Probabilistic source inversion of static displacement data using neural networks

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Introduction

We present a neural network based method for point source inversion in a Bayesian framework.

We demonstrate the method by inverting co-seismic displacement measurements provided by real-time GPS networks. The static offset is the remaining displacement after an earthquake has occurred. We aim to investigate to what extent this relatively novel observable can constrain point source estimates.

Our method has potential for earthquake early warning (EEW) systems, since it can rapidly provide source parameter estimates together with uncertainty bounds. Once a trained network is available an inversion takes only a few milliseconds on a desktop computer.

Concept

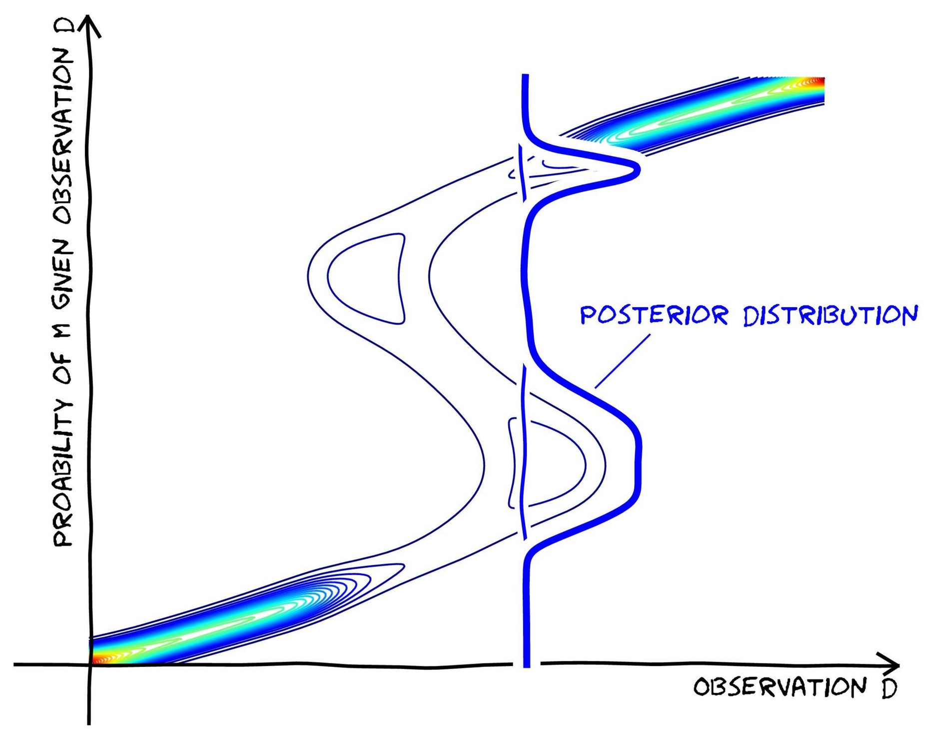

Figure 1: The solution to the inverse problem is the posterior probability distribution $p(\mathbf{m}|\mathbf{d}=\mathbf{d}_{obs})$ (Tarantola, 2005) — the conditional distribution $p(\mathbf{m}|\mathbf{d})$ evaluated at the observation $\mathbf{d_{obs}}$. In the general case, this distribution is unknown.

