Non periodic homogenization for elastic wave forward and inverse problems in seismology

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Science is such a difficult and dangerous job

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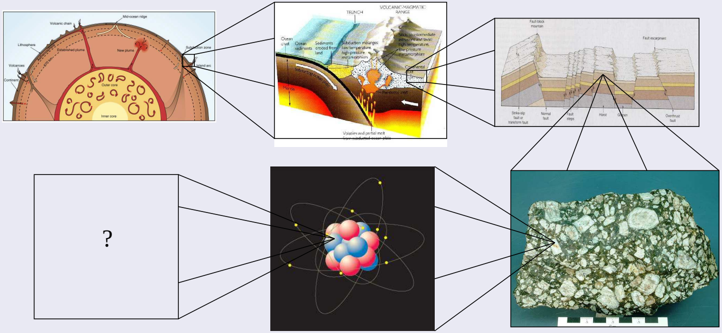
Introduction and motivations

- Seismologists often work with frequency band limited data. → the corresponding wavefield has a minimum wavelength
- Observations indicate that waves of a given wavelength are sensitive to inhomogeneities of scales much smaller than this wavelength only in an effective way. This is one of the reasons why seismic waves can be used to image the earth.

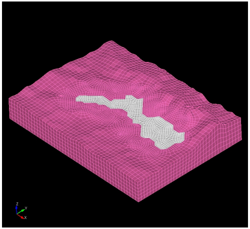
The earth contains inhomogeneities at all scales.



Small scales from the forward modeling point of view

Considering a 3-D medium, for which we can define

- a smallest inhomogeneities typical size λ_h
- a minimum wave speed α_{min}



Solving the wave equation (with FD, SEM ...) for

- N_s sources
- maximum frequency $f_{max} \Rightarrow \lambda_{min} = \frac{\alpha_{min}}{f_{max}}$

The computing time t_c is

1. in a smooth 3-D medium (that is $\lambda_h \gg \lambda_{min}$)

$$t_c \propto N_s \lambda_{min}^{-4}$$

2. in a rough 3-D medium (that is $\lambda_h \ll \lambda_{min}$)

$$t_c \propto N_s \lambda_h^{-4}$$

In a rough medium, the objective of homogenization is to bring back the numerical cost from $N_s \lambda_h^{-4}$ to $N_s \lambda_{min}^{-4}$

Two scale homogenization : the non-periodic case

Homogenization is computing a smooth effective elastic tensor for a given minimum wavelength λ_{min} :

$$\mathbf{c}^{*,k_0} = \mathcal{H}^{k_0}(\mathbf{c})$$

where $k_0 > k_{max}$ (typically $k_0 = 2k_{max}$, with $k_{max} = 1/\lambda_{min}$)

$\mathcal{H}^{k_0}$ is computed with a non-periodic homogenization technique. It implies

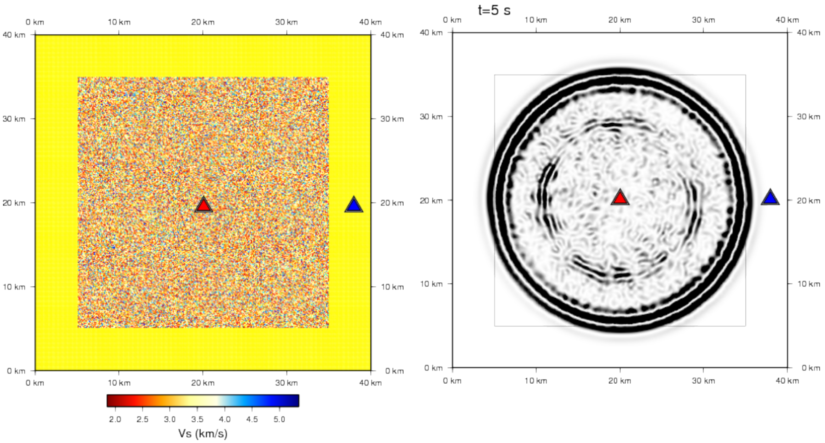
- solving a set of static loading problems (a PDE : the cell problem)
- use of smoothing operator (a lowpass spatial filter).

