Attenuation in High-Frequency Axisymmetric Wave Propagation - Methods and Applications -

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QUEST · MARIE CURIE ACTIONS · SEVENTH FRAMEWORK PROGRAMME

Motivation

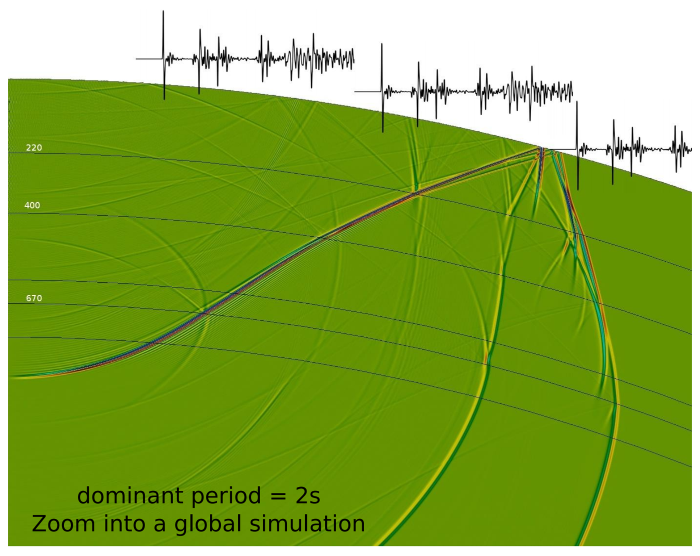
Attenuation will in future be responsible for a larger fraction of the cost of numerical wave propagation for two reasons:

- increasing **bandwidth** requires more memory variables for the same accuracy in representation of Q
- increasing **frequency** and consequently increasing **number of travelled wavelength** require a more accurate representation of Q

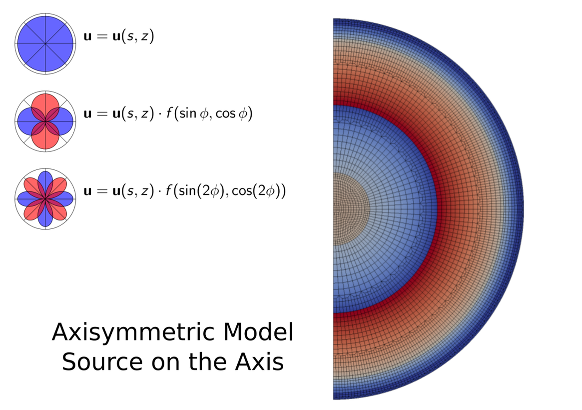
As we solve the **3D seismic wave equation** in axisymmetric media, we are pushing the edge both in bandwidth (2-3 decades) and number of traveled wavelength (on the order of 1000). Given that we use the **spectral element method** on an unstructured grid for the 2D problems involved, the methods we propose are directly applicable to full 3D SEM.

