Sensitivity Optimisation of Seismic Observables

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1. Abstract & Theory

Coupling between structural parameters plays a major role in seismic tomography but limited information usually prohibits the simultaneous inversion for the complete parameter space of inverse problems. We therefore propose a multi-observable approach that optimally decouples structural parameters. Performing a principal component analysis we find linear combinations of seismic observables that optimise the sensitivity with respect to subspaces of the parameter space. These optimal observables provide high sensitivity power in terms of the volume integral of their squared Fréchet kernels with respect to selected structural parameters and simultaneously low sensitivity power to unresolved parameters. In a first application we optimise the sensitivity of traveltime measurements with respect to density heterogeneities.

Optimisation problem:

We consider a set of arbitrary seismic observables χ_i and a set of structural parameters m_j. Our aim is to combine the observables χ_i to one optimal observable χ^{opt} with maximal sensitivity to m_1 and minimal sensitivity to $m_{j\neq 1}$ in the sense that

$$\chi^{opt} = \sum_{i=1}^{n} w_i \chi_i \qquad (1)$$

To identify the weights w_i we quantify a change in χ^{opt}, due to a relative model perturbation $\delta\ln(\mathbf{m})$, by its Fréchet derivative

$$\nabla_{\mathbf{m}}\chi^{opt}\,\delta\ln(\mathbf{m}) = \int_G \sum_{i,j} w_i K_{ij}(\mathbf{x})\,\delta\ln(m_j(\mathbf{x}))\,d^3\mathbf{x}$$

where G denotes the relevant section of the Earth model and K_{ij} the Fréchet kernel of the observable χ_i with respect to m_j.
We define the sensitivity power P_j of the optimal observable with respect to m_j as

$$P_j(\mathbf{w}) := \int_G [\sum_{i=1}^{n} w_i K_{ij}(\mathbf{x})]^2\,d^3\mathbf{x} \quad \text{with} \quad \sum_{i=1}^{n} w_i^2 = 1$$

Hence the optimisation problem consists in finding the weights w_i that maximise P_1 and minimise $P_{j\neq 1}$.

Solution of the optimisation problem:

We choose the linear optimisation criterion

$$\mathbb{P}(\mathbf{w}) = \sum_{j=1}^{p} b_j P_j(\mathbf{w}) \quad \text{with} \quad \sum_{j=1}^{p} b_j^{\,2} = 1$$

where we maximise the joint sensitivity power **IP** with the balancing factors $0<b_1<1$ and $-1<b_{j\neq 1}<0$ so that the maximum of **IP** corresponds to a maximum of P_1 and a minimum of $P_{j\neq 1}$. For the maximisation of **IP** we apply the standard techniques of a principal component analysis[1] reducing the problem to the eigenvalue problem

$$\mathbf{Mw} = \lambda\mathbf{w} \quad \text{with} \quad M_{li} := \sum_{j=1}^{p} b_j \int_G K_{lj}(\mathbf{x}) K_{ij}(\mathbf{x})\,d^3\mathbf{x}$$

It can be shown that the solution of the optimisation problem is now equal to the maximal eigenvalue λ_{max} and the corresponding eigenvector **w** of the real and symmetric matrix **M**.
The balancing factors b_j prevent the optimisation from disregarding or overvaluing certain parameters and are defined via

$$\frac{P_1[\mathbf{w}(\mathbf{b})]}{\prod_{j=2}^{p} Pj[\mathbf{w}(\mathbf{b})]} \stackrel{!}{=} \max$$

2. Sensitivity Optimistaion of Traveltime Measurements

The Earth's density structure is still poorly resolved in seismic tomography although it is a critical parameter for studying the thermal and chemical state of the Earth[2]. A central problem in density inverse schemes is the difficulty to relate the major source of seismic information, i.e. traveltime data, to density variations. Depending on the parameterisation of the Earth model density is either strongly coupled to the elastic parameters or hardly influences traveltime measurements. Considering the isotropic case the former problem corresponds to a model parameterisation in terms of the bulk modulus (κ), the shear modulus (μ) and density (ρ). We apply the developed decoupling algorithm to maximise the sensitivity of surface wave traveltimes with respect to ρ, while minimising the sensitivities to κ and μ. Using a combination of spectral-element and adjoint methods[3,4] we compute Fréchet kernels for cross-correlation time shift measurements[5] of vertical component Rayleigh waves with respect to the structural parameters ρ, κ and μ. The corresponding synthetic scenario is shown in figure 1.

