuation: temperature, fluids
  - Density: key input for geodynamical simulations

- Obtain bulk, complementary information on the earthquake source process of giant earthquakes

- Continue to assemble high-quality normal mode data: instrumentation, measurements

- Use sophisticated modelling strategies (e.g., full coupling)

Normal modes - current directions

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University of East Anglia

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Normal modes - current directions

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University of East Anglia

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See posters :
D. Al-Attar (thursday); M. Meschede (thursday)