How to seperate intrinsic and artificial anisotropy

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Introduction

The effect of anisotropy on seismic waves and on the inversion for 3D tomographic models of velocity and anisotropy is not negligible and is used for different applications in geodynamics for both regional and global scale (Montagner, TOG, 2007).The exact determination and interpretation of anisotropy (amplitude and orientation) are quite difficult because the observed or inverted anisotropy is usually a mixture of intrinsic and artificial anisotropies, which may partly hide the true properties of the medium. The *artificial anisotropy* is due to two reasons: first of all, to the the fact that seismic waves do not see the real details of medium but a "filtered" (and imperfect) version of the earth model and second of all, to the inversion technique. Our objective is to seperate the intrinsic and artificial anisotropy.

Artificial anisotropy (I)

It is shown that an isotropic medium with seismic discontinuities (such as Mohorovicic discontinuity, LAB, 220km, 410km, or 660km) will be "seen" as a transversely isotropic medium with a vertical symmetry axis (VTI). Backus (1962) provided the formulae to calculate such effective elastic parameters for the 1-D horizontally layered isotropic medium.

$$M_1=\begin{bmatrix}\lambda+2\mu & \lambda & \lambda & 0 & 0 & 0\\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0\\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0\\ 0 & 0 & 0 & \mu & 0 & 0\\ 0 & 0 & 0 & 0 & \mu & 0\\ 0 & 0 & 0 & 0 & 0 & \mu\end{bmatrix}\quad M_2=\begin{bmatrix}A & A-2N & F & 0 & 0 & 0\\ A-2N & A & F & 0 & 0 & 0\\ F & F & C & 0 & 0 & 0\\ 0 & 0 & 0 & L & 0 & 0\\ 0 & 0 & 0 & 0 & L & 0\\ 0 & 0 & 0 & 0 & 0 & N\end{bmatrix}$$

M_1 is the elastic tensor for a 1-D isotropic model, matrix M_2 is what seismic waves "see" from M_1 and corresponds to an anisotropic model. From matrix M_2,we get anisotropy parameter ξ=N/L=$\langle\mu\rangle\langle 1/\mu\rangle$ which is actually the artifical seismic anisotropy.

We try to estimate the amplitude of artificial radial anisotropy associated with "filtering" effect which denoted as ξ^{err}_{filter}, and ξ^{err}_{filter}=$\xi-1$.

Suppose we have a periodic, isotropic, two layered (PITL) model: a medium periodic in the vertical direction and consisting of alternating isotropic layers of thicknesses h_1, h_2, having constant Lame parameters λ_1, μ_1, and λ_2, μ_2, and constant densities ρ_1, ρ_2 (Backus, 1962).

Define the dimensionless parameter θ=$\frac{\mu}{\lambda+2\mu}$ which is the square of the ratio of shear velocity to compressional velocity, the fraction p_1=$\frac{h_1}{h_1+h_2}$, and the fraction p_2=$\frac{h_2}{h_1+h_2}$ for the PITL model, then its effective anisotropic parameter: ξ=$(p_1\mu_1$+$p_2\mu_2)(p_1\mu_1^{-1}$+$p_2\mu_2^{-1})$. Furthermore, define the ratio of the shear modulus α=μ_2/μ_1 of the PITL model, then ξ=$(1$+$(\alpha$-$1)\,p_1)(\alpha$-$(\alpha$-$1)p_1)/\alpha$.

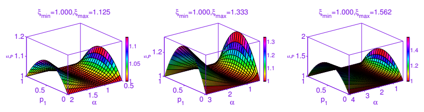

Figure 1: The effective anisotropic parameter ξ of the PITL model under different range of α: $\alpha\in[1/2,2]$, $\alpha\in[1/3,3]$ and $\alpha\in[1/4,4]$.

Inversion Method

By Rayleigh's principle (Smith and Dahlen, 1973), we have

$$\delta(d)=d-d_0=g(p)-g(p_0)=\int_0^R\sum_p\left(\frac{\partial g}{\partial p}.\delta p(r)\right)$$

and its linearization form is $g(p)-g(p_0)=G(p-p_0)$. Tarantola and Valette (1982) use the least square method to minimize the cost function F as

$$F=(d-Gp)^TC_d^{-1}(d-Gp)+(p-p_0)^TC_p^{-1}(p-p_0)$$

where d_0 and p_0 are initial parameters, C_d and C_p are respectively prior covariance matrices for data and parameter spaces.

Artificial anisotropy (II)

We use a 1-D continuous isotropic 1066A model (called "REF") and its Vsv perturbed model (called "REAL") to estimate the amplitude of artificial radial anisotropy associated with the inversion technique which denoted as ξ^{err}_{inv}.