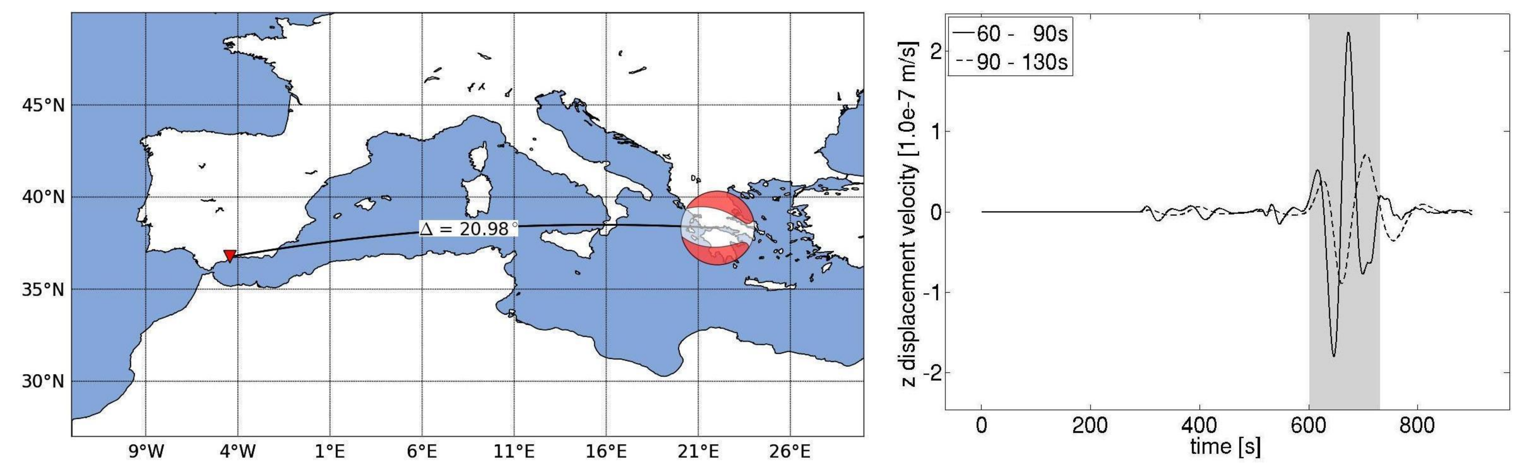


Fig. 1: Left: source/receiver geometry for an event beneath southern Greece and a receiver located in Malaga, Spain. The epicentral distance is 20.98°. The source mechanism is plotted at the epicentre. Right: vertical component seismograms of the displacement velocity. The frequency bands are 60 - 90 s (solid line) and 90 - 130 s (dashed line).

We include four frequency bands from 30 - 40 s, 40 - 60 s, 60 - 90 s and 90 - 130 s in the optimisation process. Figure 2 displays vertical slices of the Fréchet kernels corresponding to the cross-correlation time shifts for the frequency band from 60 - 90 s. The similar shaped Fréchet kernels with respect to ρ and μ reflect the strong parameter coupling effect in this scenario.

