Calculation of synthetic seismograms and phase velocities.

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1 Introduction

The objective of this practical is to show how to calculate normal mode eigrenfrequencies and eigenfunctions, synthetic seismograms, and at the end, to determine the phase velocity of surface waves between source and receiver.

When designing this practical, we started with routine codes, but we rapidly realized the necessity to simplify them in order for students to apply them during the short period of time of the course.

Here are the different steps of the whole procedure:

- Choice of the scientific target: investigation of source or structure.

In both cases, we need to calculate synthetic seismograms, so basic informations on the source and the receiver a re necessary.

-1- Choice of the stations (a specific station or the whole FDSN network)

receiver: location θ_R, ϕ_R, elevation h_R, starting time t_{D_0} and ending time t_{D_f}, transfer functions (Amplification, poles and zeros).

-2- Extraction of data (SEED file)

For a given earthquake, we need: location θ_S, ϕ_S, depth h_S, origin time t_0, half duration, CMT $M_{rr}, M_{\theta\theta}, M_{\phi\phi}, M_{r\theta}, M_{r\phi}, M_{\theta\phi}$

→ http://www.seismology.harvard.edu/CMTsearch.html

you can extract data from different data centers:

→ http:// www.iris.edu

→ http://geoscope.ipgp.jussieu.fr

→ http://www.orfeus-eu.org

Manuals and tutorials are available on these different web sites and it is very easy to make a request and get data.

- Transformation of seed file into your preferred format (AH, SAC, SEGY, Ascii, ...)

Use of the code rdseed (downloaded from any data center)

- Verification of the quality of data seismograms (by plotting them).

-3- Calculation of synthetic seismograms by normal mode summation (`nms`)

(See section 3 and DOC in the package for Normal Modes, modeslib0.2)

a) calculation of eigenmodes and eigenfrequencies:

codes `generate_1Dmodel, minosy`

b) calculation of synthetic seismograms

code `nms`

-4- Application of the transfer function to synthetic seismograms + filtering of data and equalization of starting time for synthetic and real seismograms. (See section 4)

codes `station_response, filter_qin`

-5- Calculation of phase velocity (See section 4) code `vphase_qin`

Visualization of phase and phase velocity dispersion curve.

At this stage, you can start to make science, by comparing different phase velocity curves according to the tectonic environment.

Due to the lack of time, only steps 3 to 5 are presented here. For step 3, download the package **modeslib0.2** with `doc` included . For steps 4-5, download the package **Vphase** with `doc` included. A set of real data seismograms is provided, because the connection with data centers can be slow and sometimes problematic.

2 Brief theoretical background

For a given spherically symmetric Earth model, eigenfunctions for a given eigenfrequency ${}_n\omega_\ell$ are written as

$$ {}_n\mathbf{u}_\ell^m(\mathbf{r}) = \left[{}_nU_\ell(r)\mathbf{e}_r + {}_nV_\ell(r)\boldsymbol{\nabla}_1 - {}_nW_\ell(r)(\mathbf{e}_r \times \boldsymbol{\nabla}_1)\right] Y_\ell^m(\theta,\phi) $$

where n is the radial order, ℓ the angular order and m the azimuthal order. Note that, because of the spherical symmetry, eigenfunctions and eigenfrequencies do not depend on m. $\boldsymbol{\nabla}_1 f(\mathbf{r}) = \partial_\theta f(\mathbf{r})\,\mathbf{e}_\theta + (\sin\theta)^{-1}\partial_\phi\, f(\mathbf{r})\,\mathbf{e}_\phi$. Y_ℓ^m is the spherical harmonic. The mode normalization is such that

$$ \int_0^{r_{\boldsymbol{\Omega}}} \rho(r)\left({}_nU_\ell^2(r) + \ell(\ell+1){}_nV_\ell^2(r)\right) r^2\,\mathrm{d}r = \int_0^{r_{\boldsymbol{\Omega}}} \rho(r)\,\ell(\ell+1){}_nW_\ell^2(r)\,r^2\mathrm{d}r = 1\,, $$

where $r_{\boldsymbol{\Omega}}$ is the Earth radius. The code `minosy` computes ${}_n\omega_\ell$, ${}_nU_\ell(r)$, ${}_nV_\ell(r)$ and ${}_nW_\ell(r)$ for a given Earth model.

Synthetic seismogram $\mathbf{u}(\mathbf{r}_r,t)$ at receiver $\mathbf{r}_R$ is obtained by summation of all modes ${}_nu_\ell^m exp(i_n\omega_\ell t)$. Usually the triplet n, ℓ, m is noted k:

$$ \mathbf{u}(\mathbf{r},t) = \sum_k a_k(\mathbf{r}_S)\,{}_n\mathbf{u}_\ell^m(\mathbf{r}_R)e^{i_n\omega_\ell t} $$

Where $a_k(\mathbf{r}_S)$ is the excitation coefficient due to a seismic source at $(\mathbf{r}_S)$.

