Modelling seismic noise by normal mode summation

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in collaboration with
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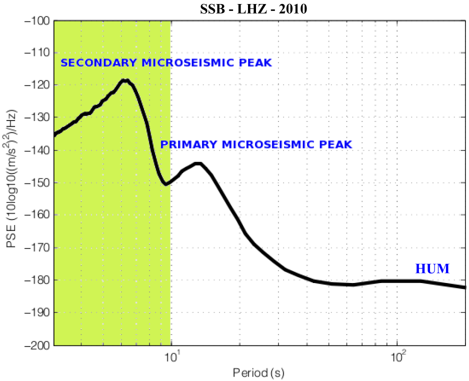
IV QUEST workshop - May 20th 2013

1-Modelling secondary microseismic noise

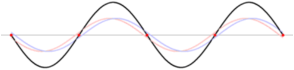
2-Modelling body waves sources

3-Modelling long period noise



Noise sources discretisation and synthetic seismogram

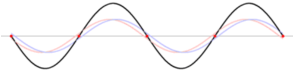
Theory by *Longuet-Higgins, 1950* and *Hasselmann, 1963*:



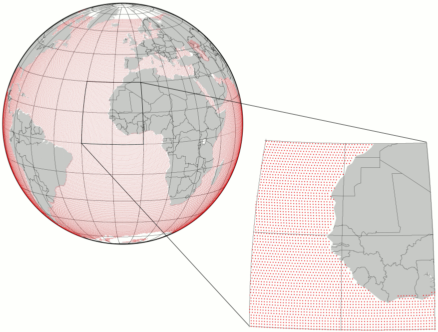
Seismic noise sources: **single vertical forces close by the ocean surface**

Noise sources discretisation and synthetic seismogram

Theory by *Longuet-Higgins, 1950* and *Hasselmann, 1963*:



Seismic noise sources: **single vertical forces close by the ocean surface**



Synthetic seismogram by using a single vertical force:

$$\boxed{\mathbf{u}(\mathbf{r},\theta,\phi) = \gamma_l^2 \; F_r \; v_r \; \mathrm{U}_k(\mathbf{r_s}) \; \mathrm{U}_k(\mathbf{r_r}) \; Y_k^0(\Theta_s,\Phi_s) Y_k^0(\Theta_r,\Phi_r) \exp(i\omega_k t)}$$

Modelling noise sources as vertical forces

Vertical force amplitude
(WAVEWATCH III^R - Ardhuin et al., 2011):

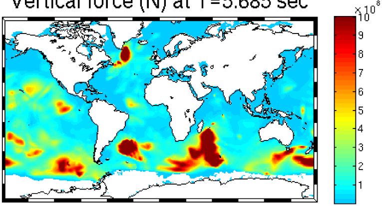
$$\boxed{F(f_s, dS, df_s) = 2\pi\sqrt{F_p(K \simeq 0, f_s) \times dS \times df_s}}$$

where

$$F_p(K \simeq 0, f_s) = {\rho_w}^2 g^2 f_s E^2(f) I(f)$$

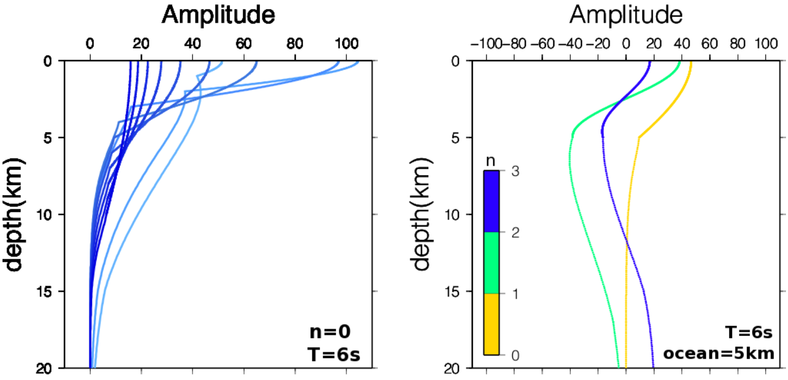
is the equivalent wave-induced pressure spectrum at ocean surface.
$E(f)$ is the surface elevation variance of the two ocean trains.