The whole process is non-linear (it is not just a linear filter of slowness or elastic parameters). If $\mathcal{F}^{k_0}$ is simple lowpass filter operator $(\mathcal{F}^{k_0}(g)(\mathbf{x}) = w^{k_0} * g(\mathbf{x}))$, then

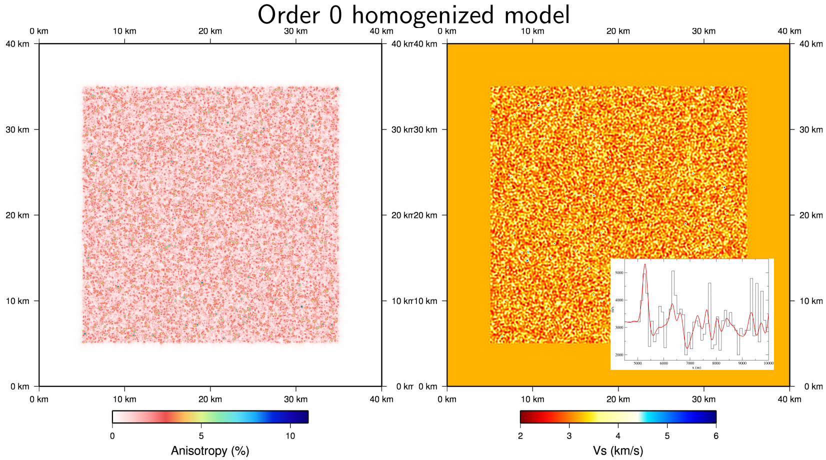
$$\mathcal{H}^{k_0}(\mathbf{c}) \neq \mathcal{F}^{k_0}(\mathbf{c})$$

A 2-D random example

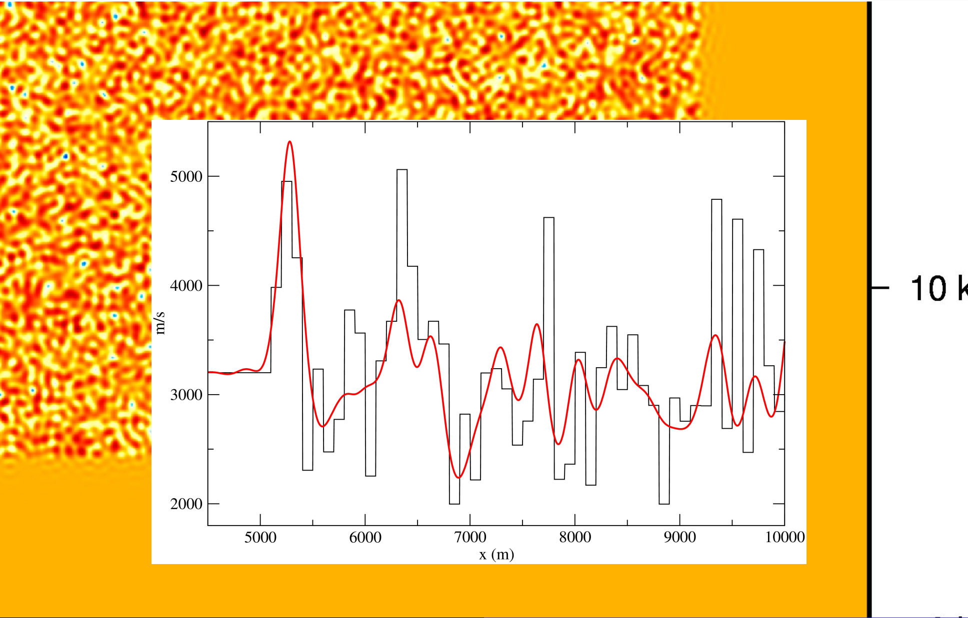
The inner square is a 300×300 homogeneous squares cells (100×100 m^2). In each cell, the density, λ and μ are determine randomly with contrast up to 50%.



