# Elastic wave propagation in complex heterogeneous earth structures: numerical modelling by using a poly-grid spectral element method


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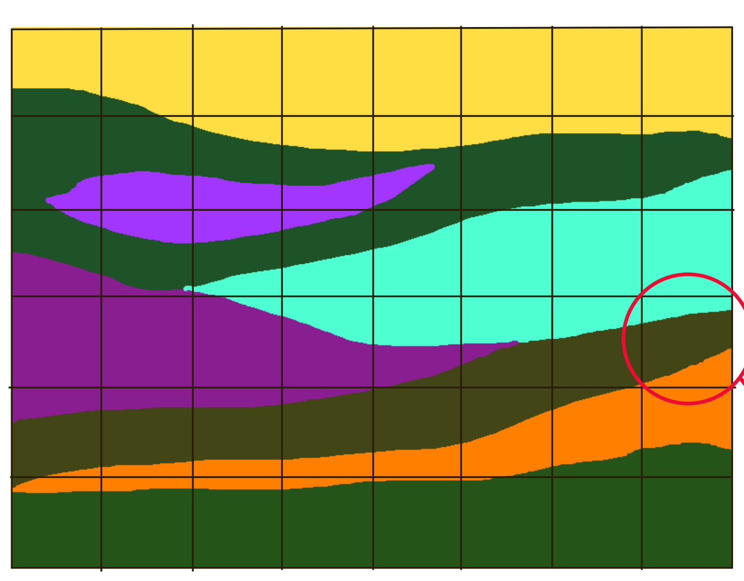
## Summary

Elastic wave propagation in complex heterogeneous earth structures requires highly accurate and computationally efficient algorithms. The spectral element methods (SEM) have excellent properties of accuracy and flexibility. In the standard SEM approach, the computational domain is discretized by using very coarse meshes with constant-property elements. The accuracy and the computational efficiency may be seriously reduced in the case of complex earth structures characterized by fine layering or property fluctuations shorter than the minimum wavelength.

A poly-grid Chebyshev spectral element method (PG-CSEM) allows to overcome this limitation. Temporary auxiliary grids are introduced which avoid the need of using large meshes, and at the macroscopic level the wave propagation is solved in a coarse mesh maintaining the SEM accuracy.

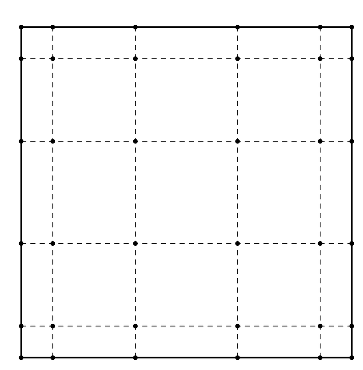

## Introduction

- In wave propagation modeling:
  - **heterogeneous properties** must be correctly reproduced,
  - **very complex structures** must be correctly modelled,
  - **numerical algorithms** must be computationally efficient.
- The Poly-Grid Method allows for:
  - **large** elements with high order (high accuracy),
  - **simple** geometry of elements (easy discretization),
  - property changes **inside** each element (high variability),
  - changes that can be **continuous** or **abrupt**,
  - changes that can be **smaller** than minimum wavelength.
- Discretization of each element with **poly-grid** (set of independent local grids)

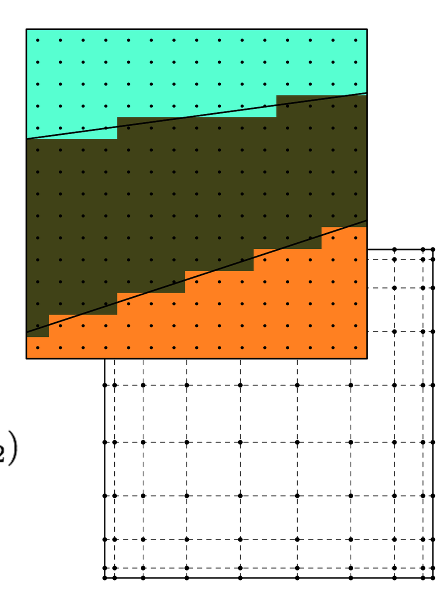


**Primary grid**

$$\tilde{u}^e(\xi_1,\xi_2,t) = \sum_{i_1,i_2=0}^{N} u^e_{i_1 i_2}(t)\,\varphi_{i_1}(\xi_1)\,\varphi_{i_2}(\xi_2)$$



