Bayesian inference of linear mixed models for Parkfield 2004 M6.0 Earthquake: Rectangular and self parametrizing partition

V. Rezanezhad, M. Holschneider, L. Wang

Potsdam University

vahid.rezanezhad@math.uni-potsdam.de

1. Introduction

When a large shallow earthquake happens, there will be notable surface displacements around the focal area. The inverse problem of reconstructing the slip distribution over the fault from irregular surface observations of dislocation is considered. For known fault geometry the inversion is linearand for unknown fault geometry it is non-linear. In the present study we apply linear mixed modeling to the 2004 M6.0 Parkfield earthquake with both rectangular partition and self parametrizing partition. This is an extension of previous work [1], where a strong priori constraints had to be imposed on the slip-field. Moreover, we can use the full available data and all displacement directions on the fault all which previously led to unstable results. In the second step we apply Partition modelling[4]. Partition modelling is a statistical method for nonlinear regression and classification. The method is an ensemble inference approach within a Bayesian framework. Here we extend this method for source modeling. The procedure involves a dynamic parametrization for the model which is able to adapt to an uneven spatial distribution of the information on the model parameters contained in the observed data.

2. Mixed Models

We can model the $n \times 3$-dimensional vector of observed surface displacments $\delta X_i, \delta Y_i, \delta Z_i$ with a stochastic model that relates slip of discretized fault to surface displacement linearly as

$$d = Gs + \epsilon$$

The measurement errors are supposed to be independent and known $\sigma_X = \sigma_Y = 3mm$, $\sigma_Z = 10mm$. The matrix elements of G can be computed from elastostatic greens functions for a dislocation [3]. Decomposing s into 2 dimensional fixed effects (uniform,tapered slip) and random effects (patchwise dislocations)

$$s_k = \sum M_{k,f}\beta_f + u_k, \quad k = 1, \dots, \#\text{patch}$$

we obtain a linear mixed model [2]

$$d = F\beta + Gu + \epsilon = g(m) + \epsilon, \quad F = GM$$

ϵ and u are assumed to be independent and have distirbutions as

$$u \sim N(0, \lambda^2\sigma^2\Omega)$$

$$\epsilon \sim N(0, \sigma^2\Sigma)$$

The fixed effects have flat prior. Bayesian theorem allows computation of posterior distirbution as

$$\mathbb{P}(m|d) \sim exp\left(\frac{||d - g(m)||^2 + \lambda^{-2}u^T\Omega^{-1}u}{-2\sigma^2}\right)$$

The estimated field is given by the posterior mode (best linear unbiased prediction [BLUP]).

3. Estimation by BLUP and Cross-Validation for rectangular partition

Here we estimate slip distribution using (BLUP) and we do K-Fold Cross-Validation to fix hyperparameter(λ). Results for synthetic data and Parkfield 2004 M6.0 earthquake are shown in the following. The fault is partitioned to 200 rectangular patches, 20 and 10 along strike and dip respectively and since parkfield 2004 M6.0 mechanism of slip is strike we have only one fixed parameter, also vertical error is big then we use only horizontal data of 14 GPS sites. The geometry of fault is $137°$ strike angle, $83°$ dip angle, length and width 40 km and 14.5 km along strike and dip respectively.