When isolating a single kind of wave, such as surface wave, a data seismogram for a given component and an epicentral distance Δ, can be expressed as:

$$ u_i(\Delta,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\Delta,\omega)exp[-i\omega(t - \frac{\Delta}{c(\omega)}) + i\phi(\omega)]d\omega $$

$\phi(\omega)$ is the sum of several terms, source phase ϕ_S, instrumental phase ϕ_I and initial phaseϕ_0, so the total data phase obtained by Fourier transform is:

$$ \Phi_d = \frac{\Delta}{c(\omega)} + \phi_S + \phi_I + \phi_0 $$

By the way , if these different source and instrumental phases can be correctly assessed, we do not need a synthetic seismogram, and phase velocity $c(\omega)$ can be easily derived from the previous expression. However, the calculation of a synthetic seismogram largely simplifies the calculations. The synthetic seismogram can be expressed on the same way, except that the phase velocity $c(\omega)$ is now the reference phase velocity $c_0(\omega)$ (obtained from `minosy` for PREM). We assume that the instrumental response and the source CMT are accurately determined, so that ϕ_S and ϕ_I are well taken into into account in the synthetic seismogram. Consequently, the difference between the real phase Φ_d and the synthetic phase Φ_s is simply:

$$\Phi_d - \Phi_s = \frac{\Delta}{c(\omega)} - \frac{\Delta}{c_0(\omega)}$$

We easily derive $c(\omega)$ from this equation.

3 Normal modes and Synthetic Seismograms: Package of Yann Capdeville

modeslib0.2

Codes useful for the calculation of phase velocity.

- `generate_prem`: little program to generate different kind of Earth models to be used by `minosy`.
- `minosy`: this program computes normal modes (eigenfunction and eigenfrequencies) for a given 1D reference Earth model
- `nms`: this program compute synthetic seismograms for a given list of receivers and sources using the normal mode catalog produced by `minosy`.

  For applying this code to the Chile earthquake, the input files `nms.dat, receivers.dat, sources.dat` were modified. See in the directory TEST-chile.

  When retrieving the CMT from Harvard, be aware that the different components of the moment tensor M_{ij} are usually given in $dyne.cm$, whereas we are using the I.S. standards, $N.m$, so there is a difference of $10-7$ between them.
- `get_fctp`: little program to extract a given normal mode from the output of `minosy`. It can be useful to visualize the eigenfunctions.

4 Calculation of Phase velocity along a path

At this stage, we know how to calculate synthetic seismograms, with **nms**. We assume that data from your favorite data center, in your favorite format (AH, SAC, SEGY, SAC ascii, ...) were recovered.

We will illustrate all calculations leading to phase velocity measurement, for the record of chile earthquake in CAN. Only the vertical component is considered, but this code is also valid for Love waves, provided that you applied a rotation of the horizontal components into longitudinal and transverse components.

For simplicity, we will use AH format, but will convert data files in AH into AH-ascii files. In the directory AH (AH-MAC for MacIntosh), there is a simple code ah2asc.c easy to compile, making this conversion.

• Real data for Chile earthquake in AH format and AH-ascii format are stored in the directory `DATA-chile` for 2 stations CAN (Canberra in Australia) and UNM (Mexico).

If you want to generate an ascii file with 2 variables (time, amplitude) easily plotted with `xmgr`, `xmgrace` or any $x - y$ plotting routine, a script $c\_ahasc2nh$ is shown in the package directory `DATA-chile`

Example of header in the data file UNM__VHZ19952110511.AH_asc

————————————————————— station information

code: CAN
channel: VLPZ
type: modet
latitude: -35.320999
longitude: 148.998993
elevation: 650.000000
gain: 6856710166473256992768.000000
normaliztion: 1.000000
calibration information
pole.re pole.im zero.re zero.im
1.000000e+01 0.000000e+00 3.000000e+00 0.000000e+00
-1.233950e-02 1.234320e-02 0.000000e+00 0.000000e+00
-1.233950e-02 -1.234320e-02 0.000000e+00 0.000000e+00
-3.917570e+01 4.912340e+01 0.000000e+00 0.000000e+00
-3.917570e+01 -4.912340e+01 0.000000e+00 0.000000e+00
-3.034990e+01 7.868120e+00 0.000000e+00 0.000000e+00
-3.034990e+01 -7.868120e+00 0.000000e+00 0.000000e+00
-2.220730e+01 2.208850e+01 0.000000e+00 0.000000e+00
-2.220730e+01 -2.208850e+01 0.000000e+00 0.000000e+00
-8.135970e+00 3.016620e+01 0.000000e+00 0.000000e+00
-8.135970e+00 -3.016620e+01 0.000000e+00 0.000000e+00
0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
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event information
latitude: -24.160000
longitude: -70.690002
depth: 20.000000
origin_time: 1995 7 30 5 11 21.000000
comment: null
record information
type: 1
ndata: 448
delta: 1.000000e+01
max-amplitude: 0.000000e+00
start_time: 1995 7 30 5 11 4.169300
abscissa_min: 0.000000e+00
comment: null
log: modhead;
extras:
0: 8.000000e+26
1: -1.070000e+28
2: 9.999999e+27
3: 4.000000e+26
4: -6.000000e+26
5: -1.320000e+28
6: 3.690000e+01
7: 2.770000e+01
8: 0.000000e+00
9: 0.000000e+00
10: 0.000000e+00
11: 0.000000e+00
12: 0.000000e+00
13: 0.000000e+00
14: 0.000000e+00
15: 0.000000e+00
16: -9.000000e+01
17: 0.000000e+00
18: 0.000000e+00
19: 0.000000e+00