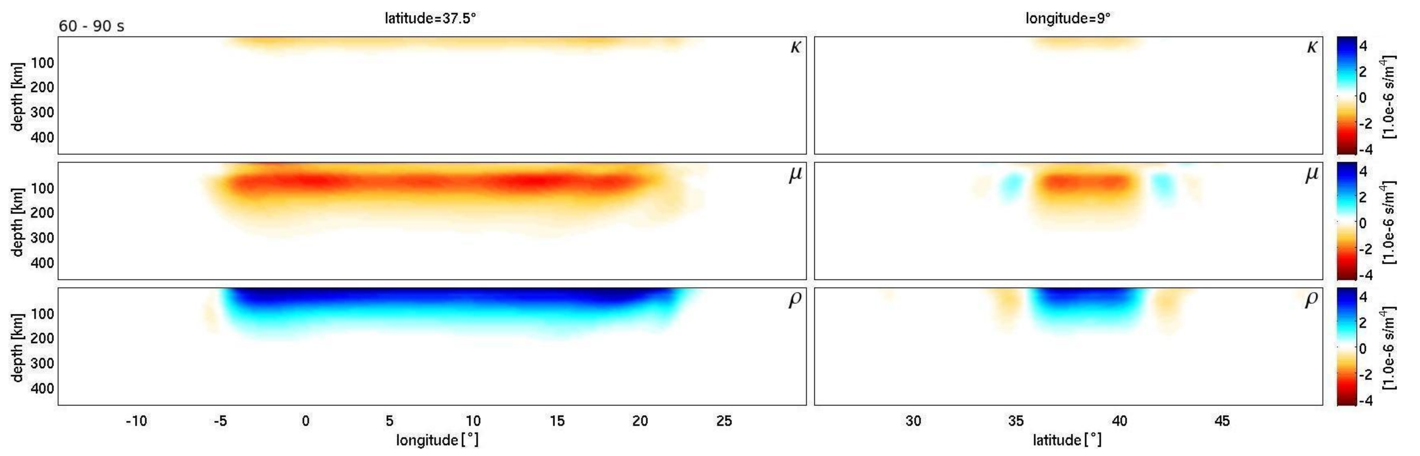
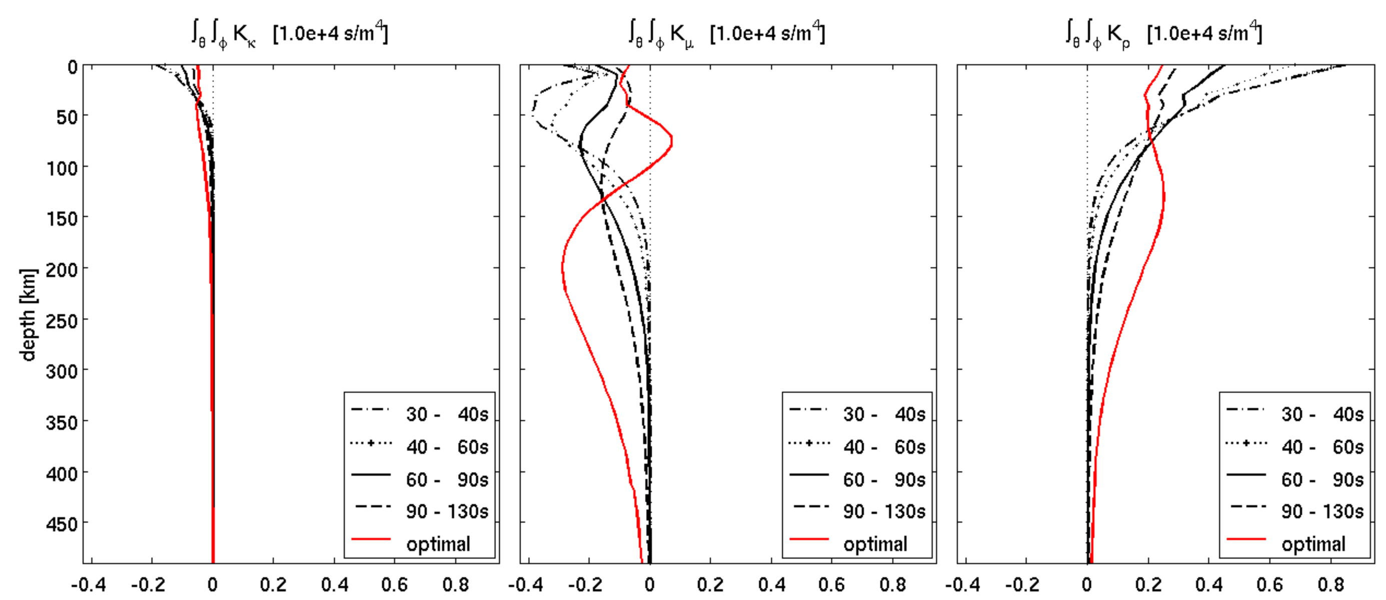


Fig. 2: Cross-correlation time shift kernels for a Rayleigh wave windowed vertical velocity seismogram with respect to the bulk modulus (first row), the shear modulus (second row) and density (third row). The vertical slices are shown at 37.5°N (left column) and 9.0°E (right column). The frequency band is 60 - 90 s.

Solving the eigenvalue problem for the involved observables and determining the balancing factors by a grid search leads to the weights $w_1 = -0.28$, $w_2 = 0.6$, $w_3 = -0.65$ and $w_4 = 0.37$. The optimal observable is then composed according to equation (1). The optimal Fréchet kernels with maximal sensitivity to ρ and minimal sensitivity to κ and μ are computed analogously. Depth profiles of Fréchet kernels for individual and optimal observables are plotted in figure 3. While the sensitivity of K_κ almost vanishes oszillations in K_μ decrease the sensitivity to large scale perturbations in the shear modulus. Concerning K_ρ we observe a significant increase of sensitivty with respect to ρ until a depth of approximately 300 km. These characteristics are also present in figure 4 that shows vertical slices of the optimal kernels.