The Challenge - And Our Solution: AXISEM

Needed: Global 3D seismic Wavefields + Seismograms at high frequencies



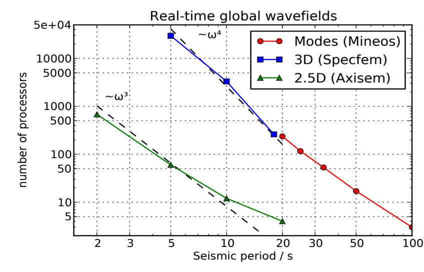
Axisymmetric Modelling

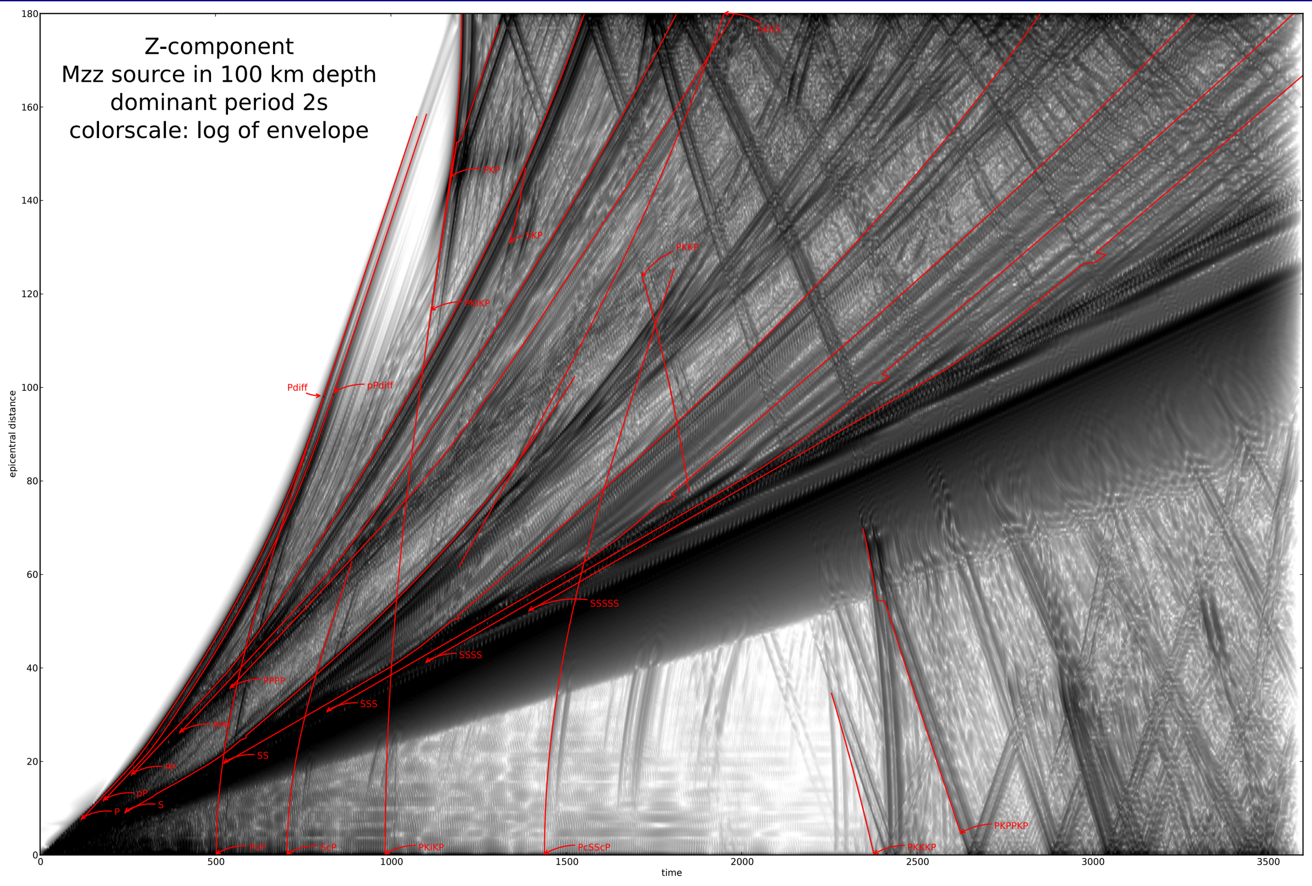


Axisymmetric Model
Source on the Axis

Performance Matters!

3 Orders of Magnitude:
1 day vs. 3 years





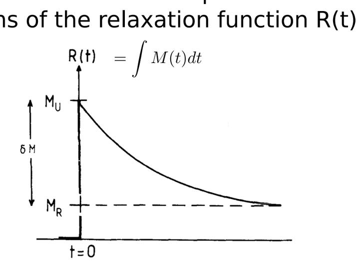
Attenuation in Time Domain Wave Propagation

Stress Strain Relation and 'Memory Variables'

