Gemini-Tutorial

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Introduction

The computation of synthetic seismograms for a layered earth is one of the basic tools for inferring the 3D–structure of the earth. This is true from a theoretical point of view because most methods which attempt to compute synthetic seismograms for a 3D–heterogeneous earth use a spherically symmetric earth model as a reference for perturbation methods. It is, however, also true from the observational point of view, since travel times of long–period body waves and phases of surface waves are often measured by cross–correlating the observed seismograms with synthetic ones.
Though the basic theory for this problem is well known for a long time, a numerical realization is still a non-trivial task and may require some computation time even on modern computers.

Essentials of wave propagation theory on a layered Earth

Wave propagation problems always start with the general elastodynamic equation. Here, we first take a Fourier transform with respect to time and we neglect effects of gravity and prestress:

$$-\rho\omega^2\mathbf{u} = \nabla\cdot\sigma + \mathbf{f}\,. \tag{1}$$

ρ is the equilibrium density, $\mathbf{u}$ the displacement from equilibrium, σ is the stress tensor due to the perturbation and $\mathbf{f}$ is the equivalent body force per unit volume due to a seismic source. Gravity is neglected because it is important only at very low frequencies. We consider a layered, transversely isotropic earth model with density ρ and the five elasticity constants A, C, F, L and N (Takeuchi & Saito 1972) depending on depth only. Attenuation is included by introducing complex elastic moduli (Müller, 1983). Boundary conditions are regularity of the solutions at infinite depth (or the earth's center), continuity of displacement and traction at solid–solid interfaces and vanishing of traction at the surface of the earth. At fluid–solid boundaries continuity is required for traction and vertical displacement only.

Separation of variables

Partial analytical solutions of this problem are available if the horizontal dependence of the wavefield can be separated from the vertical dependence. It is known that such a separation is only possible in cartesian, cylindrical and spherical coordinates. In cartesian and cylindrical coordinates the depth coordinate would be z, while in spherical coordinates the vertical coordinate would be r. Here, we denote the vertical coordinate by v to realize a common notation valid for all three cases.
A further restriction arises from the elastic symmetry of the medium which must be at least hexagonal with a vertical symmetry axis. Since such a medium is isotropic in a horizontal plane it is often called "transversely isotropic".
A separation of the horizontal from the vertical dependence of the wavefield can be achieved if the displacement vector $\mathbf{u}$, the traction vector on horizontal planes $\mathbf{t} = \mathbf{e}_v \cdot \sigma$ and the exciting force vector $\mathbf{f}$ are represented in terms of scalar potentials as follows:

$$\begin{aligned} \mathbf{u} &= U\mathbf{e}_v + \nabla_H V - \mathbf{e}_v \times \nabla_H W \\ \mathbf{t} &= R\mathbf{e}_v + \nabla_H S - \mathbf{e}_v \times \nabla_H T \\ \mathbf{f} &= F\mathbf{e}_v + \nabla_H G - \mathbf{e}_v \times \nabla_H H, \end{aligned} \quad (2)$$

∇_H is the horizontal gradient or surface gradient given by

$$\begin{aligned}
\nabla_H &= \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} \quad \text{cartesian} \\
\nabla_H &= \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\varphi \frac{1}{r} \frac{\partial}{\partial \varphi} \quad \text{cylindrical} \\
\nabla_H &= \mathbf{e}_\vartheta \frac{\partial}{\partial \vartheta} + \frac{\mathbf{e}_\varphi}{\sin\vartheta} \frac{\partial}{\partial \varphi} \quad \text{spherical}\,. \qquad (3)
\end{aligned}$$

Insertion of the above representation in terms of scalar potentials into the elastodynamic equation and ordering the result as components of "basis" vectors $\mathbf{e}_v$, ∇_H and $-\mathbf{e}_v \times \nabla_H$ leads to expressions where derivatives with respect to horizontal coordinates occur only in terms of the surface Laplacean:

$$\begin{aligned}
\nabla_H^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{cartesian} \\
\nabla_H^2 &= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} \quad \text{cylindrical} \\
\nabla_H^2 &= \frac{\partial^2}{\partial \vartheta^2} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial \varphi^2} + \cot\vartheta \frac{\partial}{\partial \vartheta} \quad \text{spherical}\,. \qquad (4)
\end{aligned}$$