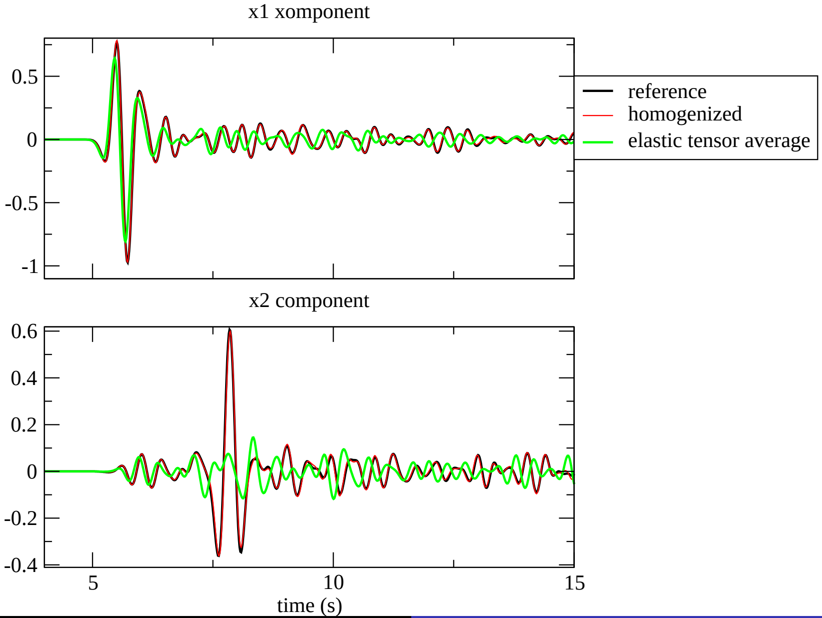
A 2-D random example



A 2-D random example



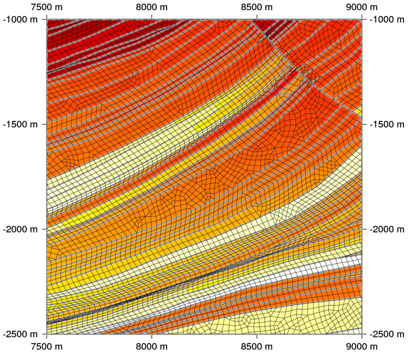
A 2-D random example : source at the center of the square



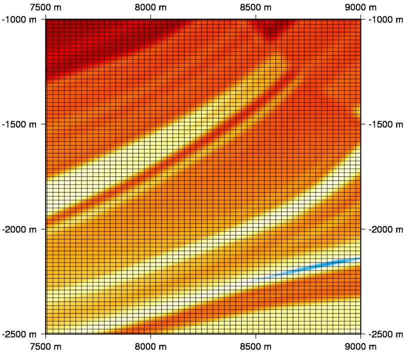
Marmousi2 example

Homogenization allows for simple meshes, even for complex media.

Mesh for the original model



Mesh for the homogenized model



Small scales from the inverse problem point of view

Inverse problem, full waveform inversion (FWI)

- FWI can only be performed in a limited frequency band.
- The best that can be recovered is what is “seen” by the wavefield. It is an effective version $(\rho^*, \mathbf{c}^*)$ of the real earth $(\rho, \mathbf{c})$.
- We expect a FWI result to be at best an effective medium.
  As a first step, we test this intuitive idea in the layered media simple case.

Residual homogenization of layered media

- Non-periodic homogenization of layered media is simple because an analytically solution exists to the cell equation. In such case, the Backus parameters are involved (still with $\varepsilon_0 = (k_0 \lambda_{min})^{-1}$) :

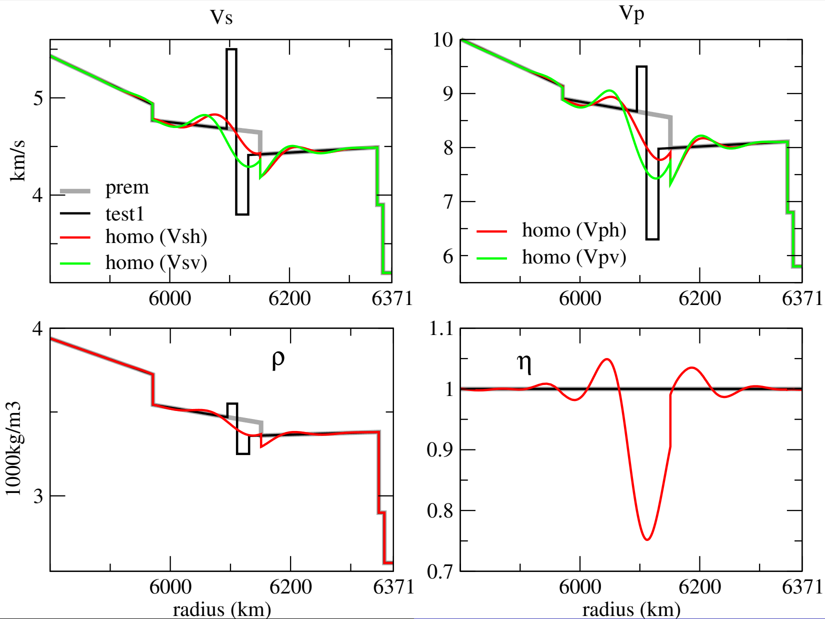
$$\rho^{\varepsilon_0 *} = \mathcal{F}^{k_0}(\rho)$$
$$\frac{1}{C^{\varepsilon_0 *}} = \mathcal{F}^{k_0}\left(\frac{1}{C}\right)$$
$$\frac{1}{L^{\varepsilon_0 *}} = \mathcal{F}^{k_0}\left(\frac{1}{L}\right)$$