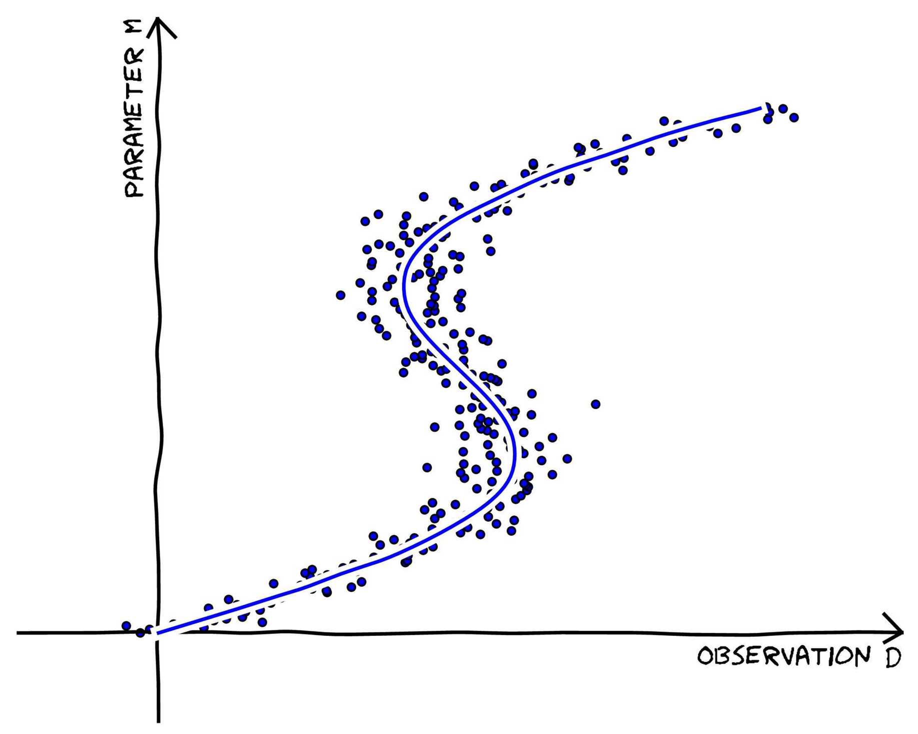

Figure 2: However, we can find samples of the conditional distribution $p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{m})p(\mathbf{d}|\mathbf{m})$ given a prior distribution on the model parameters $p(\mathbf{m})$, and a likelihood function $p(\mathbf{d}|\mathbf{m})$, that is a data noise model and a forward modelling code.

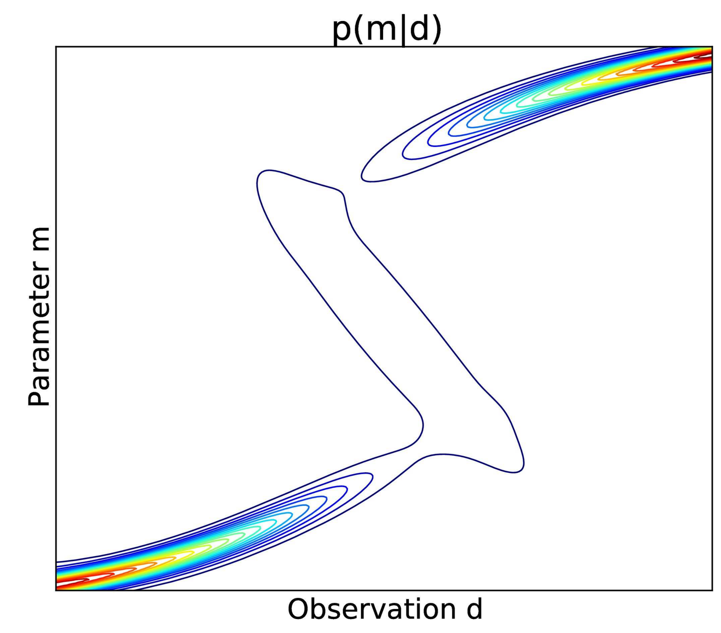

Figure 3: A mixture density network (MDN) forms a smooth, parametric approximation of the conditional probability density based on a set of training samples. Once an observation is available, we can evaluate this approximation and retrieve the posterior distribution.

Training set & synthetic data

The training set is formed by 80.000 deviatoric point sources drawn from a uniform prior distribution. Synthetic static displacements are calculated in a layered, isotropic, elastic medium using a propagator matrix method developed by O'Toole and Woodhouse (2011).

An efficient point source parametrization

Uncertainties translate to distances in parameter space. A meaningful parametrization is thus of paramount importance for probabilistic inversions. We work in the geometric moment tensor domain introduced by Tape and Tape (2012) with parameters as follows:

b	isotropic component, 0 for deviatoric sources (fixed)
γ	non double-couple component, 0 for a pure double couple
κ	strike
σ	rake
h	cosine of dip
M_W	moment magnitude

Demonstration: The 2010 El Mayor-Cucapah Event

After training and testing the networks, we present observations for a M_W 7.2, 2010 event in Baja California, which yields 1-D posterior marginal distributions.