20: 0.000000e+00
data:

In the header, for saving time during practical, we already included the earthquake information (latitude, longitude, depth, origin time). The header also provide information on the transfer function (amplification= gain,number of poles and zeros, poles and zeros), the number of points (ndata), the sampling rate (delta, here 10s), and the starting time of the record.

• Calculation of synthetic seismograms.

code `nms`

In the Directory TEST-chile, you will find the input files nms.dat, receivers.dat, sources.dat useful for calculating the synthetic seismograms in CAN and UNM stations.

4.1 Preparation of real and synthetic seismograms

Application of the transfer function to synthetic seismograms + filtering of data and equalization of starting time for synthetic and real seismograms.

Before determining the phase velocity along the path between the epicenter and the station, we have to be sure that real and synthetic seismograms have the same starting time, have the same frequency content.

For the synthetic seismogram, we have to apply the transfer function $A.\frac{\Pi(i\omega-Z)}{\Pi(i\omega-P)}$ by using the code $station\_response.f90$ where A is the gain and Z, P the complex zeroes and poles.

You can compile the code. Similarly to the normal mode summation code, you can configure your makefile according to your compiler. For example for an IBM compiler (on Mac)

```
cd SPICE_station_response
csh configure xlf90
make
cd ../TEST-chile
~/ETUSPICE05/SPICE_station_response/station_response
10.  900                        sampling rate, number of points
CAN__VHZ.asc                       data seismogram
UZ__CAN                       synthetic seismogram
```

Example of session
*****calculate the station reponse***********
../SPICE_station_response/station_response
Entrer dt et NBT:
10 900
input the name of read data in format .asc
CAN__VHZ.asc
input the name of synthetic data to be applied by station response
UZ__CAN

the result is saved in sCAN__UZ used for phase velocity calculation
****

You also have to filter the real seismogram with the same filter as the one used in calculating the synthetic seismogram (frequencies f_1, f_2, f_3, f_4 in nms). We use the code `filter_qin` in the directory `SPICE_filter_qin`

```
../SPICE_filter_qin/filter_qin
CAN__Z.asc           data seismogram file
fCAN_Z.asc          filtered data seismogram (such as the synthetic seismogram)
```

Example of session

****filter the real data****************
../SPICE_filter_qin/filter_qin
input the name of read data to be filtered in format .asc
CAN__VHZ.asc
input the name of soure wavelet (source.gnu)
source.gnu
the result is save in:
fCAN__VHZ.asc for display using xmgr
fdCAN__VHZ.asc for input of phase velocity computation

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4.2 Phase velocity calculation

Everything is ready now. You can compile the code `vphase_qin.f` in directory VPHASE, Execute it:

```
cd ../VPHASE_qin
./vphase_qin
fdCAN__VHZ.asc            name of data file
sCAN__UZ              name of synthetic seismogram
2.7 5.1 1             minimum, maximum group velocity, train number
0.0033 0.0167          minimal and maximal frequency
vph_vgrSprem_n0       File with reference phase and group velocities
end
```

Example of session

*****calculate the phase velocity*******
./ vphase_qin
name of filtered data file in format .
fdCAN __VHZ.asc
name of synthetic seismogram file with station response
sCAN__UZ
minimum and maximum group velocity and number wavetrain
2.7 5.1 1
minimal and maximal frequency in Hertz
0.00333 0.0166

name of phase and group velocity file corresponding to synthetic
vph_vgrSprem_n0

Output files:

```
deltavitphas          phase difference between real and synthetic
phase                         unrolled phase
sismo1, sismo2                both seismograms
```

The principle of the code is simple: it calculates the Fourier transform of both real and synthetic seismograms. It is assumed that the phase difference at very long periods ($T > 200s.$, $\omega < 5mHz$) is smaller than π. Since the phase given for Fourier transform is between $-\pi, +\pi$ (or $0 - 2\pi$, it is necessary to unroll the phase (see the ouput file `phase`). The phase velocity is therefore determined first at very long period (low frequency) and then by decreasing period (increasing frequency).

Afterwards you can plot the phase velocity with xmgr (xmgrace for Mac) or any $x - y$ plotting routine.

The procedure described here can be easily generalized and automated. On a routine basis, several thousands of seismograms can be processed.