**Auxiliary grids**



$$\tilde{\alpha}^e(\xi_1,\xi_2) = \sum_{l_1,l_2=0}^{L} \alpha^e_{l_1 l_2}\phi_{l_1}(\xi_1)\phi_{l_2}(\xi_2)$$

$$\tilde{f}^e(\xi_1,\xi_2,t) = \sum_{k_1,k_2=0}^{K} \mathbf{F}^e_{k_1 k_2}(t)\,\psi_{k_1}(\xi_1)\,\psi_{k_2}(\xi_2)$$

## Mathematical formulation

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} - \mathbf{D}^\top\mathbf{C}\mathbf{D}\mathbf{u} = \mathbf{f}$$

$$\begin{cases} \mathbf{M}\,\ddot{\mathbf{U}}_1(t) + \mathbf{K}_1\,\mathbf{U}_1(t) + \mathbf{K}_2\,\mathbf{U}_2(t) = \mathbf{F}_1(t) \\ \mathbf{M}\,\ddot{\mathbf{U}}_1(t) + \mathbf{K}_2^\top\,\mathbf{U}_1(t) + \mathbf{K}_3\,\mathbf{U}_2(t) = \mathbf{F}_2(t) \end{cases}$$

$$\mathbf{M}^e_{i_1 i_2 j_1 j_2} = \frac{\Delta^e_1\Delta^e_2}{4}\sum_{l_1,l_2=0}^{L}\tilde{\rho}^e_{l_1 l_2} B_{i_1 j_1 l_1} B_{i_2 j_2 l_2}$$

$$\begin{aligned}
[\mathbf{K}^e_1]_{i_1 i_2 j_1 j_2} &= \textstyle\sum_{l_1,l_2=0}^{L}\left[(\lambda_{l_1 l_2}+2\mu_{l_1 l_2})\frac{\Delta^e_2}{\Delta^e_1}\bar{\bar{B}}_{i_1 j_1 l_1} B_{i_2 j_2 l_2} + \mu_{l_1 l_2}\frac{\Delta^e_1}{\Delta^e_2} B_{i_1 j_1 l_1}\bar{\bar{B}}_{i_2 j_2 l_2}\right] \\
[\mathbf{K}^e_2]_{i_1 i_2 j_1 j_2} &= \textstyle\sum_{l_1,l_2=0}^{L}\left(\lambda_{l_1 l_2}\bar{B}_{i_1 j_1 l_1}\bar{B}_{j_2 i_2 l_2} + \mu_{l_1 l_2}\bar{B}_{j_1 i_1 l_1}\bar{B}_{i_2 j_2 l_2}\right) \\
[\mathbf{K}^e_3]_{i_1 i_2 j_1 j_2} &= \textstyle\sum_{l_1,l_2=0}^{L}\left[(\lambda_{l_1 l_2}+2\mu_{l_1 l_2})\frac{\Delta^e_1}{\Delta^e_2} B_{i_1 j_1 l_1}\bar{\bar{B}}_{i_2 j_2 l_2} + \mu_{l_1 l_2}\frac{\Delta^e_2}{\Delta^e_1}\bar{\bar{B}}_{i_1 j_1 l_1} B_{i_2 j_2 l_2}\right]
\end{aligned}$$

