Non periodic homogenization for the acoustic waves

P. Cance, Y. Capdeville

Planetology and Geodynamics, UMR-CNRS 6112, University of Nantes, France;
E-mail : philippe.cance@univ-nantes.fr

Introduction :

Acoustic interpretation of elastic datas is often used for solving inverse problems. Adressing the small scale heterogeneities problem, we check - through the homogenization method - what can be done with acoustic equations.

Non periodic homogenization method for acoustic media

Assumption : existence of a minimum wavelength for the wave field : $\lambda_m \sim \frac{V_{s_m}}{f_0}$

Elastic case :

- Initial equations :

$$\rho \ddot{u} - \nabla.\sigma = f$$
$$\sigma = \mathbf{c} : \frac{\nabla u + {}^t \nabla u}{2}$$

- Order 0 effective equations

$$\rho^* \ddot{u}^{\varepsilon_0} - \nabla.\sigma^{\varepsilon_0} = f$$
$$\sigma^{\varepsilon_0} = \mathbf{c}^* : \frac{\nabla u^{\varepsilon_0} + {}^t \nabla u^{\varepsilon_0}}{2}$$

Acoustic case :

- Initial equations :

$$\frac{1}{\kappa}\ddot{q} - \nabla.\mathbf{v} = \dot{g} \qquad f = \nabla(\kappa g)$$
$$\mathbf{v} = L : \nabla q = \frac{1}{\rho}\nabla q \qquad \kappa = \rho V_P^2$$

- Order 0 effective equations

$$\frac{1}{\kappa^*}\ddot{q}^{\varepsilon_0} - \nabla.v^{\varepsilon_0} = \dot{g}$$
$$v^{\varepsilon_0} = L^* : \nabla(q^{\varepsilon_0})$$

Practical homogenization method :

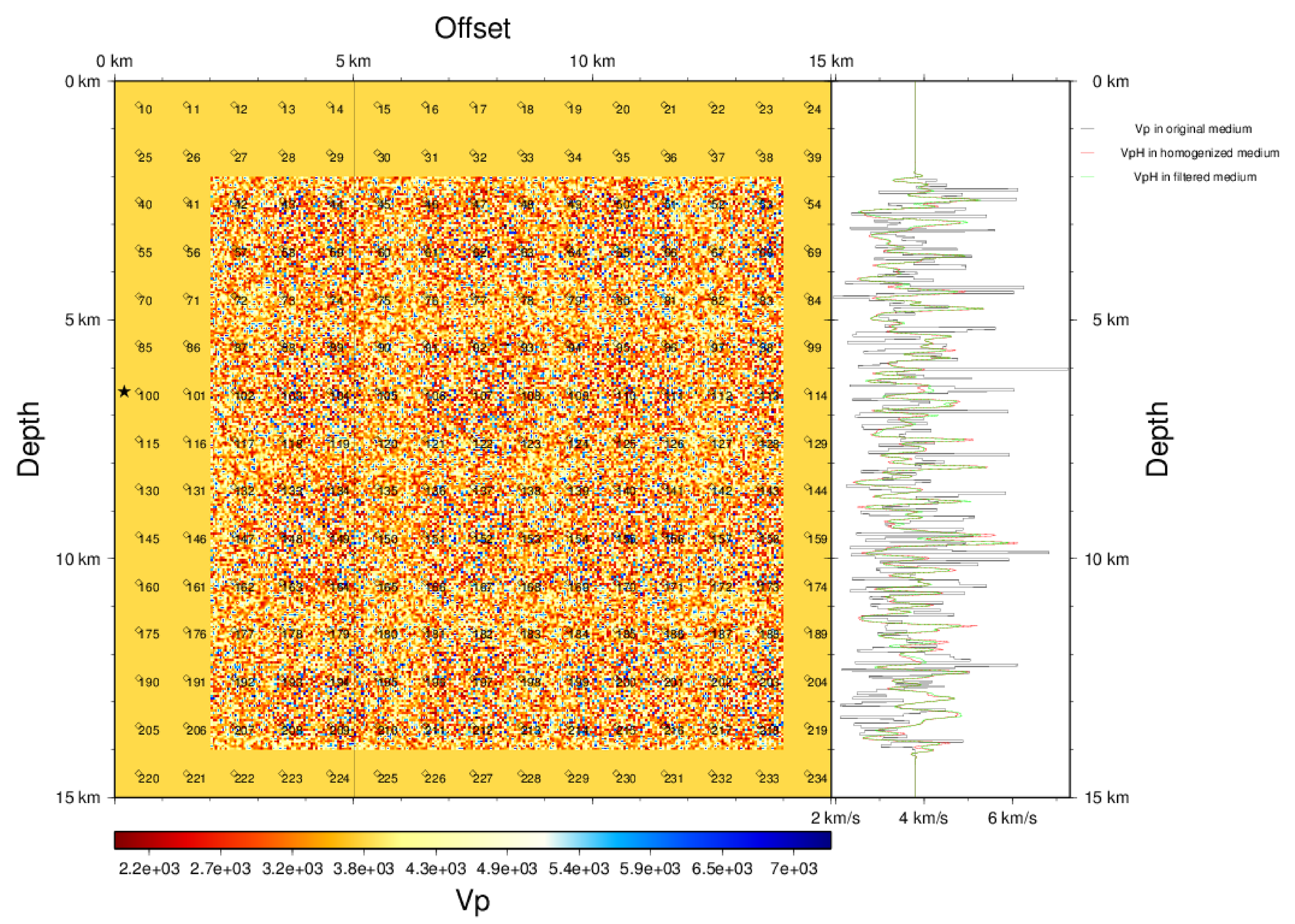
- Set up λ_0, the cell size (here the whole domain), $\mathcal{F}()$
- Cell problem : solve for χ over the cell with periodic boundary conditions