For example the $-\mathbf{e}_v \times \nabla_H$-component of the elastodynamic equation for an isotropic medium in spherical coordinates takes the form

$$-\rho\omega^2 W = \left(\frac{\partial}{\partial r} + \frac{3}{r}\right) T + \frac{\mu}{r^2}(\nabla_H^2 + 2)W + H\,. \tag{5}$$

Now we can perform the classical separation of variables: write the potentials W, T and H as product of a function depending on v (the vertical coordinate) and a function depending on the horizontal coordinates (spherical example continued)

$$W(r,\vartheta,\varphi) = w(r)Y(\vartheta,\varphi), \quad T(r,\vartheta,\varphi) = t(r)Y(\vartheta,\varphi)\,, \tag{6}$$

and obtain after some ordering and division by Y

$$\rho\omega^2 w + \left(\frac{\partial}{\partial r} + \frac{3}{r}\right) t + 2\frac{\mu}{r^2} w + h = -\frac{\mu}{r^2} w \frac{\nabla_H^2 Y}{Y}\,. \tag{7}$$

On the left hand side, we have a function of r while on the right hand side we have a product of a function of r and a function of the horizontal coordinates. To satisfy this equation for arbitrary w, t and h, the function $\nabla_H^2 Y/Y$ must be a constant:

$$\nabla_H^2 Y(\vartheta,\varphi) = -c^2 Y(\vartheta,\varphi) \tag{8}$$

In case of spherical coordinates, solutions to this equation which are regular at the poles are the spherical harmonics $Y_\ell^m(\vartheta,\varphi)$ with $c^2 = \ell(\ell+1)$.
We thus see that a general solution for the scalar $W(r,\vartheta,\varphi)$ can be written in the form

$$W(r,\vartheta,\varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} w_\ell^m(r) Y_\ell^m(\vartheta,\varphi), \tag{9}$$

with analogous expressions for the other potentials. Inserting the value $-\ell(\ell+1)$ for the ratio $\nabla_H^2 Y/Y$ leads to one of two equations for the expansion coefficients $w_\ell^m(r)$ and $t_\ell^m(r)$:

$$\frac{\partial t}{\partial r} = -\rho\omega^2 w + (\ell(\ell+1) - 2)\frac{\mu}{r^2} w - \frac{3}{r} t - h \tag{10}$$

The other equation can be derived from the stress-strain relation which connects the displacement potential w with the stress-potential t.

Very similar results can be derived for cartesian and cylindrical coordinates. Eigenfunctions of the horizontal Laplacean in cartesian coordinates are

$$Y(x,y) = \exp(ik_x x)\exp(ik_y y) \quad \text{with} \quad c^2 = k_x^2 + k_y^2 = k^2 \tag{11}$$

leading to a 2D-Fourier integral representation of the scalar potentials

$$W(z,x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(z,k_x,k_y)\exp(ik_x x)\exp(ik_y y)dk_x dk_y\,. \tag{12}$$

For cylindrical coordinates, the eigenfunctions of the surface Laplacean are Fourier-Bessel functions

$$Y(z,r,\varphi) = J_m(kr)\exp(im\varphi) \quad \text{with} \quad c^2 = k^2 \tag{13}$$

leading to a Fourier-Bessel representation of the scalar potentials

$$W(z,r,\varphi) = \sum_{m=-\infty}^{\infty}\int_0^{\infty} w_m(k,z)J_m(kr)\exp(im\varphi)kdk \tag{14}$$

System of ordinary differential equations

The above analysis is valid for wave propagation in a layered halfspace and a layered sphere. The essence is that the problem can be reduced to the solution of systems of coupled first order ordinary differential equations (SODE) with respect to the vertical coordinate for each expansion coefficient (either spherical harmonic or plane wave or Fourier-Bessel). The SODEs themselves are non-trivial as we shall see later. In the following I shall concentrate on the spherical problem appropriate for global wave propagation in the earth.
Fortunately, some of the potentials are independent from each other. There is one SODE for w and t (the $-\mathbf{e}_r \times \nabla_H$-components of displacement and traction) representing toroidal or SH motion and one SODE for u, v, r, s representing spheroidal or P-SV motion. They have the following general form:

$$\frac{\partial \mathbf{y}}{\partial r} = A\mathbf{y} + \mathbf{s}\delta(r - r_s) \qquad (15)$$

where $\mathbf{y}$ is known as the displacement-stress-vector (DSV) because two of its components are displacement potentials and two stress potentials. The excitation vector $\mathbf{s}$ is filled with the expansion coefficients of the force potentials.