$$A^{\varepsilon_0 *} - \frac{(F^{\varepsilon_0 *})^2}{C^{\varepsilon_0 *}} = \mathcal{F}^{k_0}\left(A - \frac{F^2}{C}\right)$$
$$\frac{F^{\varepsilon_0 *}}{C^{\varepsilon_0 *}} = \mathcal{F}^{k_0}\left(\frac{F}{C}\right)$$
$$N^{\varepsilon_0 *} = \mathcal{F}^{k_0}(N)$$

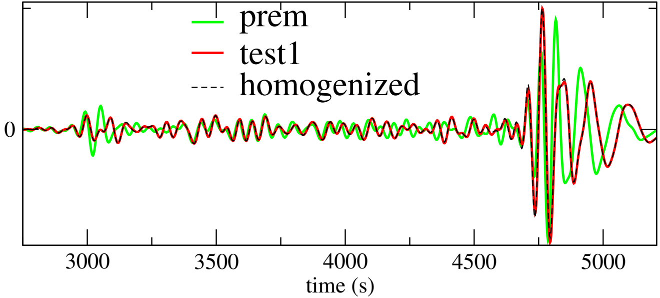
- We introduce the notion of residual homogenization, that allows to homogenize only the "difference" between a reference model and given model. For the C parameter, it gives :

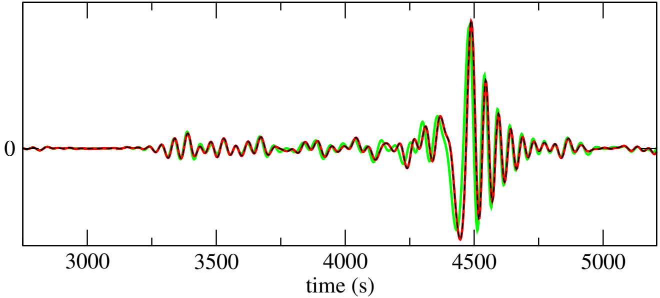
$$\frac{1}{C^{\varepsilon_0 *}} = \frac{1}{C_{\mathrm{ref}}} + \mathcal{F}^{k_0}\left(\frac{1}{C} - \frac{1}{C_{\mathrm{ref}}}\right)$$

Residual homogenization in layered media



Residual homogenization in layered media





Full waveform inversion in layered media

To test the idea that a FWI can recover at best an homogenized model

- cost function to be minimized :

$$\Phi(\mathbf{m}) = {}^{T}[\mathbf{g}(\mathbf{m}) - \mathbf{d}]\mathbf{C}_d^{-1}[\mathbf{g}(\mathbf{m}) - \mathbf{d}]$$

  with **g** : forward problem ; **m** model parameters ; **d** data set.
- Gauss-Newton iterative least square minimization :

$$\mathbf{m}^{i+1} = \mathbf{m}^i + \left({}^{t}\mathbf{G}^i {\mathbf{C}_d^i}^{-1} \mathbf{G}^i + \boldsymbol{\lambda}^i\right)^{-1} \left[{}^{t}\mathbf{G}^i {\mathbf{C}_d^i}^{-1} (\mathbf{d}^i - \mathbf{g}^i(\mathbf{m}^i))\right] ,$$

  where
  - $\mathbf{m}^i$ model at iteration i
  - $\mathbf{G}^i$ partial derivative matrix
  - λ^i damping at iteration i

We use normal modes to compute **g** in $\mathbf{m}^i$ and normal mode full coupling to compute $\mathbf{G}^i$ at each iteration.