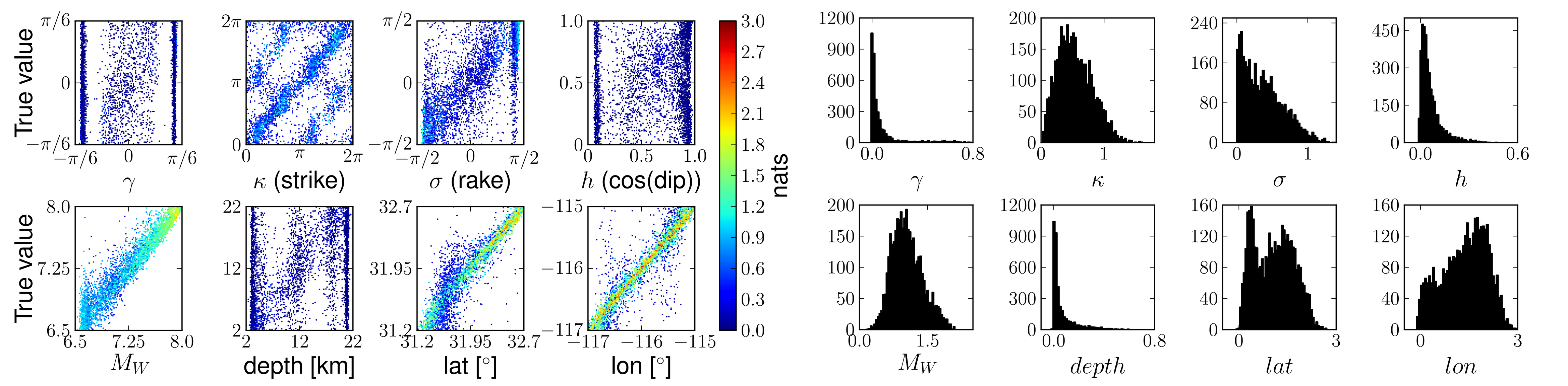

Figure 4: The network performance is assessed using another 4000 examples — the test set. **Left**: Predicted mode of the distribution vs the true value. **Right**: Histograms of the information gain — the distance between prior and posterior distribution (colour coded in the left plot). A small information gain indicates that the posterior closely resembles the prior. Not all parameters can be predicted well.