System matrices

For toroidal motion (SH) the matrix A takes the following form:

$$A_T = \begin{pmatrix} -\frac{2}{r} & \frac{1}{L} \\ -\omega^2\rho - \frac{N}{r^2}(2-\ell(\ell+1)) & -\frac{2}{r} \end{pmatrix}. \qquad (16)$$

For spheroidal motion (P-SV), its form is

$$A_S = \begin{pmatrix} -\frac{2F}{rC} & \frac{1}{C} & \ell(\ell+1)\frac{F}{rC} & 0 \\ -\rho\omega^2 + \frac{4}{r^2}(A - \frac{F^2}{C} - N) & \frac{2F}{rC} + \frac{2}{r} & -\frac{2\ell(\ell+1)}{r^2}(A - \frac{F^2}{C} - N) & \frac{\ell(\ell+1)}{r} \\ -\frac{1}{r} & 0 & \frac{1}{r} & \frac{1}{L} \\ -\frac{2}{r^2}(A - \frac{F^2}{C} - N) & -\frac{F}{rC} & -\rho\omega^2 + \frac{1}{r^2}\ell(\ell+1)(A - \frac{F^2}{C}) - \frac{2N}{r^2} & -\frac{3}{r} \end{pmatrix} \qquad (17)$$

Numerical solution of the SODE

Here, I describe how the SODE is numerically solved by the GEMINI code.

- From four basis solutions below and above the source depth, there exist two regular ones below the source and two satisfying the homogeneous boundary conditions above the source. Let us denote them by $\mathbf{b}_1^{\pm}$ and $\mathbf{b}_2^{\pm}$.
- Then, the solution can be written (omitting the indices l and m):

$$\mathbf{y}^{\pm}(r) = c_1^{\pm}\,\mathbf{b}_1^{\pm}(r) + c_2^{\pm}\,\mathbf{b}_2^{\pm}(r)\,. \tag{18}$$

- Since the jump of he DSV at the source depth is known the coefficients $c_{1,2}^{\pm}$ can be determined:

$$\mathbf{y}^{+}(r_s) - \mathbf{y}^{-}(r_s) = \mathbf{s} \tag{19}$$

$$c_1^{+}\,\mathbf{b}_1^{+}(r_s) + c_2^{+}\,\mathbf{b}_2^{+}(r_s) - c_1^{-}\,\mathbf{b}_1^{-}(r_s) - c_2^{-}\,\mathbf{b}_2^{-}(r_s) = \mathbf{s}\,. \tag{20}$$

- However, a stable solution of this equation is not always possible. For this reason, the numerical solution has to be sought in quite a different way.

Numerical solution of the SODE

- *minor vectors* are constructed from the basis solutions below and above the source:
$$m^{\pm}_{ij}(r) = b^{\pm}_{1i}(r)b^{\pm}_{2j}(r) - b^{\pm}_{2i}(r)b^{\pm}_{1j}(r) \tag{21}$$
with
$$\mathbf{m}^{\pm} = (m^{\pm}_{12}, m^{\pm}_{13}, m^{\pm}_{14}, m^{\pm}_{23}, m^{\pm}_{24}, m^{\pm}_{34}) \tag{22}$$
and $m_1(r) \propto m_6(r)$.
- Differentiating the definition of the minors one obtains
$$\frac{dm_{jk}}{dr} = A_{jl}\, m_{lk} + A_{kl}\, m_{jl}\,. \tag{23}$$
With the ordering of the minors into a vector of five elements one can setup a system of five differential equations for the minors. They can therefore be integrated directly with the same accuracy as the basis solutions. The initial values result from the initial values of the basis solutions.