The most general linear stress - strain relation is:

$$\sigma(t) = \int M(t-\tau)\cdot\epsilon(t)d\tau$$

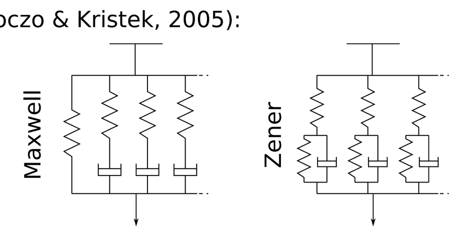
Assuming causality, fading memory, and solid behaviour in the limit of low frequencies the general form of the time dependent modulus is (in terms of the relaxation function R(t)):



Approximating this with a discrete decay spectrum:

$$R(t) = \left[M_R + \delta M \sum_{j=1}^{N} a_j e^{-\omega_j t}\right] H(t)$$

This can be equivalently interpreted as generalized Maxwell or Zener bodies (Moczo & Kristek, 2005):



The resulting stress - strain relation reads:

$$\sigma(t) = M_U\left[\epsilon(t) - \sum_{j=1}^{N}\zeta_j\right]$$

with the N additional differential 'memory variable' equations:

$$\dot{\zeta}_j(t) + \omega_j\zeta_j(t) = a_j\omega_j\frac{\delta M}{M_U}\epsilon(t)$$

The resulting frequency dependent modulus and attenuation are:

$$M(\omega) = M_R + \sum_j a_j \delta M \frac{i\omega}{i\omega + \omega_j}$$

$$Q^{-1}(\omega) = \frac{\delta M}{M_R}\frac{\sum_j a_j \frac{\omega/\omega_j}{1+(\omega/\omega_j)^2}}{1+\frac{\delta M}{M_R}\sum_j a_j \frac{(\omega/\omega_j)^2}{1+(\omega/\omega_j)^2}}$$

This is one nonlinear optimization problem in the 2N parameters a_j and ω_j, for each Q in the model.

A common linearization (equivalent to τ-Method by Blanch et al, 1994) reduces this to a single optimization problem:

$$Q^{-1}(\omega) \approx \frac{\delta M}{M_R}\sum_j a_j\frac{\omega/\omega_j}{1+(\omega/\omega_j)^2}$$

(Emmerich & Korn, 1987)

Nonlinear Optimization

Often, the decay frequencies ω_j are not inverted for, but fixed log-spaced, since this allows for a linear (hence faster) inversion for the parameters a_j.

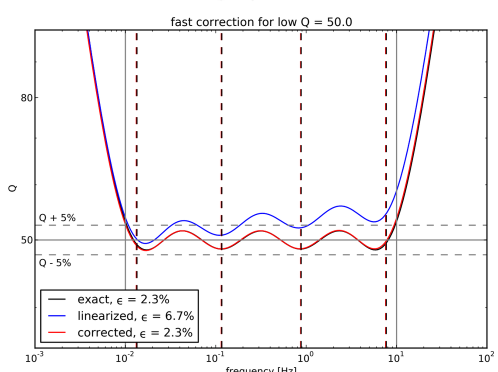
We propose a fast, iterative correction to the linearization, which combines the advantages of the linearization (a single optimization problem) with increased accuracy for low Q:

$$y_j = \frac{\delta M}{M_R}a_j$$

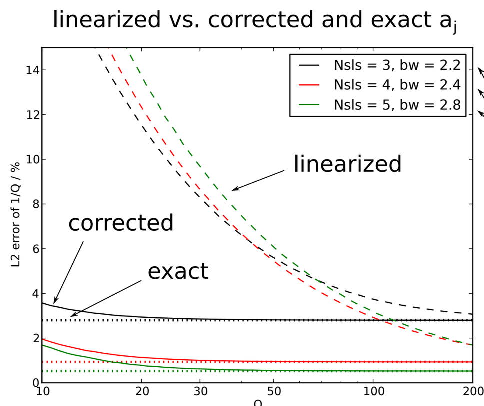
$$\begin{cases}\delta_0 &= 1 + \frac{1}{2}y_0\\ \delta_{n+1} &= \delta_n + \left(\delta_n - \frac{1}{2}\right)y_n + y_{n+1}\end{cases}$$