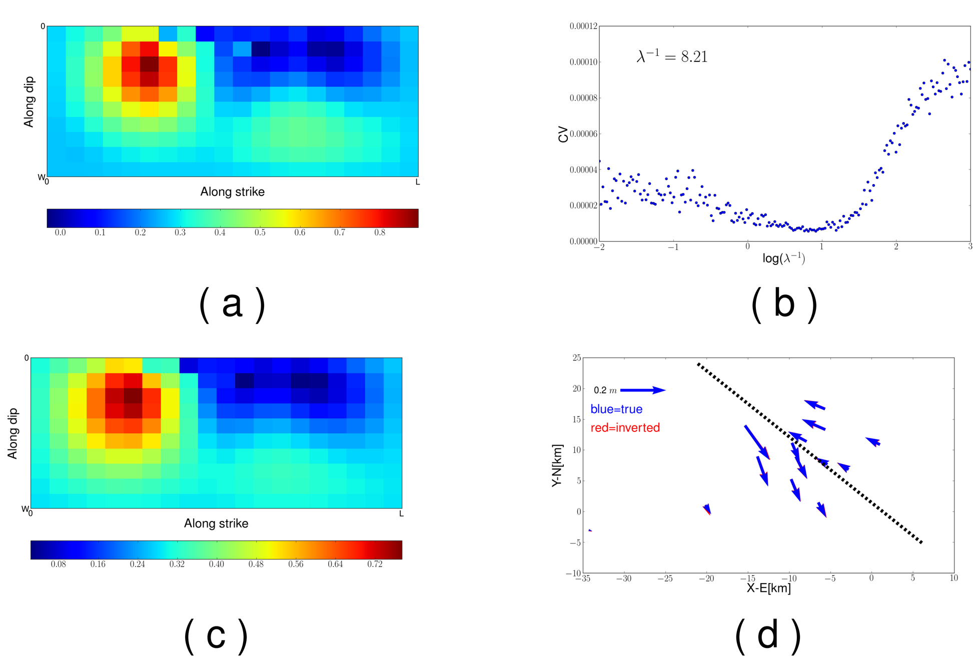

Figure 1: *Synthetic test a) Input slip distribution b) K-Fold Cross-Validation c) Inverse slip distribution d) Surface displacements.*

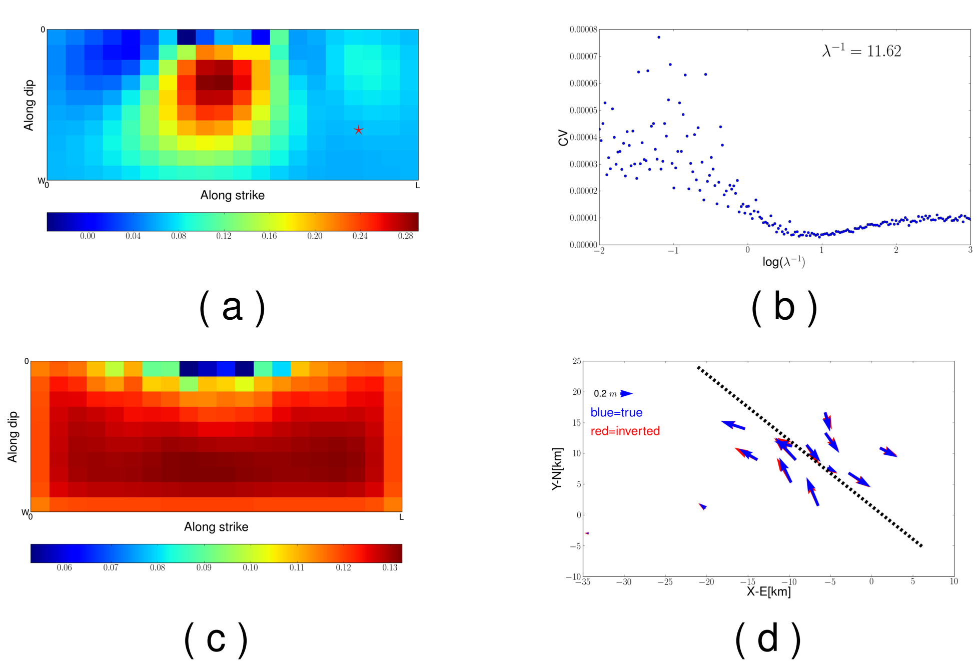

Figure 2: *Parkfield 2004 M6.0 a) Inverse slip distribution b) K-Fold cross-validation c) standard deviation d) Surface displacements.*

4. Estimation by metropolis hastings algorithm for self parametrizing partition

In the fixed rectangular partition , green function is fixed but in the case of self parametrizing partition green function also is going to be changed for any set of voronoi cells. We use flat prior for position of voronoi cells. The fault is divided to 64 voronoi cells.