Fig. 3: Profiles of Rayleigh wave Fréchet kernels with respect to the bulk modulus (left), the shear modulus (middle) and density (right) at variable depth. The profiles correspond to frequency bands from 30 - 40 s (dashed-dotted), 40 - 60 s (dotted), 60 - 90 s (solid) and 90 - 130 s (dashed). Red lines represent the depth profiles of the optimal Fréchet kernels.



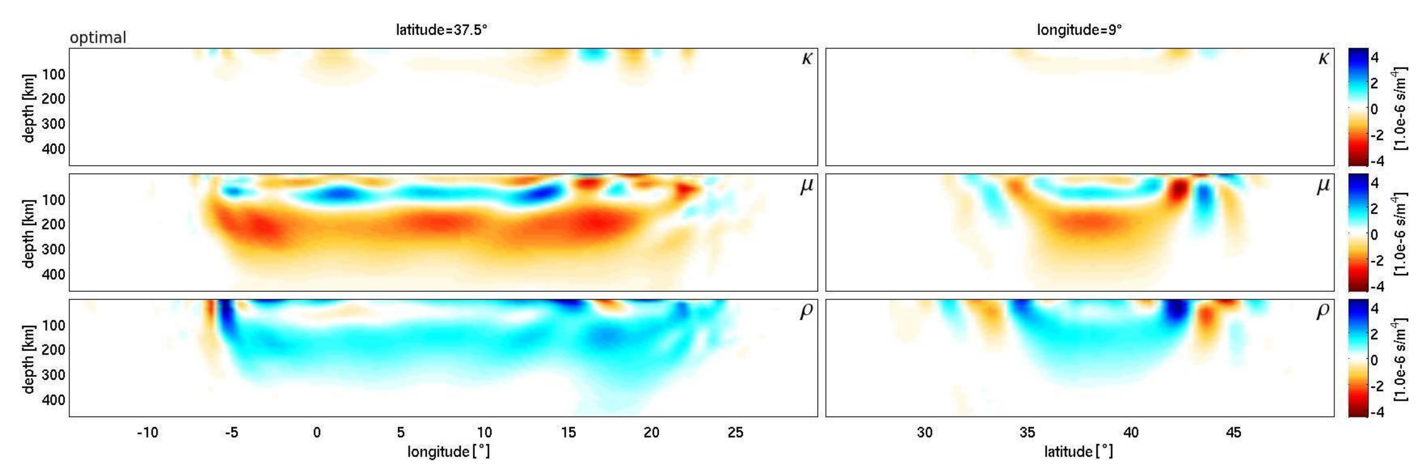


Fig. 4: Optimal Fréchet kernels for the bulk modulus (first row), the shear modulus (second row) and density (third row). The vertical slices are shown at 37.5° N (left column) and 9.0° E (right column). The frequency bands included in the optimisation process are 30 - 40 s, 40 - 60 s, 60 - 90 s and 90 - 130 s.

3. Synthetic Perturbation Test

The scenario includes ten seismic events encircling the Mediterranean Sea. The epicenter locations are plotted in figure 5 in the left panel. For recording the data we selected 43 broadband stations from the seismic networks IberArray and ISIDe. We regard again the cross-correlation time shifts of vertical component Rayleigh waves for the frequency bands 30 - 40 s, 40 - 60 s and 90 - 130 s. For the multi-receiver event kernels (one source, 43 receivers) of the current scenario the optimisation algorithm reproduces the weights that we found already for the single source/receiver kernels studied before. For the synthetic test we add a spherical perturbation centered in the Tyrrhenian Sea to the 1-D background model AK135 (figure 5, right). The perturbation reaches a depth of 300 km and attains its maximal values around 150 km depth. In a first simulation we computed time shifts for a density perturbation of maximal +15% relative to the background model. In a second simulation we replaced the density perturbation by a perturbation of the shear modulus with a maximal variation of -15% relative to the background model. The cumulative time shifts including all stations and events are displayed in figure 6 for the optimal and the individual observables. While the ratio χ[δln(ρ)] / χ[δln(μ)] attains values between 0.5 and 0.8 for the individual frequency bands the ratio corresponding to the optimal observable is 1.4. This means that the new observable is characterised by a substantially increased sensitivity with respect to density and decreased sensitivity with respect to the shear modulus.