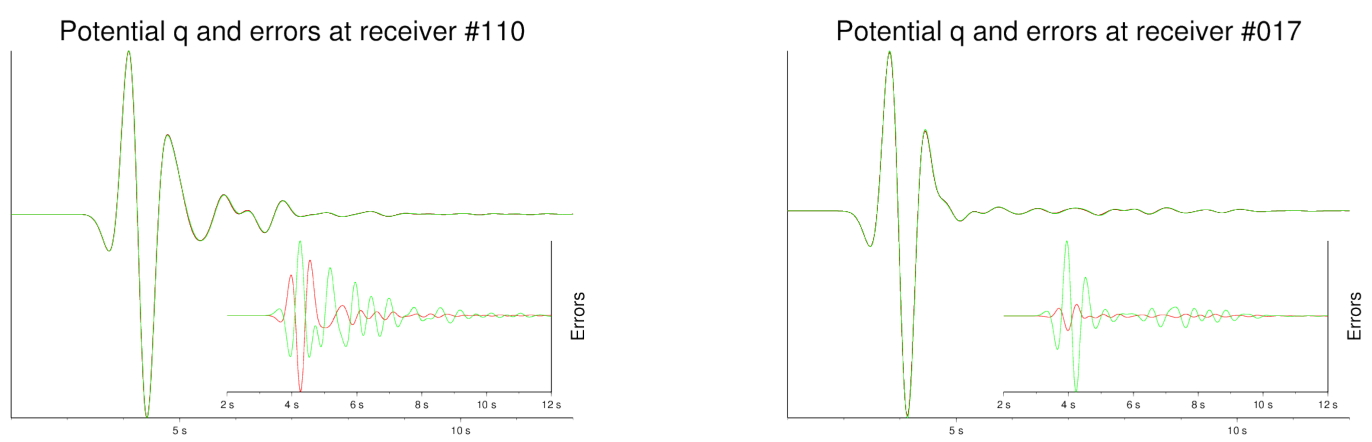
$$\nabla.H = 0$$
$$H = L : G = L : (I + \nabla\chi)$$
$$\langle\chi\rangle = 0$$

- Filtering :