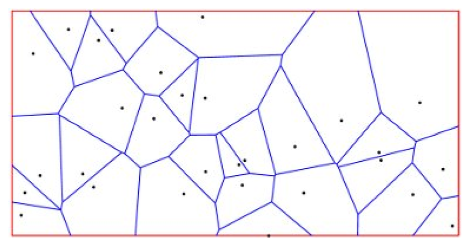

Figure 3: *Voronoi cells*

algorithm of sampling

- Randomly pick one cell
  - Even iteration: Randomly change the slip value of the cell according to a Gaussian proposal probability density $q(s_i'|s_i)$ centred at the current value s_i.
  - Odd iteration: Randomly change the position of the cell nucleus according to a 2D Gaussian proposal probability density $q(R_i'|R_i)$centred at the current position R_i. The covariance matrix for the 2D Gaussian function is proportional to the identity matrix, with the constant of proportionality(σ_c), a parameter to be chosen.
- Solve the forward problem for proposed model
- Decide whether or not to replace or update the current model with the proposed model by drawing from a uniform random variate, U, (between 0 and 1) and using an acceptance criterion which takes the form

$$p(accept) = min(1, \frac{p(m'|d)}{p(m|d)}.\frac{p(m|m')}{p(m'|m)})$$

Assuming symmetric proposal distributions (i.e. $p(m|m') = p(m'|m)$)

$$p(accept) = min(1, \frac{p(m'|d)}{p(m|d)})$$

draw a value U from the Uniform(0,1) distribution, if $U < p(accept)$: then accept m'.

After sampling we do an ensambl avreaging to get slip distribution for synthetic test and Parkfield 2004 M6.0 event that the results for different parameters of σ_c and λ^{-1} are shown in following.

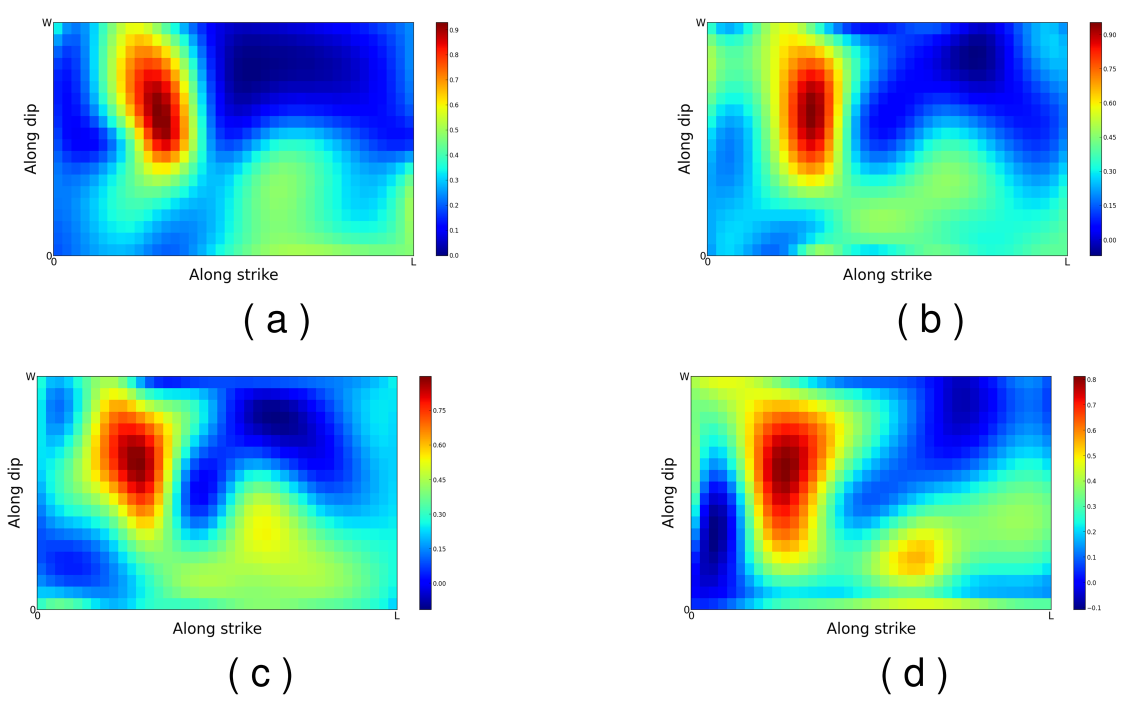

Figure 4: *Inverse slip distribution of synthetic test a)* $\sigma_c = 0.1km$, $\lambda^{-1} = 5$ *b)* $\sigma_c = 0.1km$, $\lambda^{-1} = 15$ *c)* $\sigma_c = 0.3km$, $\lambda^{-1} = 10$ *d)* $\sigma_c = 0.3km$, $\lambda^{-1} = 15$