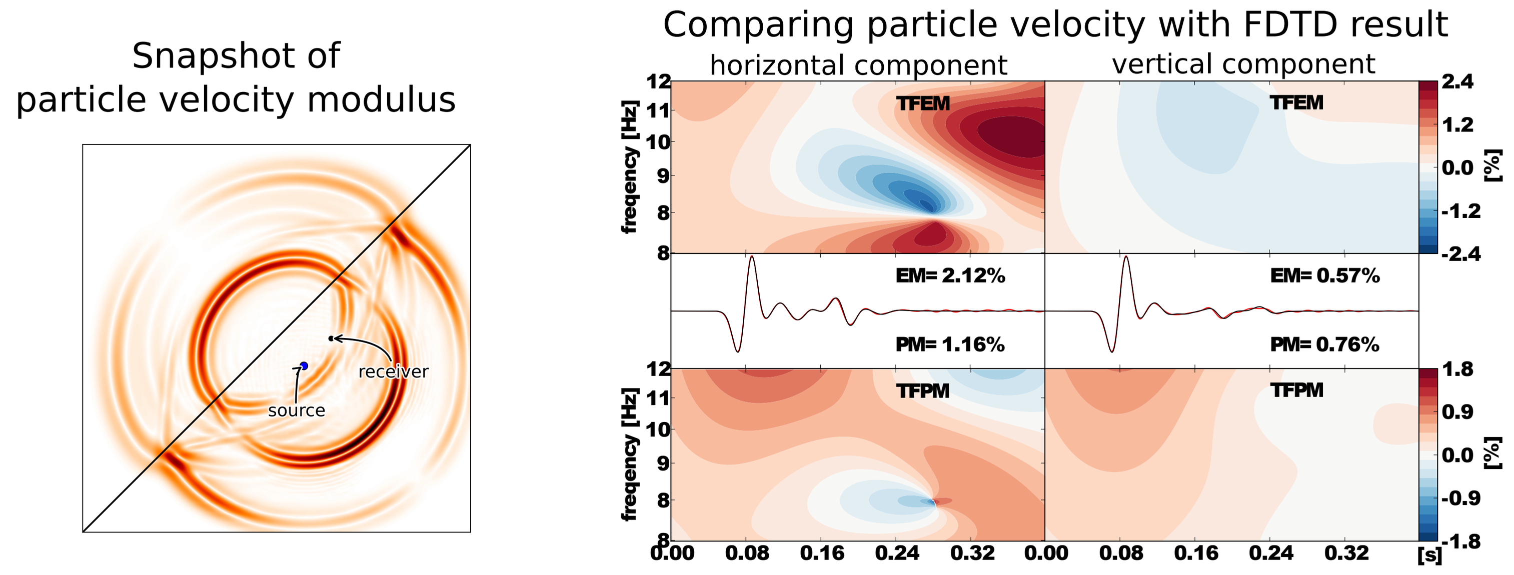
$$\mathbf{F}^e_{i_1 i_2} = \frac{\Delta^e_1\Delta^e_2}{4}\sum_{k_1,k_2=0}^{K} A_{i_1 k_1} A_{i_2 k_2}\mathbf{f}^e_{k_1 k_2}(t)$$

### Poly-grid coupling operators

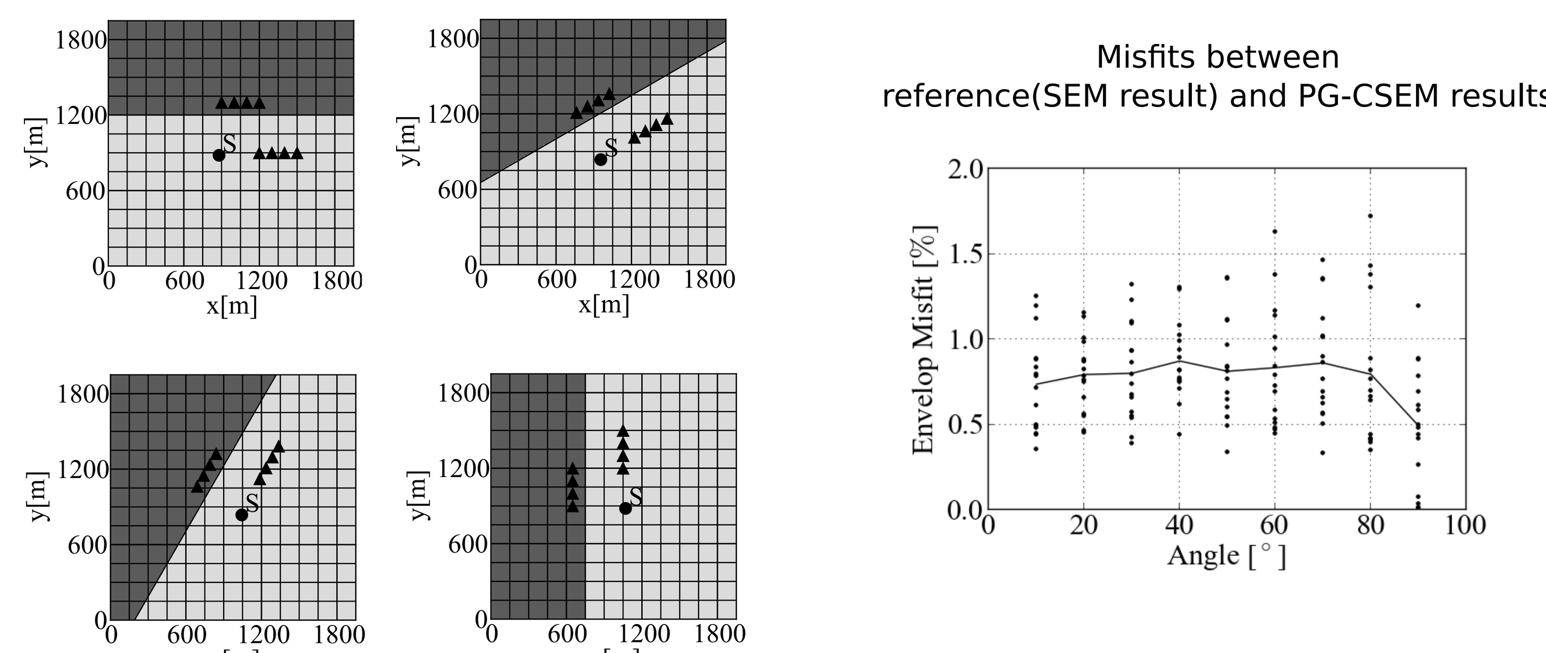
$$A_{ik} = \int_{-1}^{+1}\varphi_i(\xi)\,\psi_k(\xi)\,\mathrm{d}\xi \qquad B_{ijl} = \int_{-1}^{+1}\phi_l(\xi)\,\varphi_i(\xi)\,\varphi_j(\xi)\,\mathrm{d}\xi$$