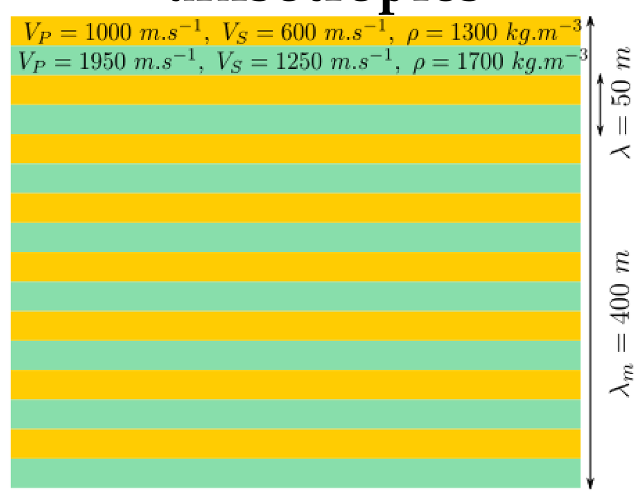
$$\frac{1}{\kappa^*} = \mathcal{F}\left(\frac{1}{\kappa}\right)$$
$$L^* = \mathcal{F}(H) : \mathcal{F}(G)^{-1}$$

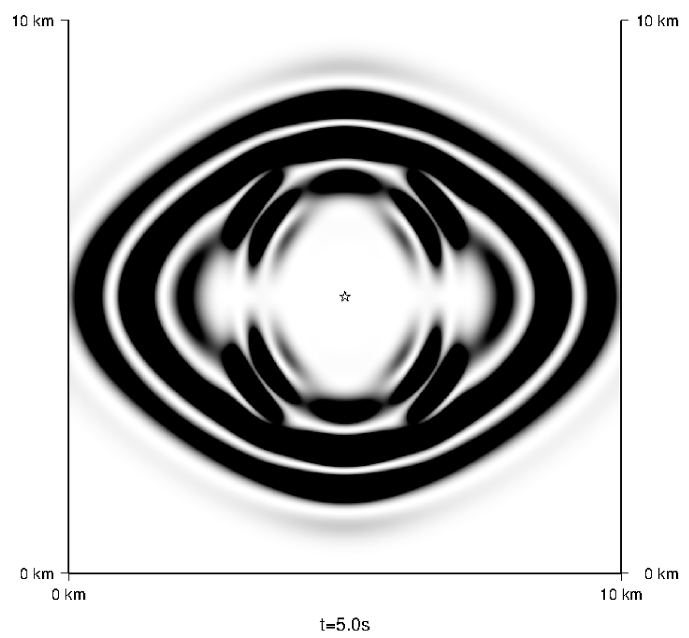
Effective anisotropy of the *density*



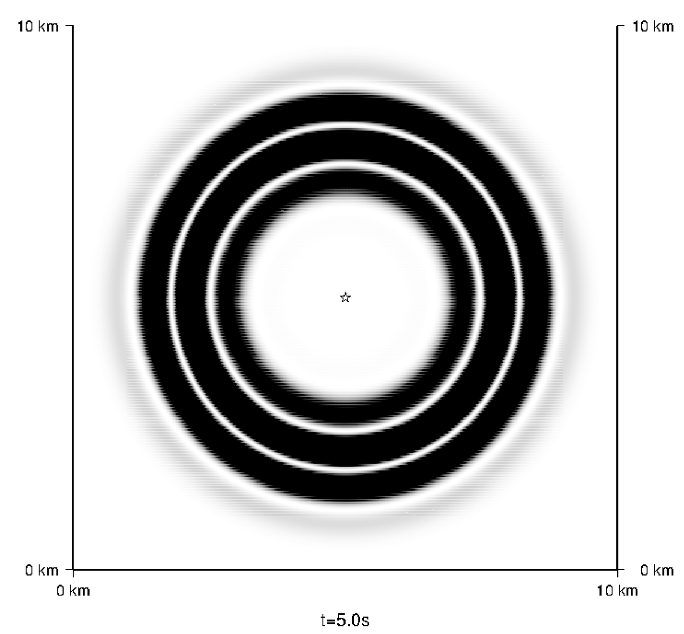


Different induced anisotropies