Vertical force (N) at T=5.685 sec



1st June 2010 (00:00-03:00)

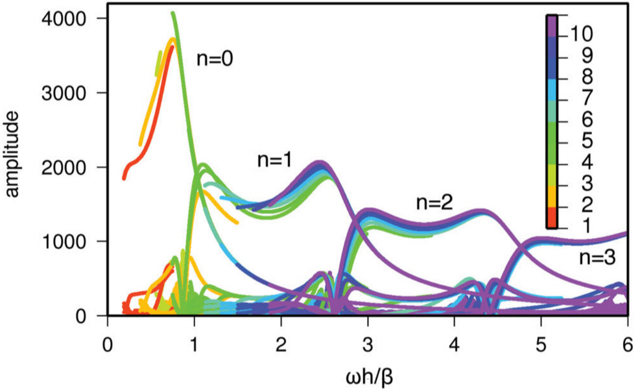
Eigenfunctions ${}_0\mathrm{U}_\mathrm{l}$, ${}_1\mathrm{U}_\mathrm{l}$, ${}_2\mathrm{U}_\mathrm{l}$ respectively at 6*s*:



by using normal modes in PREM model:

$$c_n = \frac{{}_n\mathrm{U}_l(\mathbf{r_r})\,{}_n\mathrm{U}_l(\mathbf{r_s})}{{}_n\omega_l}$$

${}_n\mathrm{U}_l(\mathbf{r_r})$, ${}_n\mathrm{U}_l(\mathbf{r_s})$ = eigenfunctions at receiver and source position; ${}_n\omega_l$ =eigenfrequency



and considering the receiver on land:

$$c_n = \frac{{}_n\mathrm{U}_l(\mathbf{r_r})\,{}_n\mathrm{U}_l(\mathbf{r_s})}{{}_n\omega_l}$$

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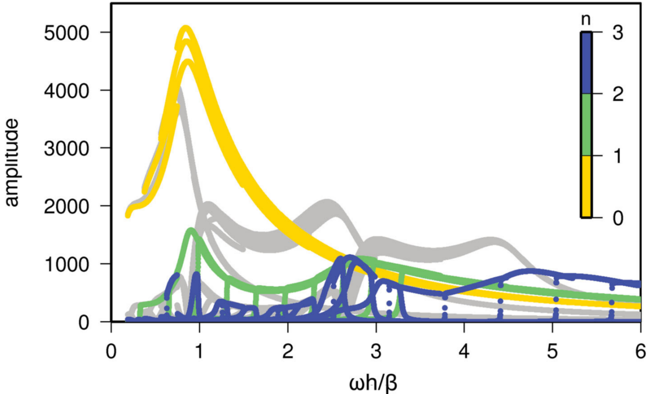
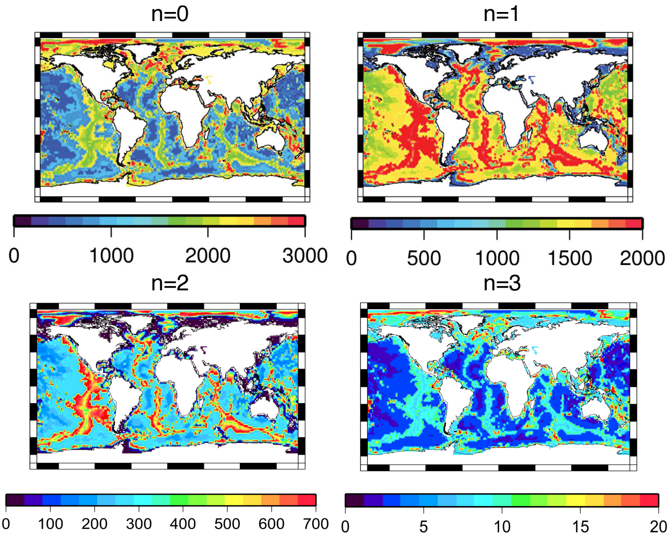
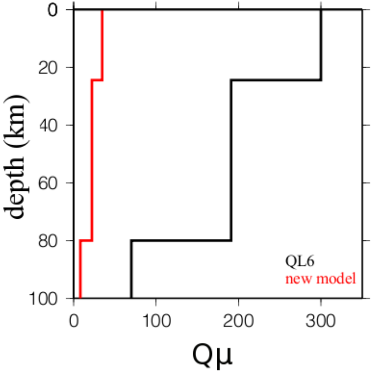