Numerical solution of the SODE

- The following must be given without derivation. See Friederich and Dalkolmo, GJI, 122, 537-550, 1995 for details. Construct from the minors the matrices

$$\tilde{\mathcal{M}}^{\pm} = \begin{pmatrix} 0 & m_1 & m_2 & m_3 \\ -m_1 & 0 & m_4 & m_5 \\ -m_2 & -m_4 & 0 & m_6 \\ -m_3 & -m_5 & -m_6 & 0 \end{pmatrix} \qquad (24)$$

- Solve in addition a system of four differential equations from the source up and down to the receivers:

$$\frac{d\mathbf{g}(r)}{dr} = -\mathcal{A}^T(r)\,\mathbf{g}(r)\,. \qquad (25)$$

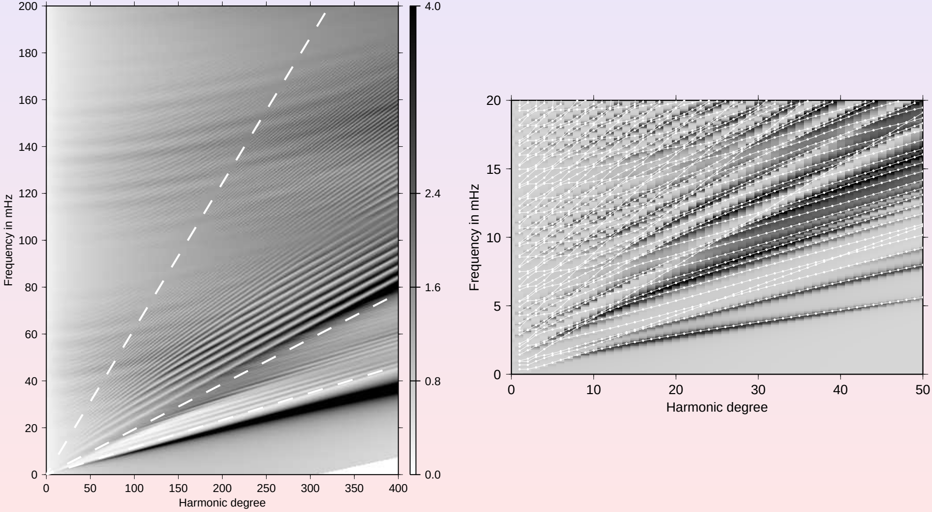
  Initial values can be obtained from the values of the minors and the jump of the DSV at the source.

- The desired solution is then given by

$$\mathbf{y}^{\pm}(r) = \pm\tilde{\mathcal{M}}^{\pm}(r)\,\mathbf{g}^{\pm}(r) \qquad (26)$$

  This way of computing the Green function for a specified point source is *stable*.

The figure shows the absolute value of the expansion coefficient $u_\ell^0(\omega, r_E)$ for an explosive point source at 33 km depth with maximum frequency of 200 mHz (left) and the same for a source at 600 km depth and maximum frequency 20 mHz (right).



Computation of displacement spectra and synthetic seismograms

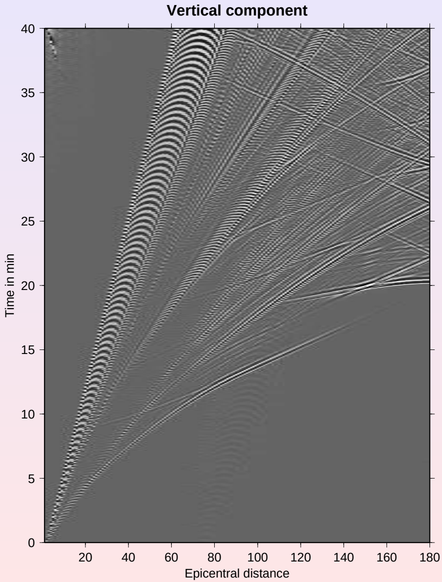
The above described major computation step of GEMINI calculates the spherical harmonics expansion coefficients of the displacement-stress potentials. Displacement spectra are obtained by the formula:

$$\mathbf{u}(r,\vartheta,\varphi,\omega) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \Big(u_\ell^m(r,\omega) Y_\ell^m(\vartheta,\varphi)\mathbf{e}_r + v_\ell^m(r,\omega)\nabla_H Y_\ell^m(\vartheta,\varphi) - w_\ell^m(r,\omega)(\mathbf{e}_r \times \nabla_H) Y_\ell^m(\vartheta,\varphi)\Big) \, . \tag{27}$$

In practice, two important simplifications apply.