$$y'_j = \delta_j \cdot y_j$$

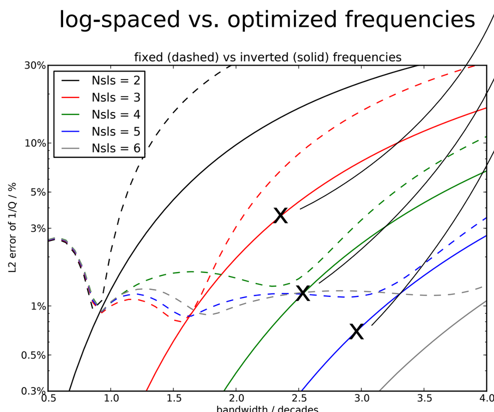
This allows to use a relatively expensive simulated annealing optimization scheme.



linearized vs. corrected and exact a_j



log-spaced vs. optimized frequencies



Optimal Q Parametrization

How much error in Q is acceptable to ensure a given maximum amplitude error in the resulting seismograms?

The amplitude effect of Q is:

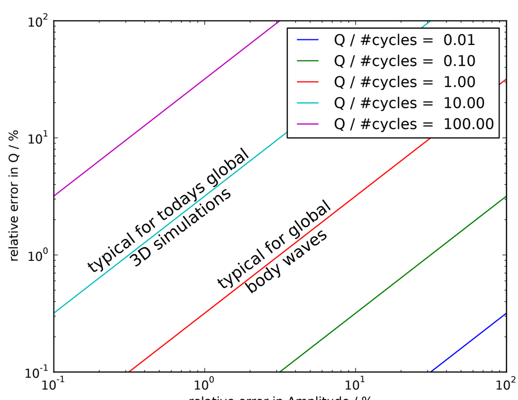
$$\delta A(\omega) = e^{-\frac{\omega t}{2Q}} \qquad Q = \frac{t}{t^*}$$

So the amplitude error in first order of the Q error is:

$$\frac{\Delta A}{A} = \frac{\Delta Q}{Q}\frac{\omega t}{2Q}$$

where the number of cycles / traveled wavelength can be identified as:

$$\#\text{cycles} = 2\pi\omega t = ft$$



Analytical Time-Stepping

The 'memory variable' equation is an ODE of the form:

$$\frac{d}{dt}R(t) + \alpha R(t) = s(t)$$

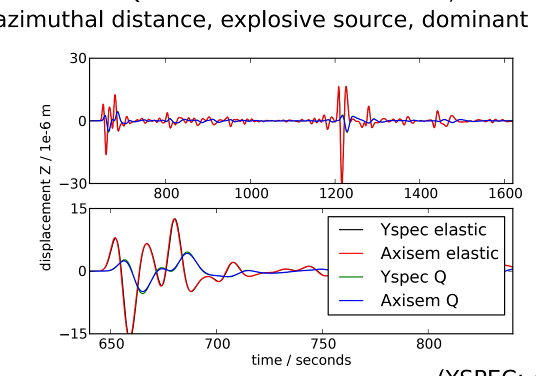
This can be solved with the standard method of multiplication with an integrating factor and the solution is:

$$R(t+\Delta t) = e^{-\alpha\Delta t}\left[R(t) + \int_t^{t+\Delta t} s(t')e^{\alpha(t-t')}dt'\right]$$

For a second-order scheme, where s(t) is known at two times only (with linear interpolation) this results in:

$$R(t+\Delta t) = R(t)e^{-\alpha\Delta t} + \frac{s(t)}{\alpha}\left[\frac{1}{\alpha\Delta t}\left(1-e^{-\alpha\Delta t}\right) - e^{-\alpha\Delta t}\right] + \frac{s(t+\Delta t)}{\alpha}\left[1 - \frac{1}{\alpha\Delta t}\left(1-e^{-\alpha\Delta t}\right)\right]$$