Maps of amplification factor - n=0,1,2,3 - T=6 s



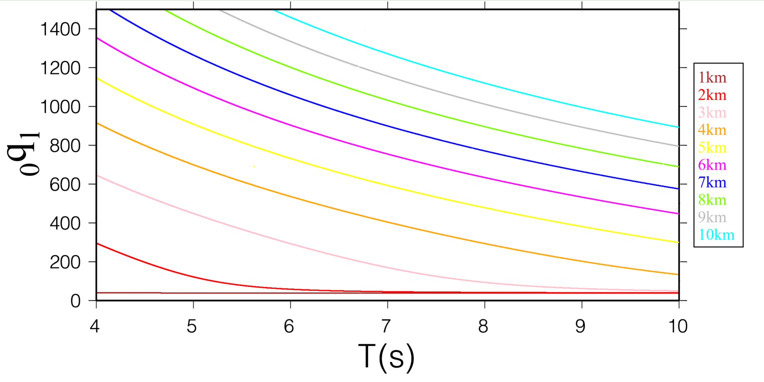
Apparent attenuation model

depth$=$ **0 $-$ 100 km**:
decreasing of all values of
QL6
↓
depth$\geq$ **100 km**:
QL6
(*Durek and Ekström*, 1996)

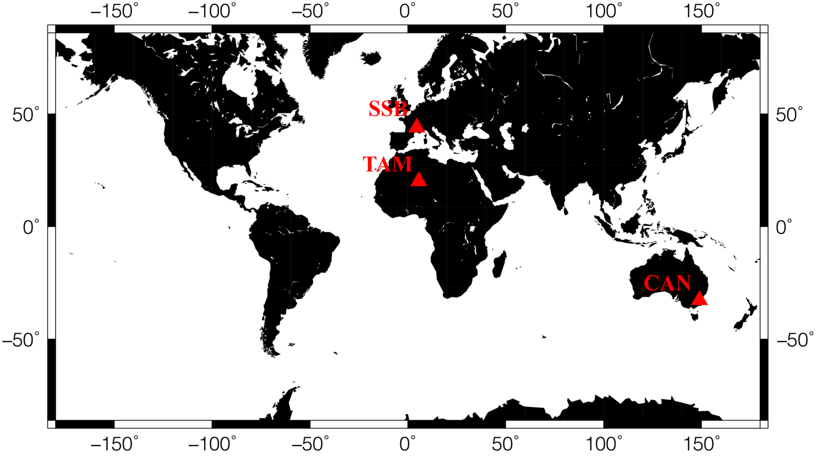


Attenuation of Rayleigh fundamental mode (n=0)

The attenuation of Rayleigh waves:
- decreases with increasing period;
- increases with increasing water depth.



Vertical components of noise spectra: Rayleigh waves modelling



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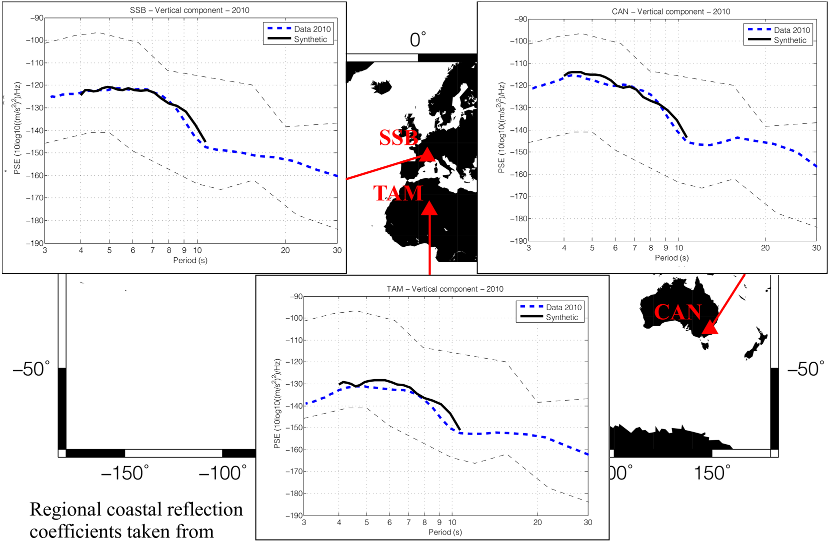
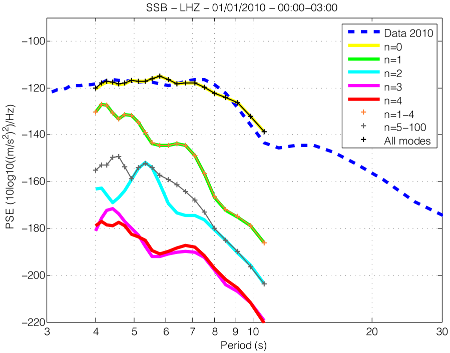