- For a moment tensor point source at the pole of the spherical coordinate system, angular orders m range only from -2 to 2.
- There is an upper bound for the harmonic degree l at fixed frequency beyond which contributions to the synthetic seismogram fall off exponentially. The sum over harmonic degree l need not be performed to infinity but only to a well-defined maximum value (see file lw200mhz).

Synthetic seismograms are obtained by a Fourier transform into the time domain.


Vertical component
Time in min
40
35
30
25
20
15
10
5
0
20
40
60
80
100
120
140
160
180
Epicentral distance

Working with the GEMINI program

The computation of synthetic seismograms with GEMINI is done by running three different programs:

- **gemini**: Computation of spherical harmonics expansion coefficients for fixed source depth, time window length and maximum frequency
- **dispec**: Computation of displacement spectra for given moment tensor and station locations
- **totido**: Transform of displacement spectra into the time domain

Input for **gemini**

```
# Choose the kind of motion you like to calculate. Set the
# variable to 1 for P-SV-motion only; set it to 2 for SH-motion only;
# set it to 3 to calculate all P-SV- and SH-motion.
Which_Motion=3

# Verbose level for monitor-output. '0' yields monitoring every
# frequency; n every degree if mod(l,n)=0
Print_Level=0

# Length of seismogram in seconds
Seismo_Length=5400

# Damping time for complex frequency (->Laplace transform). A good
# choice is a fifth of the seismogram length.
Damping_Time=1080

# Minimum frequency in millihertz. You can set it to zero, 'Gemini' will
# adjust it to at least 1/Seismo_Length
Minimum_Frequency=0

# Maximum frequency in millihertz
Maximum_Frequency=100
```

Input for **gemini**

```
# Take into account dispersion (attenuation), which leads to
# frequency-dependent elastic moduli. Set 1 for 'yes', 0 for 'no'.
Dispersion_Switch=1

# Minimum degree of spherical harmonics. We recommend '0'.
Minimum_Degree=0

# Maximum degree of spherical harmonics. 'Gemini' will not compute
# beyond this limit.
Maximum_Degree=1000

# Step in the degree-domain. Normally this is '1', because of the
# 2*Pi-periodicity. Setting this greater than '1' speeds up the
# calculation and gives nice alias-effects because the Earth will
# become 2*Pi/lstep-periodic!!
Degree_Step=1

# Depth of the source in [km]. You can set this to Zero, if you like
Source_Depth=33.
```

Input for **gemini**

```
# Accuracy which rules the performance of the integration
# algorithm (Bulirsch-Stoer). Don't be too greedy, 1.e-4 should be
# sufficient
Accuracy=1.e-4

# File name of the earth model.
Earth_Model=stutprem

# File name of the window in the frequency-degree-domain, which contains
# tabulated maximum degrees for selected frequencies.
Omega_Ell_Window=lw200mhz

# Name of output file with expansion coefficients
Output_Filename=green/bas.f50.d33.3.out

# Confirm the input
Confirmation=1
#--------------------------------------------------------------------
```

Input for **dispec**

```
# Give here the file with the basis solutions calculated by GEMINI
BasisSolutions=green/bas.f100.d33.3.out

# The following variable must tell the source file containing ONE
# set of earthquake parameters in Harvard-CMT-format
SourceFile=sources/C062003G

# This defines the source mechanism: moment tensor ('m')
# or single force ('f'):
SourceMech=m

# Set maximum order 'm' in the sum over spherical harmonics here.
# For example, set 'm=0' if the source is explosion type
MaximumOrder=2

# This file must contain the window in the omega-l-domain also needed
# by GEMINI
OmEllWindow=lw200mhz
```

Input for **dispec**

```
# Give here the length of the taper which is applied on the l-range
# to avoid cut-off-effects. This is an empirical number, don't choose
# it too large, you may cut in the surface wave branch
EllTaper=40

# This is the name of the file with station names and parameters defined
# in IRIS-DMC format
StationFile=stations/GRSN_2003

# Specify here          1 -> Station by latitude and longitude in degrees
# the receiver type:    2 -> Station by its abbreviation (e.g. PFO)
#                       3 -> all stations in file
#                       4 -> section along great circle between two points on spher
ReceiverType=2
```