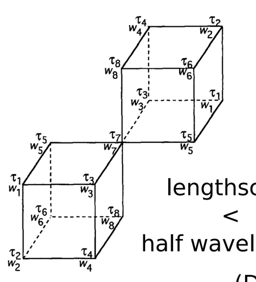
A first test with Q = 100 in the whole mantle, station at 68° azimuthal distance, explosive source, dominant period 10s:



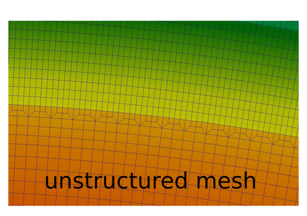
(YSPEC: Al-Attar, 2007)

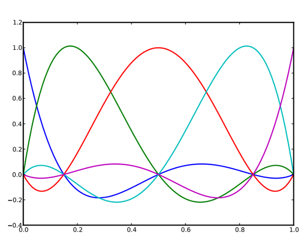
'Coarse Grained' Memory Variables in Spectral Elements on Unstructured Grids

Coarse Redistribution of the Memory Variables

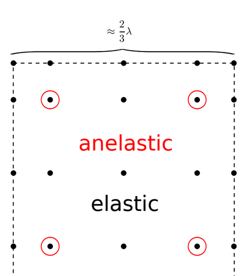


(Day, 1998)



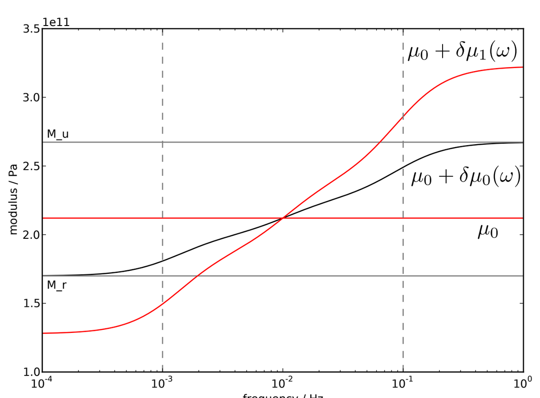


SEM Basis



Inverse Homogenization

The aim is to find an equivalent medium with heterogeneities on sub-wavelength scale such that it is computationally less expensive: mainly elastic with a few anelastic points. This is an inverse homogenization problem.



Empirically the best approximation (and exact in 1D) for the homogenization is the weighted harmonic average of the modulus (Graves & Day, 2003, Cioranescu & Donatu, 1999). This reads:

$$\mu_0 + \delta\mu_0(\omega) = \frac{1}{\frac{w_1}{\mu_0} + \frac{w_2}{\mu_0 + \delta\mu_1(\omega)}}$$

$$w_1 + w_2 = 1$$

$$\delta\mu_1(\omega) = \frac{\mu_0\delta\mu_0}{w_2\mu_0 - w_1\delta\mu_0}$$

For the generalized linear solid the modulus is:

$$\delta\mu_1(\omega) = \sum_j a_j\delta\mu_1\left(\frac{\omega_j^2}{\omega_1^2+\omega_j^2} - \frac{\omega_j^2}{\omega^2+\omega_j^2}\right)$$

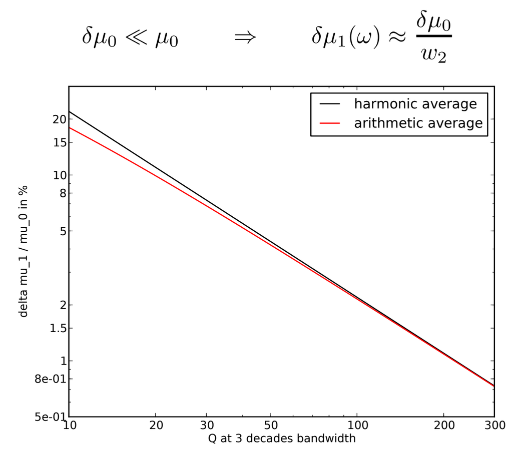
$$\sum_j a_j = 1$$

$$\delta\mu_1 = \lim_{\omega\to\infty}\delta\mu_1(\omega) - \lim_{\omega\to 0}\delta\mu_1(\omega)$$

This is a linear optimization problem to find the coefficients a_j, given the frequencies ω_j.