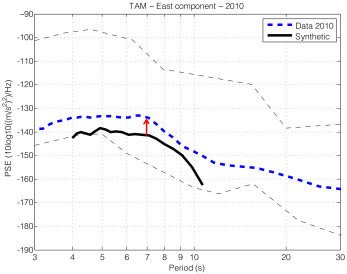
Regional coastal reflection coefficients taken from Stutzmann et al., 2012

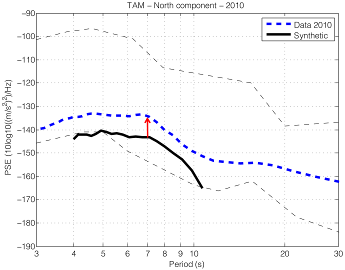
Role of the overtones



- The amplitude decreases with increasing the overtone number;
- The differences between them become smaller with increasing the overtone number;
- The amplitude computed with the fundamental mode is comparable with the amplitude computed with 100 modes.

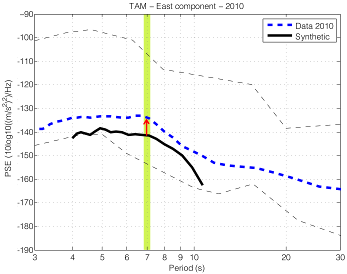
Love wave energy estimation in horizontal components

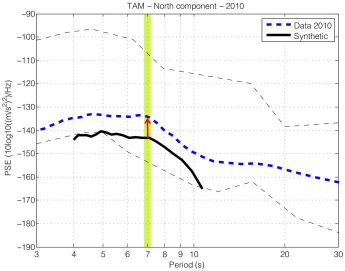




Synthetic spectra: only Rayleigh waves

Love wave energy estimation in horizontal components





Missing Love wave energy estimation: e.g. 7 *s*

Power Spectral Energy (Raileigh & Love waves) $\simeq -135\ dB$

It is necessary: $\frac{E_{Love}}{E_{Rayleigh}} \sim 0.65$

→ compatible with *Nishida et al.*, 2008.

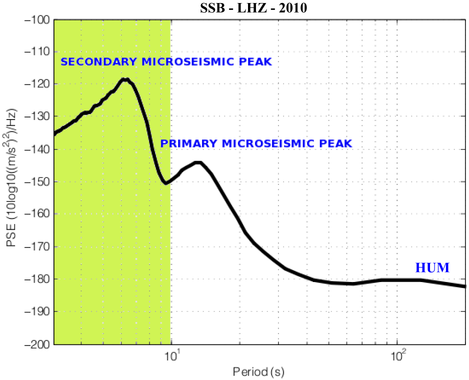
Gualtieri et al., 2013, GJI

1-Modelling secondary microseismic noise

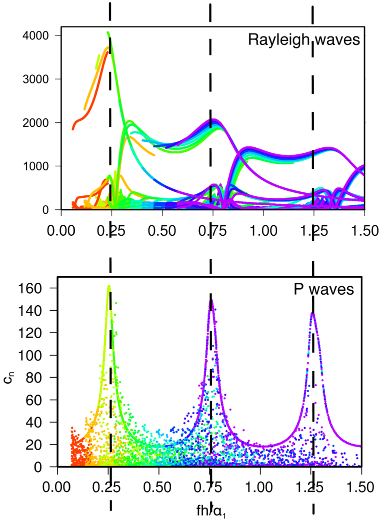
2-Modelling body waves sources:

work in progress!