$$\bar{B}_{ijl} = \int_{-1}^{+1}\phi_l(\xi)\,\varphi_i(\xi)\,\frac{\mathrm{d}\varphi_j(\xi)}{\mathrm{d}\xi}\,\mathrm{d}\xi \qquad \bar{\bar{B}}_{ijl} = \int_{-1}^{+1}\phi_l(\xi)\,\frac{\mathrm{d}\varphi_i(\xi)}{\mathrm{d}\xi}\,\frac{\mathrm{d}\varphi_j(\xi)}{\mathrm{d}\xi}\,\mathrm{d}\xi$$

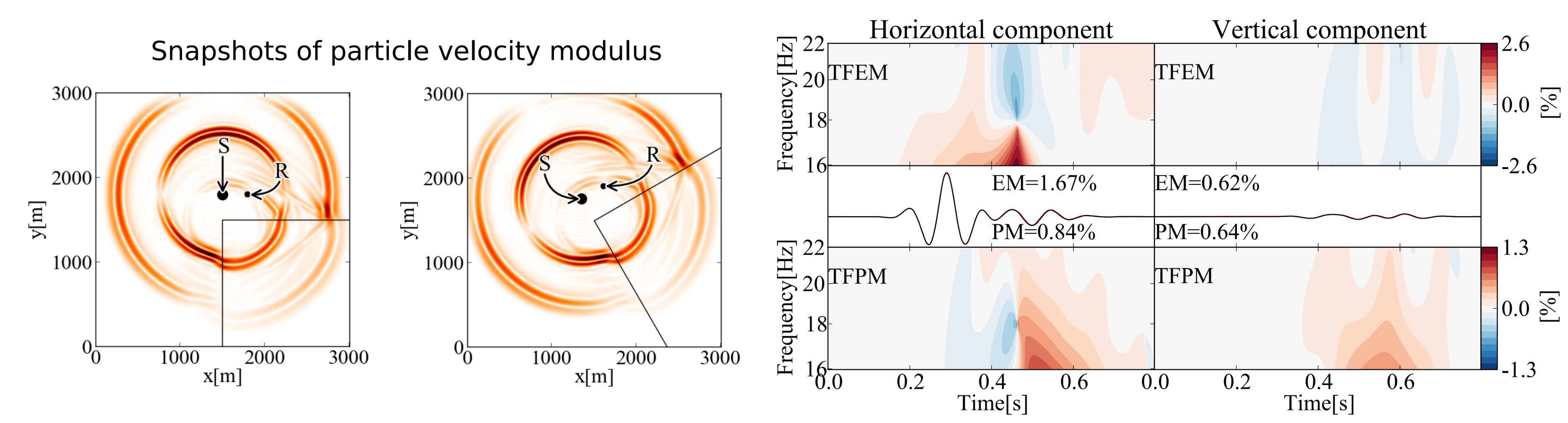
## Example: Elastic wave impinging on a planar interface