For large Q, the arithmetic average is a good approximation to the harmonic average:

$$\delta\mu_0 \ll \mu_0 \qquad \Rightarrow \qquad \delta\mu_1(\omega) \approx \frac{\delta\mu_0}{w_2}$$



Arithmetic average as approximation to the harmonic average assuming constant Q over a bandwidth of 3 decades.

Computational Benefits

Theoretical speed-up and memory reduction (in the anelastic part):

in 2D, O(4): $\frac{25}{4} = 6.25$

in 3D, O(4): $\frac{125}{8} = 15.625$

memory test in 2D (AXISEM):

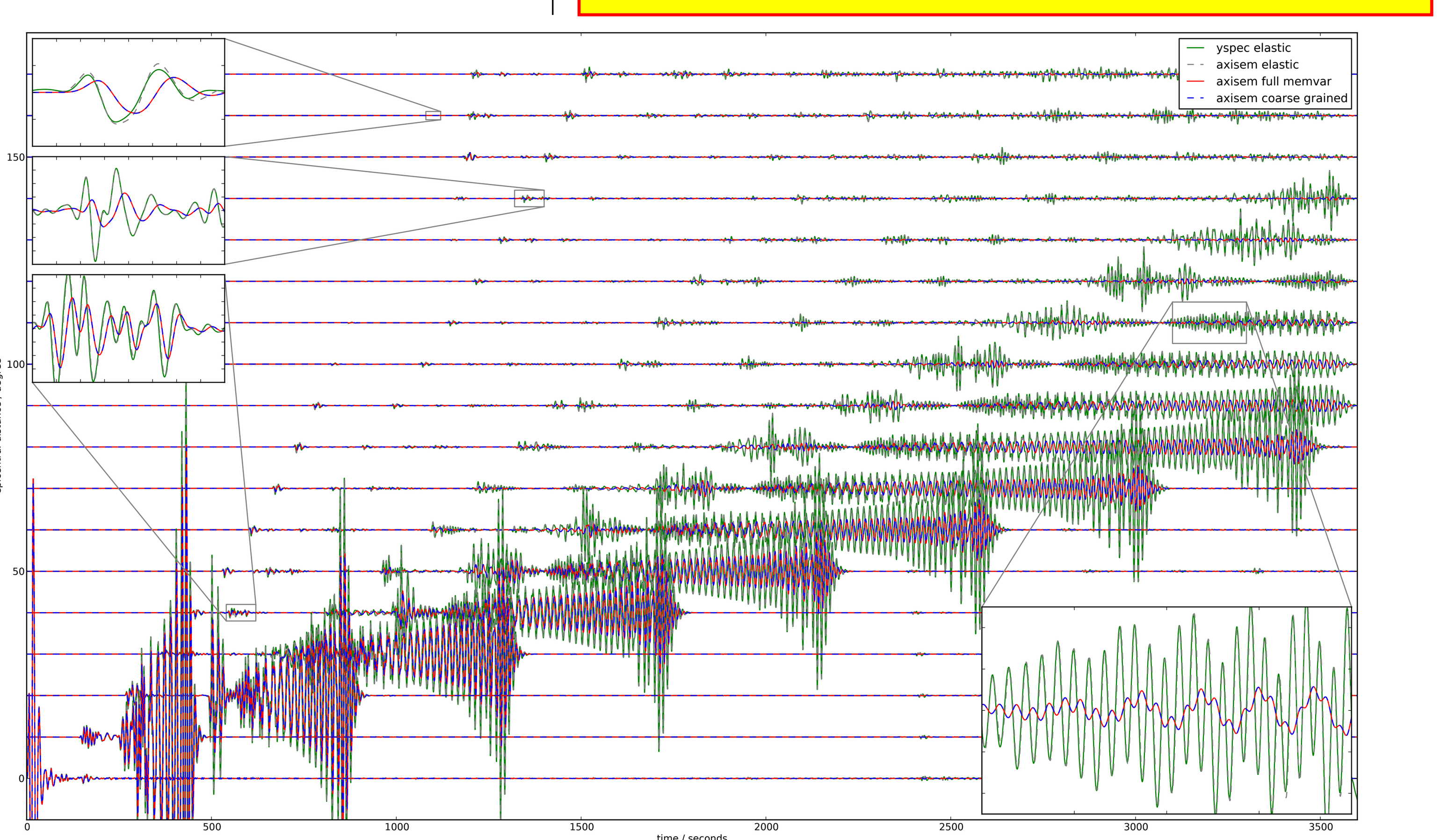
N	full	coarse grained
3	1.7	1.1
4	1.8	1.1
5	1.9	1.1
6	2.0	1.1
7	2.2	1.2
8	2.3	1.2