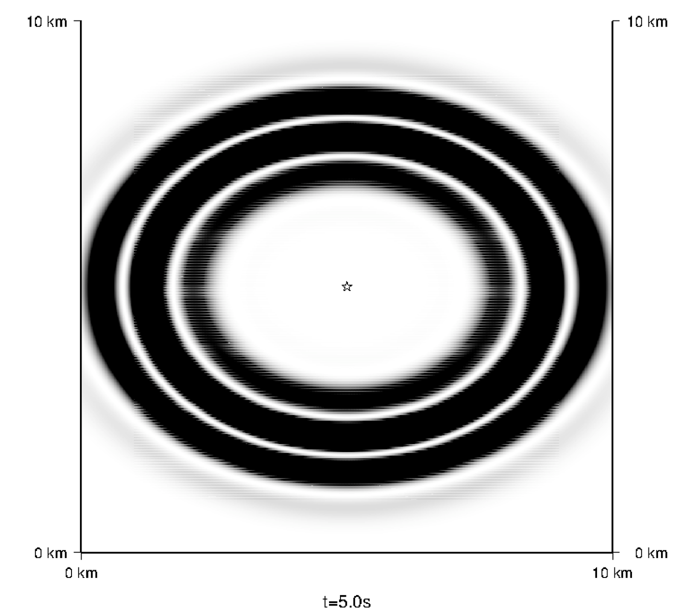




Snapshot of elastic energy



Snapshot of acoustic energy (elastic medium with $V_s = 0$)



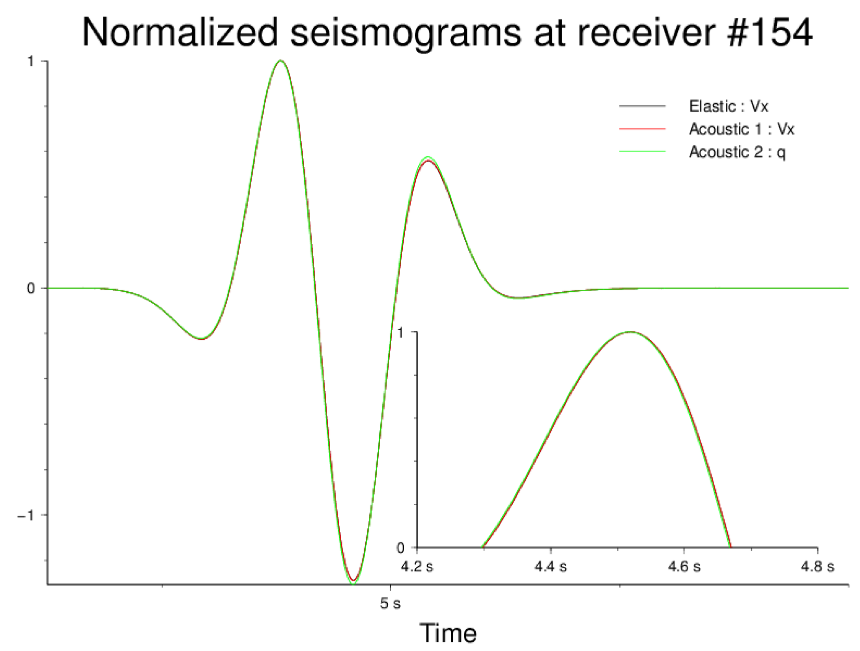
Snapshot of acoustic energy for a XZ-kinetically equivalent medium
Elliptical anisotropy due to strong ($\times 5$) small scales variations of the density.