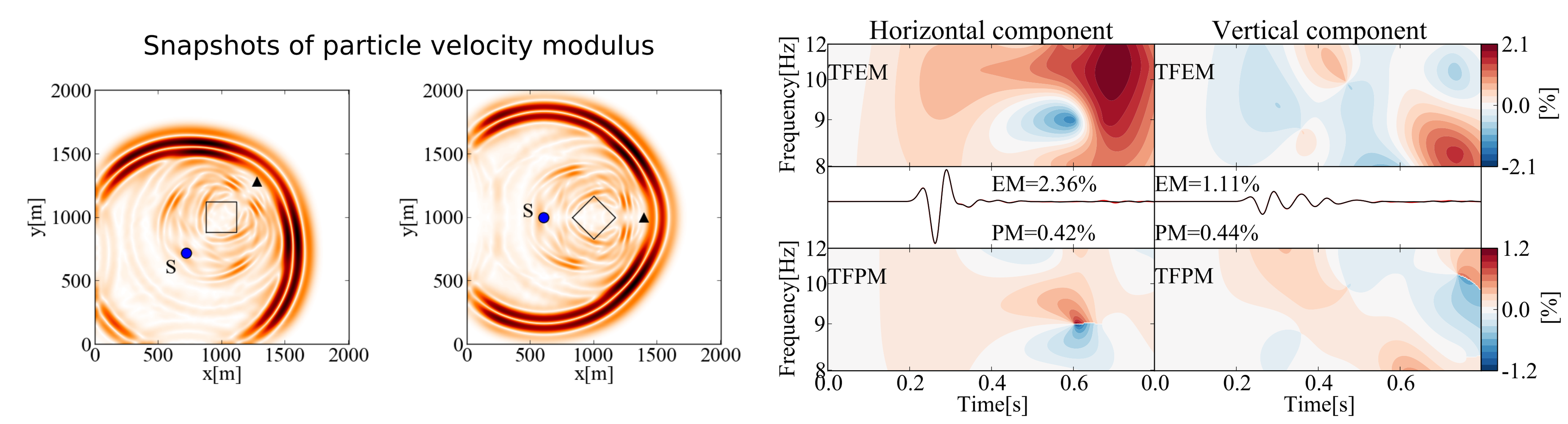

**Changing the angle of inclined interface**

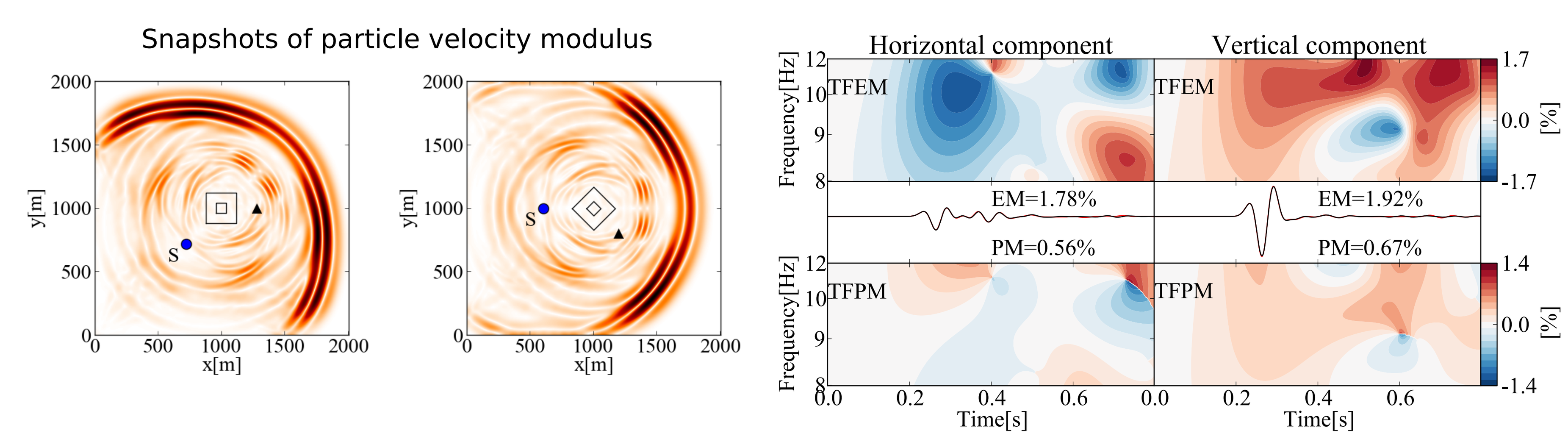



## Example: Elastic wave diffraction by a wedge




## Example: Elastic wave diffraction by square bodies




## Example: An inclined stratified fault model