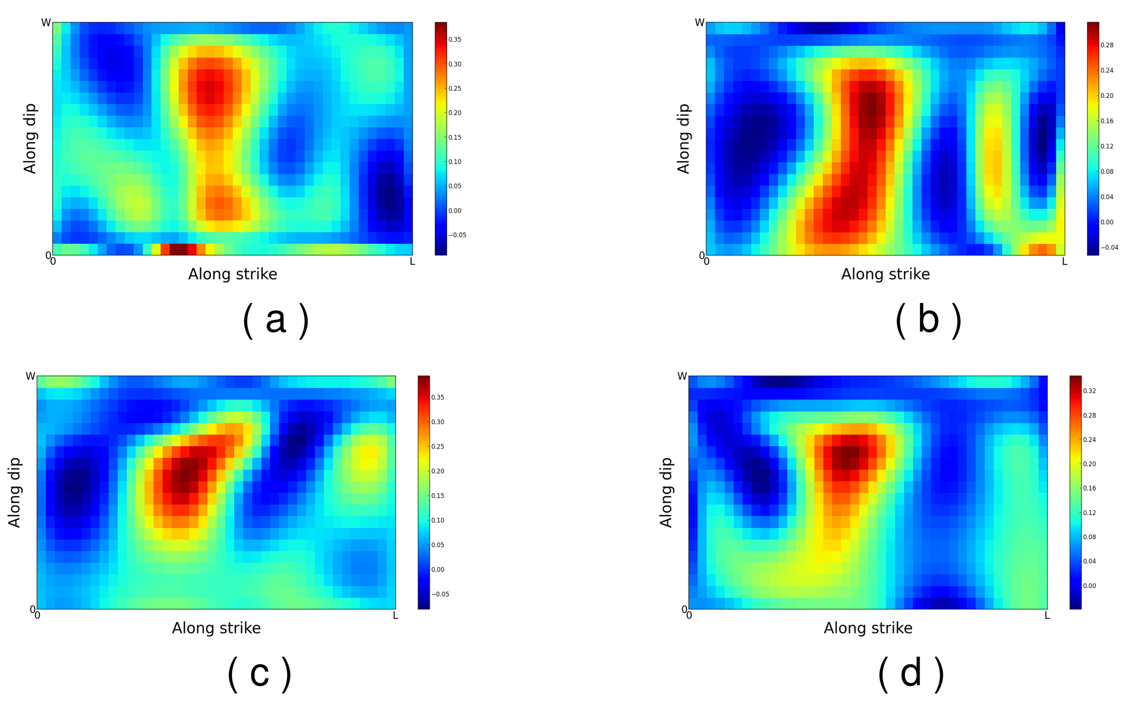

Figure 5: *Inverse slip distribution of Parkfield 2004 M6.0 earthquake a)* $\sigma_c = 0.2km$, $\lambda^{-1} = 25$ *b)* $\sigma_c = 0.3km$, $\lambda^{-1} = 0$ *c)* $\sigma_c = 0.6km$, $\lambda^{-1} = 1$ *d)* $\sigma_c = 0.6km$, $\lambda^{-1} = 10$

References

[1] Wang L., Hainzl, S., Zoeller, G. Holschneider, M. *Stress- and aftershock- constrained joint inversions for co- and post- seismic slip applied to the 2004 M6.0 Parkfield earthquake*, J. Geophy. Res., 2012.

[2] Douglas M Bates, Saikat DebRoy *Linear mixed models and penalized least squares*, J of Multivariate Analysis (2004)

[3] Okada, Y.(1985) *Surface Deformation Due to Shear and Tensile Faults in a Half-Space.* Bull. Seismol. Soc. Am., 75, 1135-1154

[4] T. Bodin, M. Sambridge and K.Gallagher(2009) *A self-parametrizing partition model approach to tomographic inverse problems.* Inverse Problems, 25, 055009.