Elastic vs acoustic

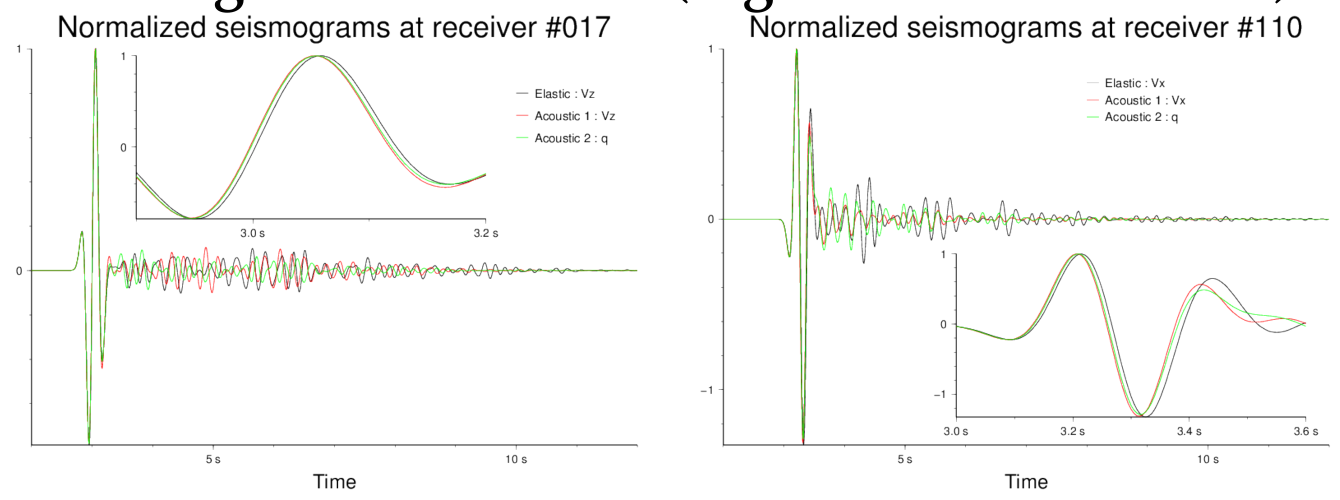
Strategies for kinematic equivalency

	Elastic	Acoustic 1	Acoustic 2
Parameter	ρ	$\frac{1}{\kappa} = \frac{1}{C_{11}}$	$\frac{1}{\kappa} = \rho$
Parameter	$\mathbf{c}$	$L = \frac{1}{\rho}I$	$L = C_{11}I$
Recording	Velocity v	Velocity v	Potential q

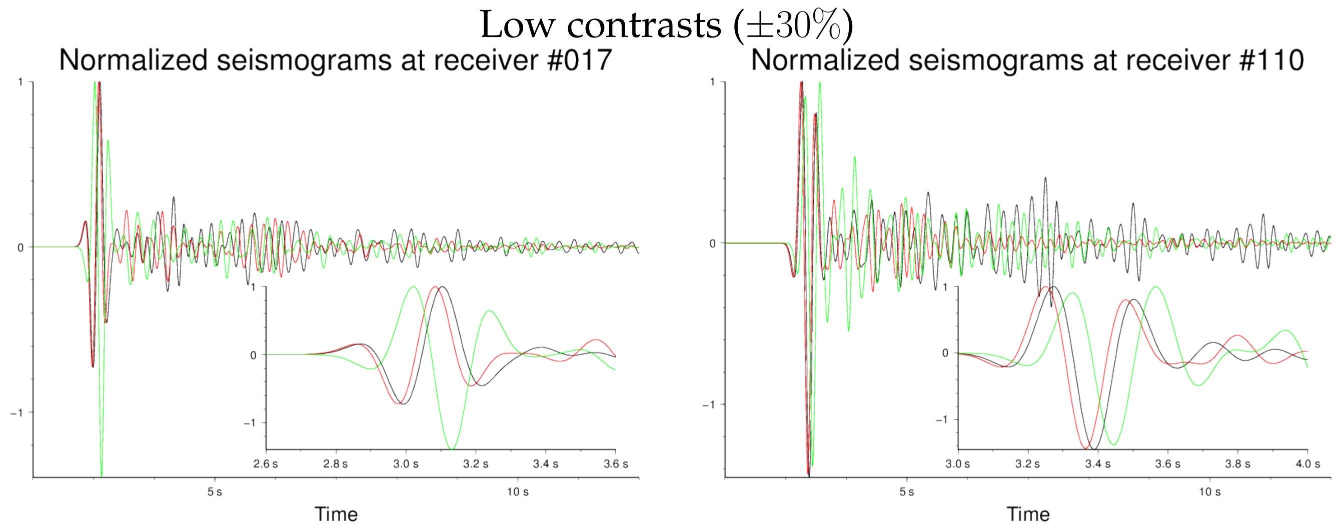
Homogeneous (or smooth) medium



Heterogeneous medium (high and low contrasts)