Input for **dispec**

```
# Now, coordiantes or station name or all stations or section?
# Example:     Type 1 :  Receiver='48.3 8.3'
#              Type 2 :  Receiver='BFO'
#              Type 3 :  Receiver='all'  (this is a dummy,
#                                         program reads format '(1x)'
#              Type 4 :  Receiver='-30. -71. 48.3 8.3 10'
#
Receiver='BFO'

# Horizontal displacement is calculated by default in source centered
# coordiantes. If you like to have station centered coordinates, i.e.
# North-South- and East-West-coordinates give here a number greater
# than zero.
NSEWCoordinates=1
#
#  The output file will have the name "spec3k" in the current directory.
#
#-----------------------------------------
```

Input for **totido**

```
# Response file containing real and imaginary part of instrument
# transfer function at the frequencies used in gemini and dispec.
# Each line of the file contains freqeuncy, real part, imaginary part.
ResponseFile=""

#  Time by which beginning of times series is delayed
TimeShift='0.0'

# To apply a Butterworth-filter (low and high pass)
# to the seismogram specify in the string
# the number n of filters and then n times  order and corner frequency.
# For example: One low pass with order 7 and corner frequency 0.01 Hz
#              Two high pass: order 3 and corner freq. 0.008 Hz AND
#                             order 2 and corner freq. 0.007 Hz
#             -------> type: 1  7  0.01
#                            2  3  0.008  2 0.007
LowpassNumber=1
LowpassOrder=5
LowpassCF=0.050
HighpassNumber=1
HighpassOrder=0
HighpassCF=0.0025
```

Input for **totido**

```
#  either a for Ascii output or s for SFF output
OutputFormat="s"

# The Fast Fourier Transform needs 2**n samples. If the spectra do not have
# such a length zeros will be appended to accomplish this. If you give here
# a number n greater than zero, the number of samples will be multiplicated
# with 2**n, resulting in interpolation/smoothing of the time series.
ZeroPadding=2

# With the following character you can choose the type of seismogram:
#     type d for displacement, v for velocity, a for acceleration or
#     g for accelerometer response.
SeismoType=v

# Specify here the length of the output time series in seconds
SecondsOut=5400
```

Harvard CMT file

```
C082103B 08/21/03 12:12:50.0 -44.97  166.91  33.97.07.0SOUTH ISLAND OF NEW ZEA
PDE BW:79189  45 MW:78195 135 DT=  10.0 0.1 -44.97 0.01  166.91 0.01  33.9 -1.0
 DUR 9.8 EX 26  5.09 0.01 -1.03 0.01 -4.05 0.01 -2.72 0.02 -4.25 0.02 -2.81 0.00
   1.74 72  83   0.02  2 178  -1.76 18 268   1.75   1 27   93 177 63   88
```

Station file

```
File last updated on
Tue Sep 2003
      03 s_station s_location s_lat        s_long       s_elev   s_site
      03 BFO       GR            48.3301       8.3296    600.0   Black Forest Obser
      03 BRG       GR            50.8732      13.9428    340.0   Berggiesshuebel
      03 BSEG      GR            53.9353      10.3169     40.0   Bad Segeberg
      03 BUG       GR            51.4406       7.2693    131.0   Bochum University
      03 CLL       GR            51.3077      13.0026    276.0   Collm
      03 CLZ       GR            51.8416      10.3724    700.0   Clausthal-Zellerfe
      03 FUR       GR            48.1650      11.2770    572.0   Fuerstenfeldbruck
      03 GRFO      GR            49.6909      11.2203    434.0   Graefenberg
      03 IBBN      GR            52.3070       7.7570      1.0   Ibbenbueren (elev?
      03 MOX       GR            50.6447      11.6156    501.0   Moxa
      03 RGN       GR            54.5460      13.3640      1.0   Ruegen (elev ?)
      03 RUE       GR            52.4800      13.7800      1.0   Ruedersdorf (elev?
      03 STU       GR            48.7708       9.1933    360.0   Stuttgart
      03 TNS       GR            50.2225       8.4473    846.0   Taunus Observatory
      03 WET       GR            49.1440      12.8782    652.0   Wettzell
```