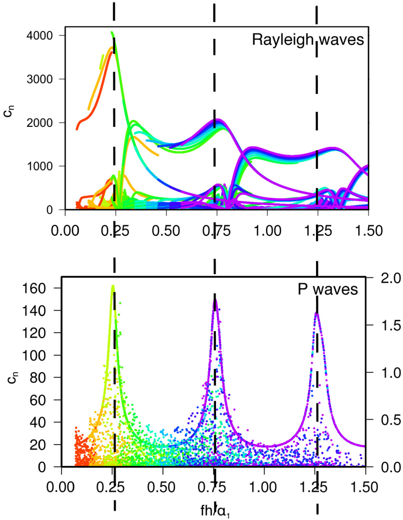
3-Modelling long period noise

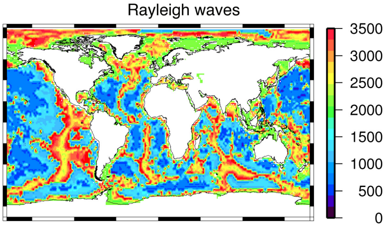


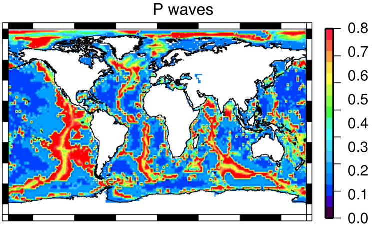
Modelling body wave sources: amplification factor



Modelling body wave sources: amplification factor T=7s







Gualtieri et al., 2013b, to be submitted

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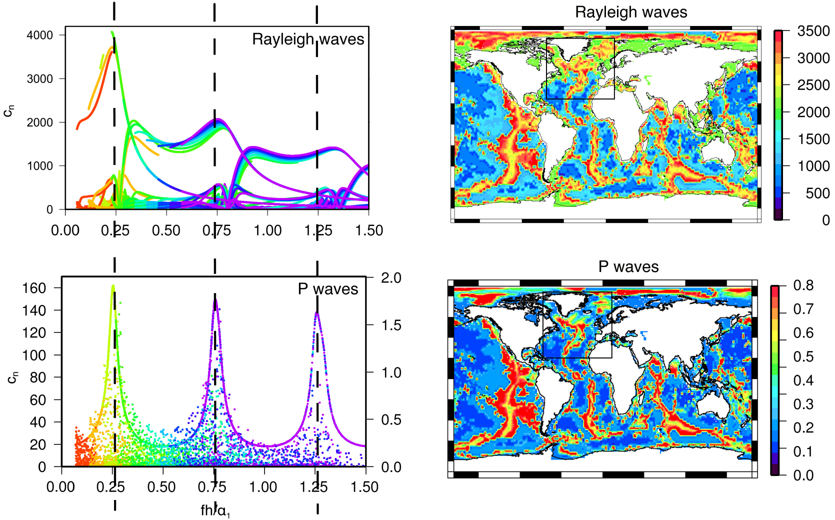

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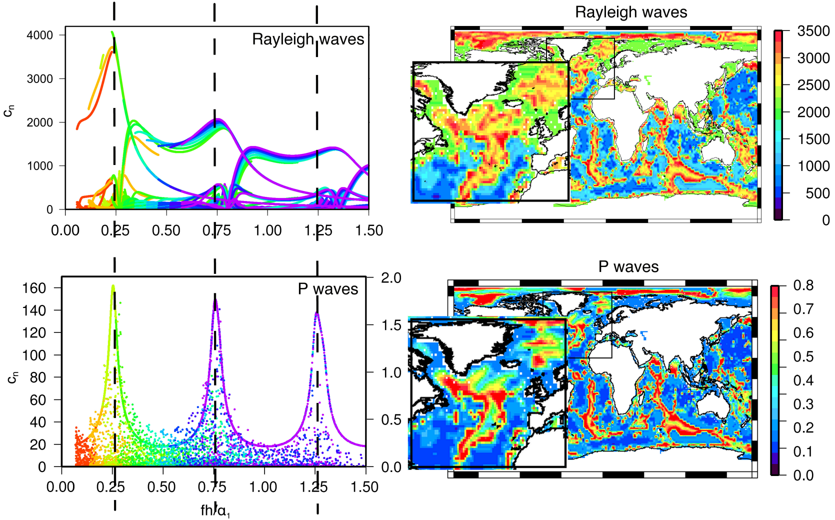