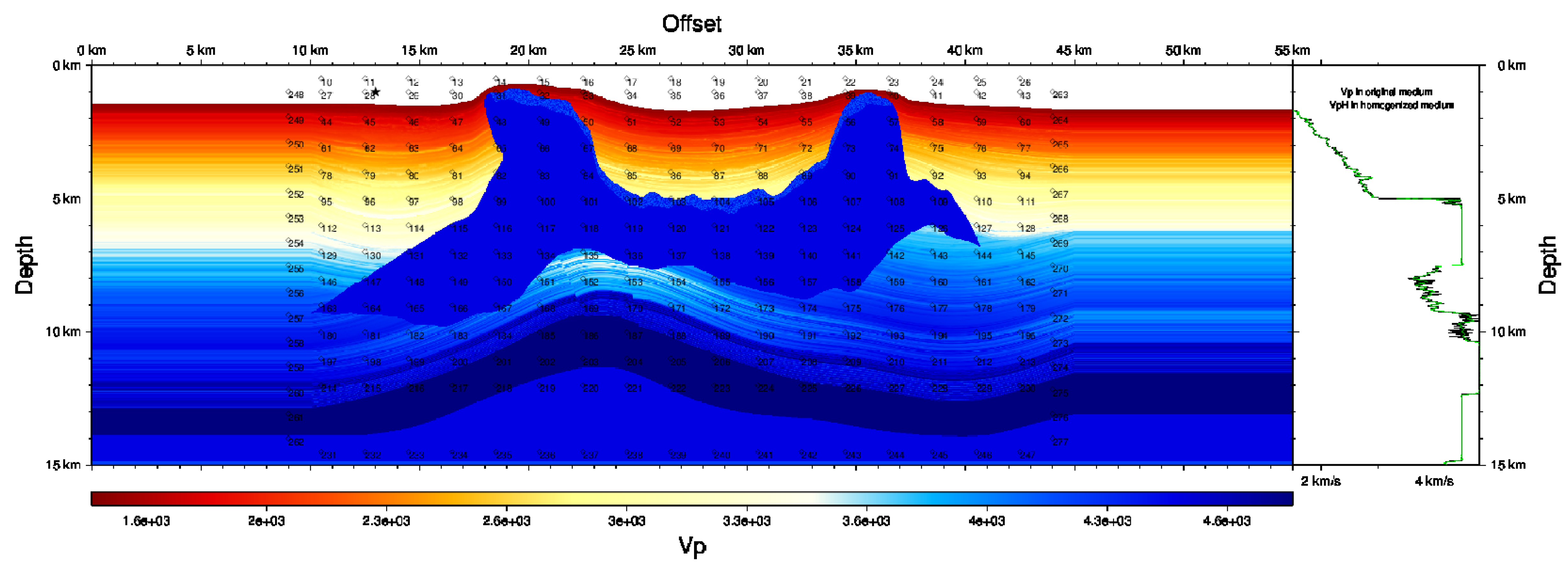


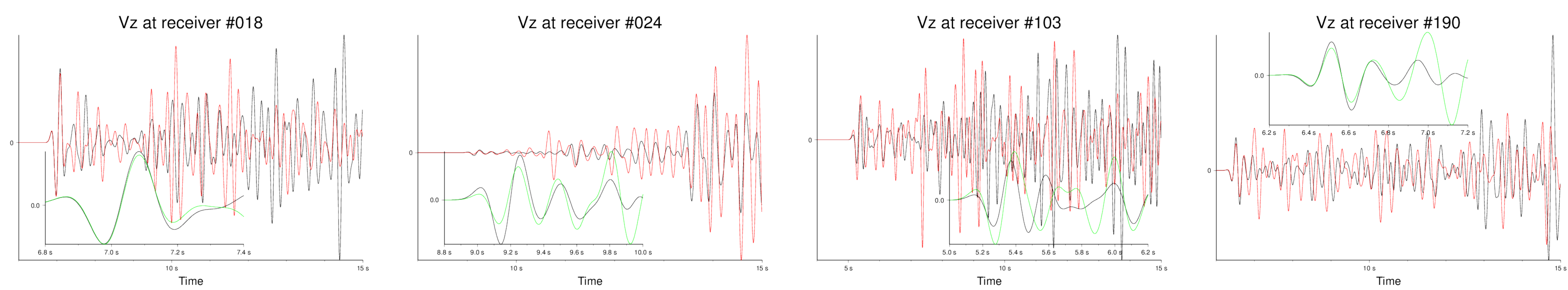
Low contrasts ($\pm 30\%$)



High contrasts ($\pm 100\%$)

Ongoing study : SEAM2D





QUEST workshop , May 19-25 2011, Benodet, France.