Full waveform inversion in layered media

$\mathbf{m}$ parameter set :

- we use the Backus parameters set $p_i(r)$ ($p_1 = \rho$; $p_2 = \frac{1}{C}$; $p_3 = \frac{1}{L}$; $p_4 = A - \frac{F^2}{C}$; $p_5 = \frac{F}{C}$; $p_6 = N$)
- we invert relative we respect to a reference model (here PREM), $\mathbf{p}_{ref}$ $\delta\mathbf{p} = \mathbf{p} - \mathbf{p}_{ref}$
- we use a radial Lagrange polynomials for the vertical parameters, which can be defined by values at given set of radius $r_1 ... r_N$.

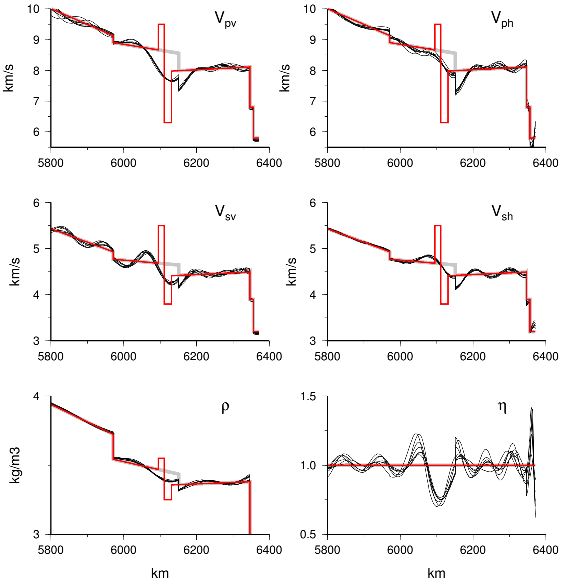
$$\mathbf{m} = (\delta p_1(r_1), ..., \delta p_6(r_1), \delta p_1(r_2), ..., \delta p_6(r_n))$$

$\mathbf{d}$ data to be inverted

- Synthetic data **d** data to be inverted are computed with normal modes in TEST1 model ;
- a random noise of 3% amplitude is added to the data ;
- we use a single 3 components receiver in the period band [33s-1000s], two sources depth (10 km and 700 km) and an epicentral distance of 130°.

Full waveform inversion in layered media : raw result

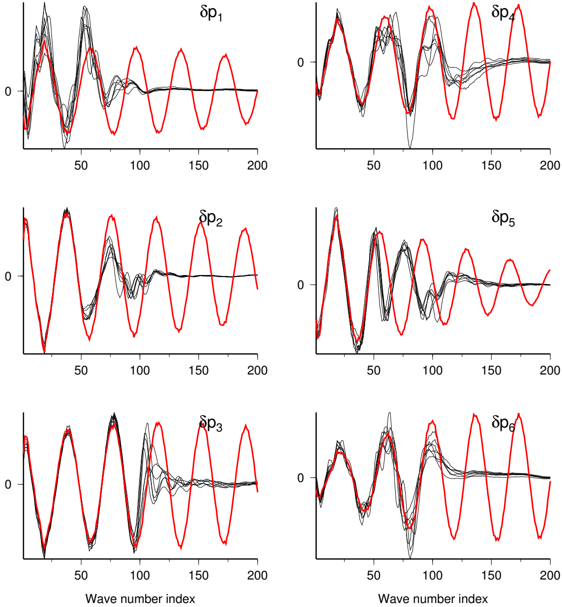
- grey line : reference model
- red line : target model
- black lines : inversion results for 8 different realizations of the 3% random noise