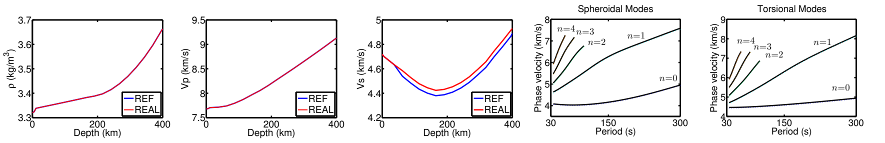

Figure 2: (**Left 3**) The parameters ρ,Vp and Vs for the 1-D isotropic "REF" and "REAL" models; (**Right 2**) The phase velocity of the fundamental and the first four overtones of Spheroidal and Torsional modes for the "REF" and "REAL" models at different periods.

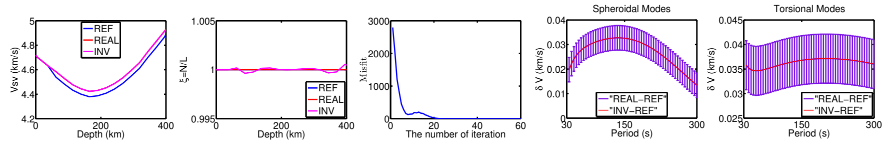

Figure 3: (**Left 2**) The inverted parameters Vsv and ξ for the 1-D isotropic 1066A model; (**The middle**) The value of the cost function at different iteration steps; (**Right 2**) Differences of phase velocity of the fundamental Spheroidal and Torsional modes between the REF model and REAL model, the inverted model and REF model.

Inversion of phase velocity data of surface wave

We generate a 1-D anisotropic PREM model (Dziewonski and Anderson, 1981), use its phase velocity of both fundamental and overtones of Spheroidal and Torsional modes to do inversion and try to retrive the real anisotropy.

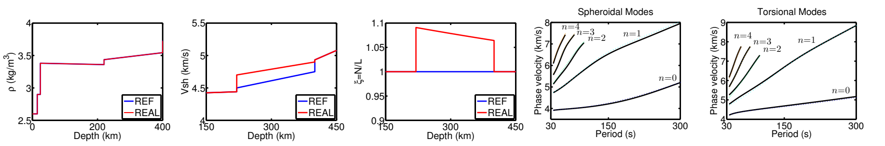

Figure 4: (**Left 3**) The parameters ρ, Vsv and ξ for the 1-D isotropic reference and anisotropic real PREM models; (**Right 2**) The phase velocity of the fundamental and the first four overtones of Spheroidal and Torsional modes for the "REF" and "REAL" models at different periods.

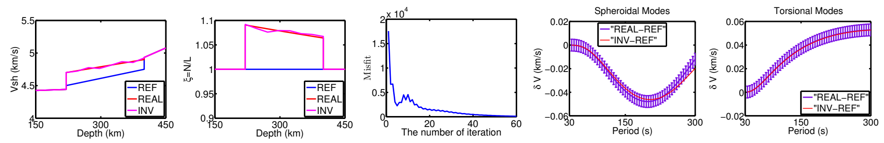

Figure 5: (**Left 2**) The inverted parameters Vsv and ξ for the 1-D anisotropic PREM model; (**The middle**) The values of cost function at different iteration steps; (**Right 2**) Differences of phase velocity of the fundamental Spheroidal and Torsional modes between the REF and REAL model, the inverted and REF model.

Inversion Method

The procedure of inversion can be summarized as:

- **Forward problem**
  - To compute the phase velocity $V_R(T)=g_1(p)$, $V_L(T)=g_2(p)$ of the spheroidal and torsional modes ($n\in[0,4]$) of the reference and the real models at different periods T (p are the parameters of the models such as density ρ, $A=\rho V^2_{PH}$, $C=\rho V^2_{PV}$, $L=\rho V^2_{SV}$, $N=\rho V^2_{SH}$, $F=\eta/(A-2L)$, subscript R refers to the Rayleigh wave, L refers to the Love wave and $T\in[35,300]$ s)
  - To compute the partial derivatives of the eigenperiod ${}_0T_l$ with respect to parameters p (ρ, A, C, L, N, F) for the reference model, then $(p/T)(\partial T)/(\partial p)$ are converted to phase velocity partial derivatives by using $\frac{p}{V}(\frac{\partial V}{\partial p})_T=-\frac{V}{U}\frac{p}{T}(\frac{\partial T}{\partial p})_k$ (V is the phase velocity and U is the group velocity)
- **Inversion at depth**

  We use an iterative quasi-Newton method together with the generalized minimal residual (GMRES) method to solve the last inverse problem in a least square sense.

Conclusion

- **The "filtering effect" is explored through analytical solution, and its corresponding amplitude of artifical anisotropy ξ^{err}_{filter} can be reached to a large amount (eg. 56%).**
- **The amplitude of artificial anisotropy of our inversion technique ξ^{err}_{inv} is about 1% when tested by the 1-D continuous 1066A isotropic model.**
- **The inverted results of the 1-D anisotropic PREM model show that we can retrieve the intrinsic anisotropy through phase velocity data of surface wave.**
- **Actual seismic data for the 3D problem is left to the future for demonstrating the separation of intrinsic and artificial anisotropy.**