Earth model file

```
#----------------------------------------------------------------------
#              Earth model STUTPREM
#       ( isotropic PREM without ocean )
#
# Quality factors are taken at 1 Hz !!
# The ocean is removed by enlarging the thickness its underlying solid
# layer by the ocean thickness.
#-------------------------------------------------------------
#
# +++++++ Do not insert any comments below this line !! +++++++++++++++
12                         Number of layers
 3 4 4 4 4 2 2 2 2 2 1 1   Polynomal coefficients of each layer
1.0                        Reference frequency of quality factors (Hertz)
0                          Tranversal isotropic?    1=y, else=n
---------------------------------------------------------------------------
Radius Density    V-Pv     V-Ph     V-Sv     V-Sh     Qmu     Qk     Eta
---------------------------------------------------------------------------
   0.0  13.0885  11.2622  11.2622   3.6678   3.6678   84.6   1327.7  1.0
         0.0      0.0      0.0      0.0      0.0                     0.0
        -8.8381  -6.3640  -6.3640  -4.4475  -4.4475                  0.0

1221.5  12.5815  11.0487  11.0487   0.0      0.0      -1.0  57823.0  1.0
        -1.2638  -4.0362  -4.0362   0.0      0.0                     0.0
        -3.6426   4.8023   4.8023   0.0      0.0                     0.0
        -5.5281 -13.5732 -13.5732   0.0      0.0                     0.0

3480.0   7.9565  15.3891  15.3891   6.9254   6.9254  312.0  57823.0  1.0
        -6.4761  -5.3181  -5.3181   1.4672   1.4672                  0.0
         5.5283   5.5242   5.5242  -2.0834  -2.0834                  0.0
        -3.0807  -2.5514  -2.5514   0.9783   0.9783                  0.0
```

Earth model file

```
3630.0   7.9565  24.9520  24.9520  11.1671  11.1671  312.0  57823.0  1.0
        -6.4761 -40.4673 -40.4673 -13.7818 -13.7818                  0.0
         5.5283  51.4832  51.4832  17.4575  17.4575                  0.0
        -3.0807 -26.6419 -26.6419  -9.2777  -9.2777                  0.0

5600.0   7.9565  29.2766  29.2766  22.3459  22.3459  312.0  57823.0  1.0
        -6.4761 -23.6027 -23.6027 -17.2473 -17.2473                  0.0
         5.5283   5.5242   5.5242  -2.0834  -2.0834                  0.0
        -3.0807  -2.5514  -2.5514   0.9783   0.9783                  0.0

5701.0   5.3197  19.0957  19.0957   9.9839   9.9839  143.0  57823.0  1.0
        -1.4836  -9.8672  -9.8672  -4.9324  -4.9324                  0.0

5771.0  11.2494  39.7027  39.7027  22.3512  22.3512  143.0  57823.0  1.0
        -8.0298 -32.6166 -32.6166 -18.5856 -18.5856                  0.0

5971.0   7.1089  20.3926  20.3926   8.9496   8.9496  143.0  57823.0  1.0
        -3.8045 -12.2569 -12.2569  -4.4597  -4.4597                  0.0

6151.0   2.6910   4.1875   4.1875   2.1519   2.1519   80.0  57823.0  1.0
         0.6924   3.9382   3.9382   2.3481   2.3481                  0.0

6291.0   2.6910   4.1875   4.1875   2.1519   2.1519  600.0  57823.0  1.0
         0.6924   3.9382   3.9382   2.3481   2.3481                  0.0

6346.6   2.9000   6.8000   6.8000   3.9000   3.9000  600.0  57823.0  1.0

6356.0   2.6000   5.8000   5.8000   3.2000   3.2000  600.0  57823.0  1.0

6371.0
------------------------------------------------------------
```

Maximum harmonic degree vs frequency (File lw200mhz)

```
9
0.   0. 150.
25.   0.   400.
50.   0.   770.
75.   0.   1050.
100.   0. 1400.
125.   0. 1750.
150.   0. 2100.
175.   0. 2400.
200.   0.   2750.
```

Descendants of GEMINI

- Green functions for shallow seismic media
- Marine version with water layer for shallow applications (e.g. Scholte modes)
- Full space version with radiation condition at the top of the model. Used to compute seam waves
- Eigendegrees and eigenfunctions at fixed frequency for toroidal and spheroidal motion for global and shallow seismic applications
- Version that uses reciprocity relation to compute Green functions for one receiver and many sources