Full waveform inversion : raw result in the spectral domain

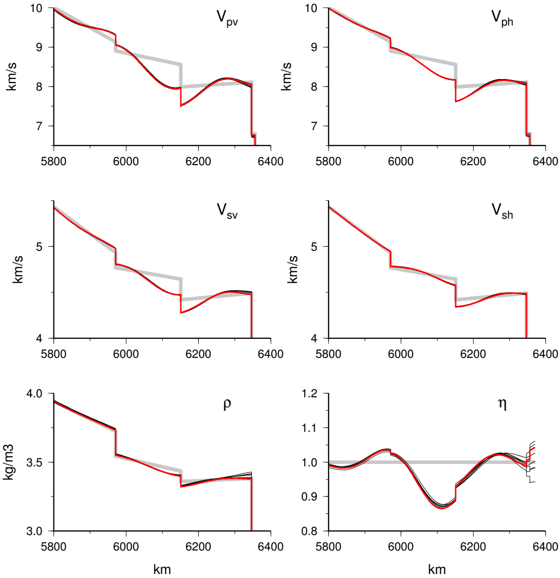
- red line : target model
- black lines : inversion results for 8 different realizations of the 3% random noise

$$p_1 = \rho$$
$$p_2 = \frac{1}{C}$$
$$p_3 = \frac{1}{L}$$
$$p_4 = A - \frac{F^2}{C}$$
$$p_5(r) = \frac{F}{C}$$
$$p_6 = N$$



Full waveform inversion : homogenized result

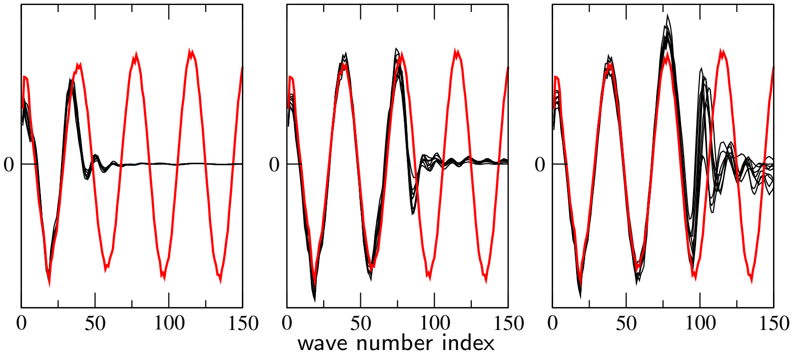
- grey line : reference model
- red line : homogenized target model
- black lines : homogenized inversion results for 8 different realizations of the 3% random noise



Full waveform inversion : the inversion is multi-scale

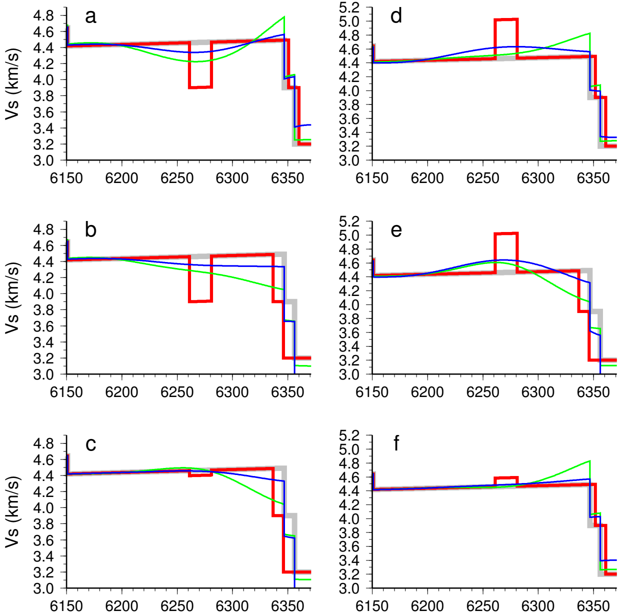
The inversion is designed such that the frequency band increases along with the iteration number.

Results for the δp_3 residual ($p_3 = \frac{1}{L}$) in the spectral domain for three iterations and therefore for 3 different frequency bands.



red line : target model ; black lines : inversion results for 8 different realizations of the 3% random noise

Full waveform inversion : effect of a wrong a priori crust



Conclusions and perspectives

Conclusions

- So far, homogenization technique, by removing small scales, is useful for the forward problem (by easing the meshing problem for example).
- for the inverse problem, homogenization can allow to build a multi-scale inversion and a consistant parametrisation.
- an elastic model obtained from a FWI is indeed an homogenized version of the traget model (at least in the layered case)

perspectives

- develop the down-scaling inverse problem (ongoing project)
- develop the higher dimensions (than layered) inverse problem (soon)
- up to what extent homogenization can lead to unicity of the inverse problem solution ?