PREM @ 20s, serial job, memory usage in time-loop relative to elastic (116MB)

speed-up Test in 2D (AXISEM):

PREM @ 20s, serial job, time-loop only, runtime relative to elastic (44s)

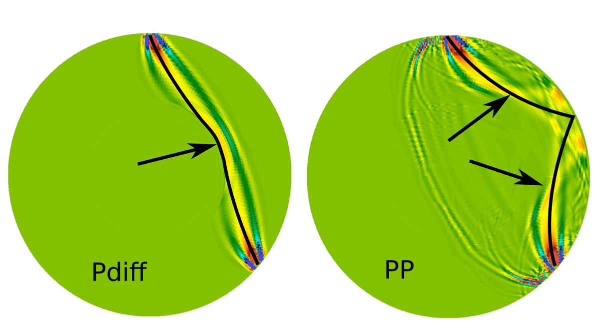
N	elastic runtime	full runtime	coarse grained runtime	total speed-up	anel stiffness speed-up	anel time step speed-up
3	1	2.55	1.35	1.9	5.3	4.3
4	1	2.75	1.38	2.0	5.2	4.5
5	1	3.0	1.41	2.1	5.3	4.7
6	1	3.16	1.44	2.2	5.5	4.7

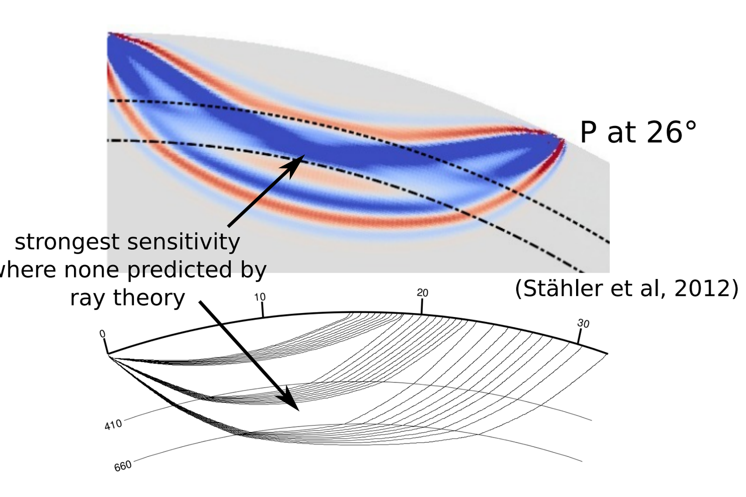


comparison of full memory variables to coarse grained (arithmetic averaging), PREM, Mzz source in 30km depth, dominant period 5s:

Finite Frequency Kernels and First-Order Forward Modeling

Breakdown of Ray Theory: Diffraction, Caustics and Triplications





(Stähler et al, 2012)

Severeral phases for which ray based kernels (Dahlen et al, 2000) cannot be computed accurately. Wave based kernels as computed with AXISEM will help to improve resolution especially in D'' using Pdiff and in the upper mantle using triplicated phases (Stähler et al, 2012). Also, the amplitude of the kernel can accurately be computed in caustics (e.g. in PP).