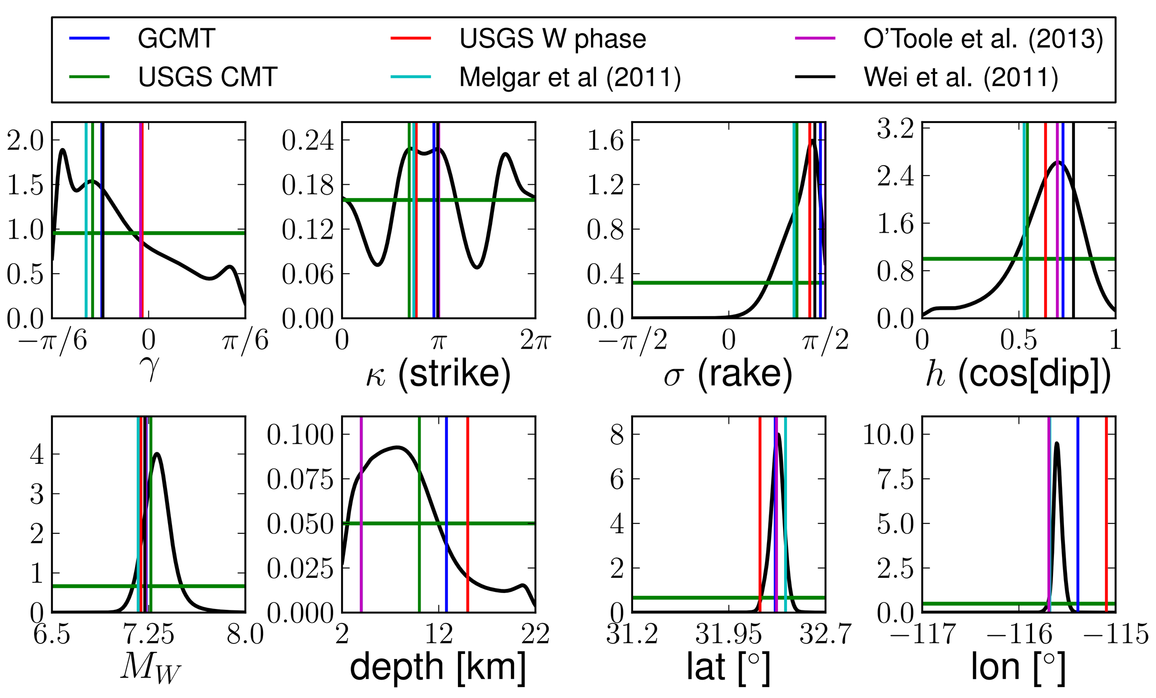

Figure 5: Inversion results for the 2010 M_w 7.2 El Mayor-Cucapah event. Vertical lines denote the position of other published moment-tensor point source solutions (Melgar et al., 2012; O'Toole et al., 2012; Wei et al., 2011). Prior distributions are shown in green. Our uncertainty estimates comprise most other solutions.

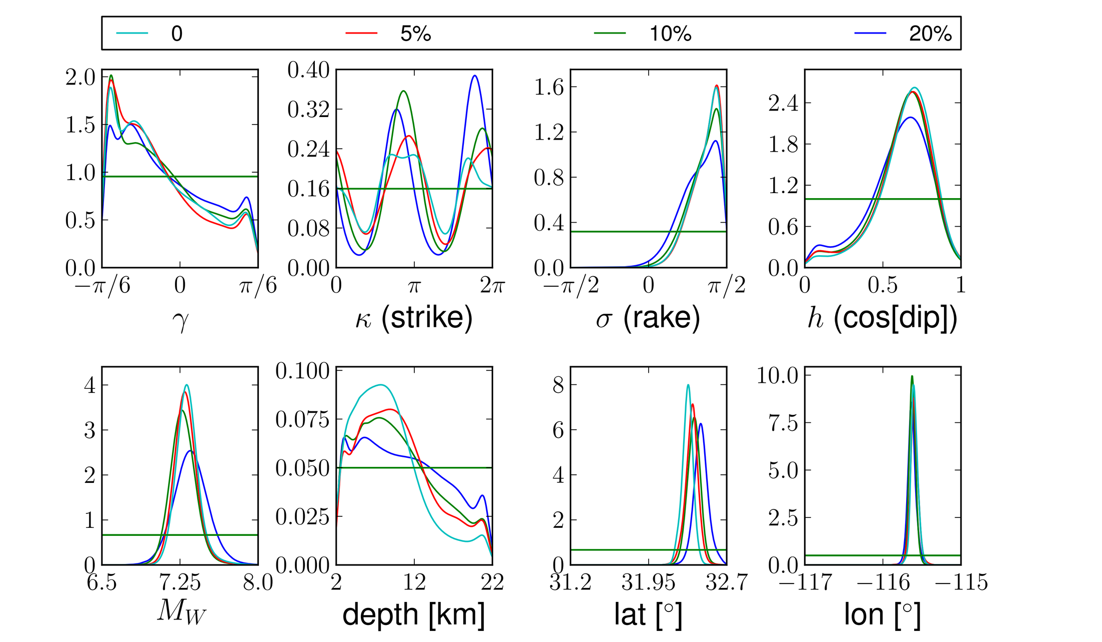

Figure 6: Influence of different amounts of perturbations of the 1-D crustal Earth model on the posterior marginals. The sensitivity with respect to the Earth model seems minimal. Most parameters are unaffected up to unrealistically large variations of 20%.

Conclusions

A probabilistic neural network inversion yields realistic uncertainty estimates and is able to deal with non-unique and multi-modal mappings.
Computational demands are low, once a trained network is available, making the method suitable for EEW purposes.
Static displacements provide robust information on location and magnitude and show little sensitivity to the crustal model.
The flexible treatment of input data makes it possible to perform joint inversions of different data types, such as displacement waveforms and strong-motion accelerograms. A potential that still remains to be explored.

References

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