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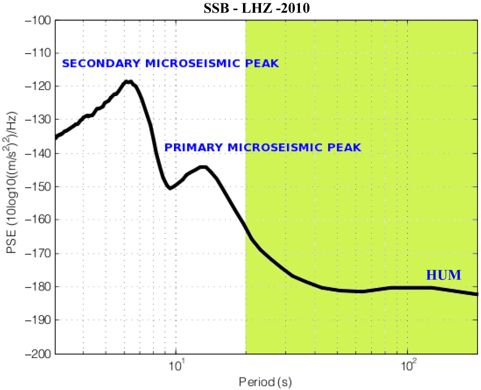
1-Modelling secondary microseismic noise
2-Modelling body waves sources
3-Modelling long period noise: work in progress!



Long period noise: T=20-500 s

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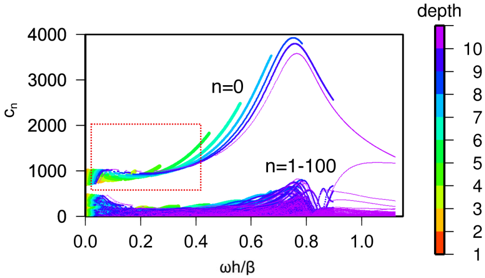

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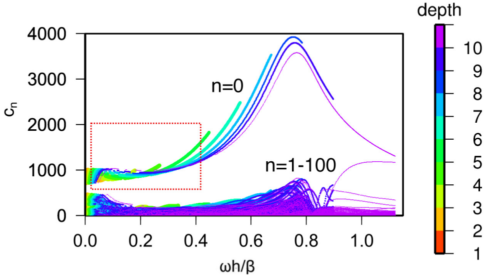
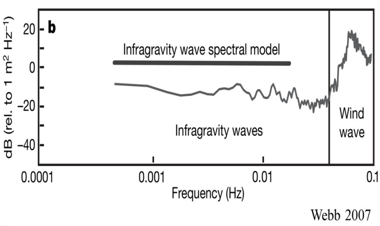
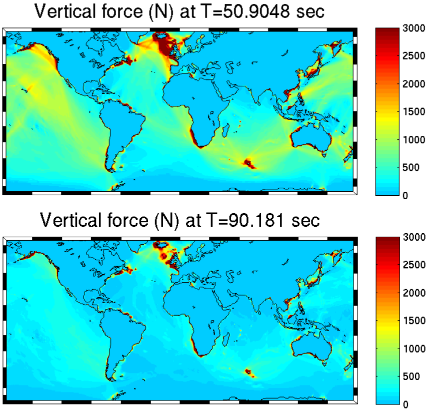
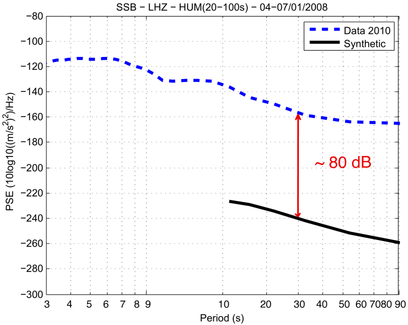




Webb 2007

Modelling long period (T=20-100 s) vertical spectra by considering interaction of infragravity waves





First computations: noise levels is too low by ∼ 80dB

Conclusions

- We model noise **Rayleigh wave** amplitude considering:
  - sources as vertical single forces;
  - source amplification coefficients in a realistic Earth model;
  - an empirical attenuation model.
- We estimate missing **Love wave** energy on the horizontal spectra;
- We model seismic noise **body wave** sources;
- We observe that **long period noise** sources are not frequency dependent;
- TO BE DONE:
  modelling noise body waves, Love waves and hum.