Linearized Forward Solution for Smooth and Small Amplitude Perturbations: A 1D Toy Problem

1D Wave Equation:

$$\rho_0\ddot{u} - \partial_x(\mu\partial_x u) = f$$

Model Perturbation:

$$\mu = \mu_0 + \epsilon\mu_1$$

Perturbation of the Wavefield:

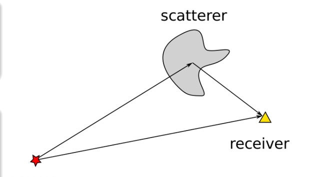
- Born: $u = u_0(1+\phi)$
- Rytov: $u = u_0 \cdot e^{\phi}$

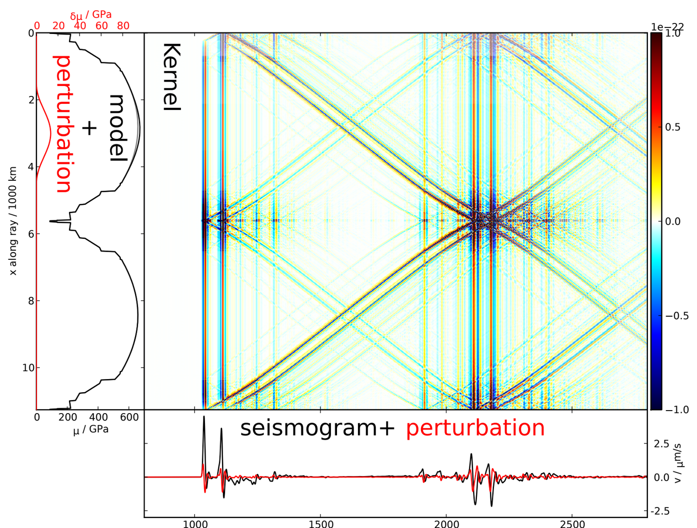
First Order Perturbation:

$$u_0 \cdot \phi = \int K_\mu(x, x')\mu_1(x')\,dx' + O(\epsilon^2)$$

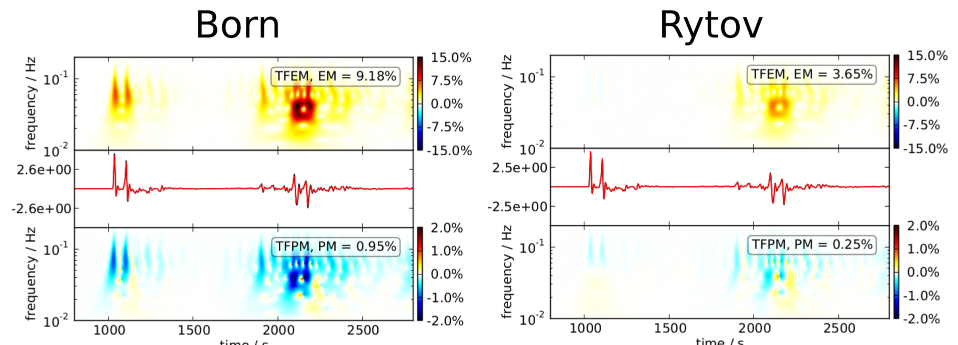
Kernel (time domain):

$$K_\mu(x_r, x, t) = \underbrace{\partial_x G_0(x_r, x, t)}_{\text{backward strain}} \star \underbrace{\partial_x u_0(x, t)}_{\text{forward strain}}$$





Seismograms computed using Born and Rytov approximation in the model with perturbation compared to the reference solution (Time Frequency Envelope and Phase Misfits, Kristekova et al. 2006). Background model is PREM projected on a PP ray.



(wavefields and seismograms computed with SPECFEM1D)

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