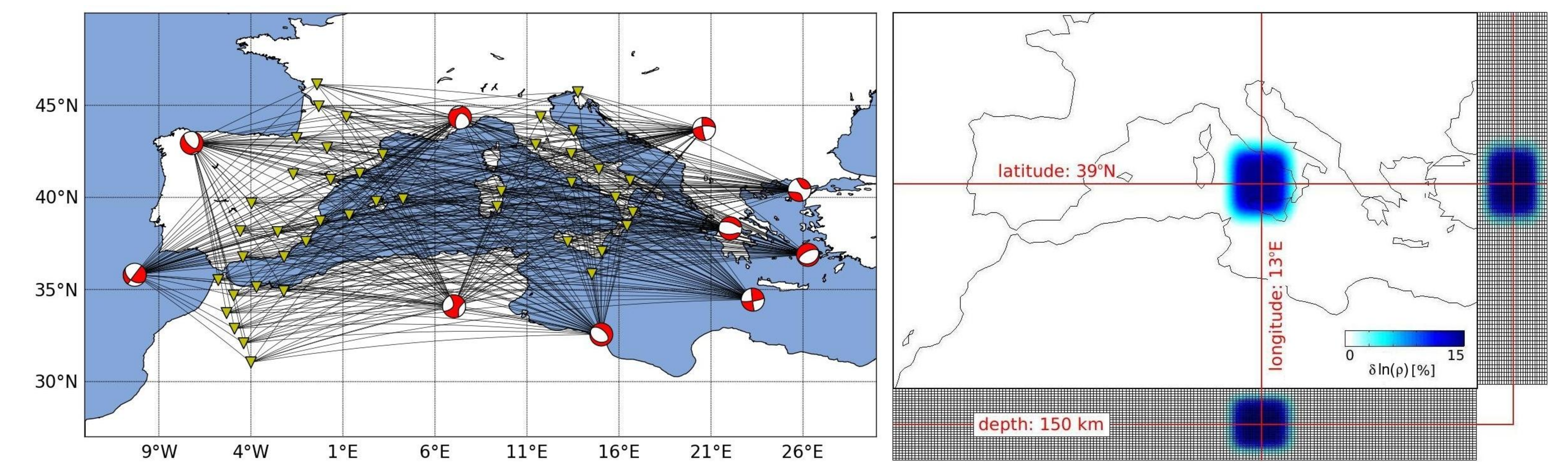
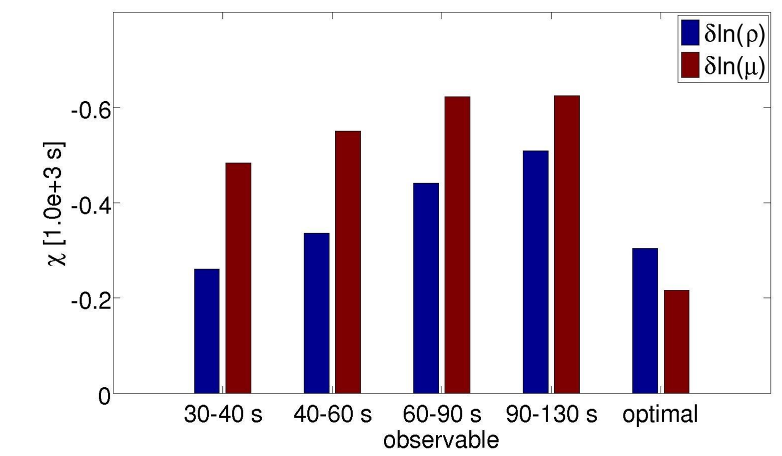


Fig. 5: Left: Source/receiver geomery of the synthetic tomography scenario. Ten seismic events visualised by the corresponding source mechanisms encircle the Mediterranean Sea. The 43 broadband receiver locations are marked by yellow triangles. Great circle paths are represented by black lines between each source/receiver pair. Right: Perturbation centered in the Tyrrhenian Sea at 13° E and 39° N. The perturbation corresponds to a density variation of maximal +15% relative to the 1-D background model AK135 at 150 km depth.

Fig. 6: Cummulative time shift measurements for the indiviual and the optimal observables including all stations and events of the scenario. Blue bars represent measurements for density variations while red bars correspond to perturbations in the shear modulus. The measurements of the optimal observable at the right show substantially increased sensitivity to δln(ρ) in relation to δln(μ).



4. Conclusions

Facing a major problem in seismic tomography - the coupling of seismic parameters - we developed a decoupling algorithm that is applicable to a wide range of seismic data processing scenarios. In a first example we constructed an observable that is primarily sensitive to density variations which is a remarkable result for a data set that is exclusively based on traveltime measurements. Furthermore, we emphasise that the linear formulation of the optimisation criterion with balancing factors is necessary to control numerical costs. As an increasing number of observables only increases the number of eigenvalues to be computed from the real, symmetric matrix **M** this method is clearly not limited to a reasonable number of observables.

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